

More than half of the pre-service secondary mathematics teachers' interpretations of De Villiers' (1990) roles of proof are often aligned on communication and explanation compared to other roles of proof.

Background

- Over the past decade, increased attention has been given to teaching and learning proof in school mathematics, as its central roles in mathematics include verification, explanation, discovery, systemization, and communication (e.g., De Villiers, 1990).
- In addition, proving and proof are fundamental to doing mathematics, communicating mathematical ideas, and developing mathematical knowledge (Stylianides, 2007).
- Given that pre-service secondary mathematics teachers (PSMTs) play a critical role in shaping their future students' experiences with proof and proving, we seek to better understand how PSMTs' interpretations of the verification, explanation, discovery, systemization, and communication roles of proof align or misalign with that of the discipline (as represented in De Villiers' (1990) seminal work).

Study

- This study involved 29 PSMTs in the transition-to-proof course titled "Discrete Mathematics" during two semesters ($n = 11$ in Fall 2020 and $n = 18$ in Fall 2021) at a Midwestern University in the United States.
- While gaining an appreciation of these five roles that proof can play in mathematics was one of the course goals, the instructor who is the first author did not explicitly name these five roles as a topic of study with students in class until the study week before final exams.
- The primary source of data for this study was PSMTs' written responses to the take-home portion of the final exam, where they were asked to read De Villiers's (1990) article and describe the five given roles of proof using their own words.
- The first author and one research assistant read all PSMTs' responses individually and coded each participant's self-description as **not in alignment (N)**, **in partial alignment (P)**, or **in alignment (Y)** with De Villiers's (1990) explanation.

Learner vs. Mathematical Perspectives: Pre-Service Secondary Mathematics Teachers' Interpretations of Roles of Proof

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Tables, References, & Contact Information



De Villiers' Roles of Proof & Sample Responses

Verification Proof serves as a means to obtain conviction and establish the truth of a mathematical statement.	Y	<i>This role deals with the justification behind mathematical proofs. It reasons with the truth of the statements and is used as conviction to proof.</i>
	P	<i>To be able to verify a proof can be a difficult task. "Absolute certainty also does not exist in real mathematical research..." (18). This is not the only way verification could be used in fact most are taught to check the answer at the end of a problem. This form of verification can be extremely effective in easing the doubts of an incorrect conclusion.</i>
	N	<i>Verification can be used to show that something is incorrect or find mistakes in the problem.</i>
Explanation Proof serves as a means to promote understanding and illumination of why underlying mathematical concepts are true.	Y	<i>We must also say why something is true. Many believe the function of explanation is far more important than verification because it gives us an insight into the proof and in return makes us more intelligent.</i>
	P	<i>Explanations in proofs play the part of describing why a statement might be true. They are the validation of a proof's truthfulness. The explanation of a proof also helps strengthen the verification of the proof itself.</i>
	N	<i>Explanation doesn't necessarily aim to prove something true. It is more about giving reason or explanation as to why it is true.</i>
Systemization Proof serves as a means to structure unrelated definitions and previously-proven results to gain a global perspective of mathematical concepts.	Y	<i>Another function of a proof is as a means of systematization. Systematization helps show the logical relationships between statements. These statements are deductive systems such as theorems and definitions. Systematization makes it easier for all of the audience to understand how the proof is built.</i>
	P	<i>The systemization of a proof is the organizational process. This role helps simplify a proof and combine similar ideas into a more uniform one. Systemization can help strengthen a proof as well.</i>
	N	<i>Systematization is the organization of results. It can help identify errors within the work, connect similar ideas to promote understanding and simplifications, it provides a "global perspective" which I interpreted as a greater understanding within multiple views, and it can provide applications for many things.</i>
Discovery Proof as a means to expose unexpected results beyond the given mathematical scope or context	Y	<i>A proof that is written for the sake of discovery follows logical deductions to come to a conclusion that was not necessarily already known or thought to be true. This type of proof can also be used as a way to generalize something—if someone discovers the reasoning behind why a certain hypothesis is true while they are working through examples, that knowledge can then be used to construct a more general proof than before. Proofs of discovery do not always have to be done this way, however. They can also be done just by analyzing the properties of objects that are given.</i>
	P	<i>According to de Villiers, there is always a discovery of mathematical results when writing a proof. There are also many instances that new results are found for a proof. Many times, a mathematician has not gone and verified their theorems making their results different.</i>
	N	<i>The fourth role of a proof is discovery, in which some concepts and theories are actually discovered on accident through proof. This is not just for new theories to the world, but also new theories to the student. Students can stumble upon untaught theories through class work in proofs.</i>
Communication Proof serves as a means to expose unexpected results beyond the given mathematical scope or context.	Y	<i>The final role is communication, where the real value of proofs is seen through debates about the topic. Through debates, we learn new perspectives on topics that we would not have known otherwise and can implement these perspectives into our own work.</i>
	P	<i>The article by de Villiers points out that a proof is a way of communicating among people who are working on mathematics. The argument made in the proof is addressed to a human audience. We are trying to convince the audience our results are correct meaning in our explanations we need to use communication so they can understand it.</i>
	N	<i>Communication is key to any problem. This means explaining the reasoning good enough to communicate to the reader that it is a proof. It goes hand in hand with explanation. Being able to transcribe the meaning behind the proof to get the reasoning across.</i>