

How Undergraduates Make Decisions When Co-Validating Mathematical Proofs

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Mathematical proof is central to the work of mathematicians (e.g., Weber, 2008), and is a center point of the undergraduate curriculum for mathematics majors (Stylianides, G., 2007). As undergraduates learn to prove, they need to learn various proving processes, not the least of which are construction and validation (e.g., Ko, & Rose, 2022; de Villiers, 1990; Segal, 2000). Construction and validation of proofs go hand-in-hand, yet they involve unique skills and processes (Kirsten & Greefrath, 2023; Selden & Selden, 2003)). Therefore, it is crucial to understand how undergraduates make decisions about proof construction and proof validation. In recent work, we explored how students in an inquiry-based Introduction-to-Proof (ITP) course make decisions when it comes to *co-constructing* proofs in small groups (Bleiler-Baxter et al., 2023). In this poster, we seek to extend the findings from that research by exploring situations where students are *co-validating* proofs in small groups. We ask: To what aspects of proof do students in an undergraduate inquiry-based ITP course attend when co-validating mathematical proofs within small groups?

This study involves students enrolled in an ITP course at a public university in the southeastern United States in the fall of 2023 taught by the second author. Students in this class were asked, at three separate time points during the semester (early, middle, late), to review sample arguments and to determine if those arguments should “count” as mathematical proof. They were given independent think-time to make their initial validations of the given arguments, and then they were tasked with coming to a consensus in their pre-assigned small-groups, as to whether each argument was or was not a proof and why. We explore the transcripts of audio data of small group discussions across these various points in the semester to investigate how students make decisions about validating proofs.

To analyze the small-group discussions, we use a three-part theoretical coding scheme informed by A. Stylianides’ (2007) definition of proof: set of accepted statements, modes of argumentation, and modes of argument representation. We use this definition as a coding scheme to be able to discern what students prioritize when making decisions on what counts as proof. For example, students may focus on the way the definition of divisibility is used in an argument (i.e., set of accepted statements), the way proof by contradiction is employed (i.e., mode of argumentation), or the way a table is used to organize an argument (i.e., mode of argument representation).

Submitting as a poster will allow us to include various student excerpts from transcripts highlighting each of the codes we have used as well as how their frequency changes over the course of the semester. We hope to gain from attendees at the conference what questions they may have about our data and other data analysis methodologies to consider as we begin finalizing our data analysis in the spring.

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