Predictive Effects of Students' Grades in Prior Mathematics Courses

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DFW rates in first-year mathematics courses are a major concern for undergraduate institutions. In this study, we explored statistical trends in grades for students who repeat mathematics courses or transition to subsequent courses. We analyzed trends among final grades in courses from Intermediate Algebra through Calculus II over the past five years at a major midwestern metropolitan university. Our results show that students are statistically more likely to pass a class on their first attempt, and that DFW rates among students who repeat a course hover around 50% for subsequent repetitions of the same course. We also found that final grades are a strong predictor of grades in subsequent courses, with students who earn an A in the previous course being 3 and 4 times more likely to pass the subsequent course than students who earn a B or C, respectively. We conclude by discussing the implications of our results.

Keywords: DFW rates, First-year mathematics courses, Course trajectories

DFW rates in first-year mathematics courses are a concern for universities, four-year colleges, and community colleges alike. Many factors contribute to student success in a course, including past mathematical experiences and prior mathematical knowledge. In this preliminary study, we explore trends in student grades as students transition from prerequisite mathematics courses into subsequent courses along a standard course trajectory. In particular, we focus on a sequence of first-year courses ranging from Intermediate Algebra to Calculus II and analyze the relationship between students' grades in prior courses and DFW rates in subsequent courses.

Background

Enrollment in first-year mathematics courses is increasing. The College Board of Mathematical Sciences reported that estimated enrollments increased by 21% for precollege level mathematics courses, by 16% for introductory level courses, and by 8% for calculus level courses between fall 2010 and fall 2015 at four-year institutions (Blair, Kirkman, & Maxwell, 2018, p. 1). Over the same time period, Bressoud et al. (2015) found that DFW rates in Calculus 1 courses ranged from 22% to 38% among two-year and four-year institutions in the USA. Notably, no systematic data collection has yet taken place to provide statistics on DFW rates in precollege level mathematics courses or introductory level mathematics courses at the college or university level in the United States. Yet, administrators at many institutions cite their college algebra courses as significant challenges to student retention (e.g., Callahan & Belcheir, 2017; Gordon, 2008).

Success in a student's first college mathematics course is especially important for retention and persistence. In one study, Callahan and Belcheir (2015) found that only 55% of students earning a D, F, or W in their first mathematics course were still enrolled in college one year later (p. 168). To ensure that students succeed in passing their first college mathematics course, appropriate placement is critical (Bressoud et al., 2015; Harrell & Lazari, 2020). In their report

on characteristics of successful calculus programs, Bressoud et al. (2015) highlighted the importance of effective placement procedures on student success and, in particular, the attention paid to students near the cutoff. It was found to be essential that students who were identified as "ready for calculus" but more at-risk than their peers, as well as students who did not place into calculus, were given special attention and placed into programs that suited their individual needs.

Our work adds to the body of research on factors affecting success in first-year mathematics courses. Rather than focusing on a single course in isolation, we instead examine a sequence of courses that STEM-intending students typically take as they progress through lower-level undergraduate mathematics courses. While we acknowledge that student "success" in mathematics encompasses many variables beyond just a student's final grade in the course, our preliminary work focuses on two primary research questions:

1. How does a student's grade in a prior course affect their likelihood of earning a passing grade in a subsequent course?
2. For students who do not pass a course, what is their likelihood of passing the course in a repeated attempt?

Methods

For our study, we gathered institutional data for all undergraduate students enrolled in any mathematics course at a major midwestern metropolitan university from Fall 2018 to Spring 2023, along with the enrollment records of any previous mathematics courses students had taken for a period of at least two years prior. Individual students were referenced within our dataset by anonymized identification numbers (pseudo-ID) along with the term, course, section, and registrar grade for all enrollments.

The first step of our analysis was to take this list of student enrollments and use it to build a data structure which captured the variety of paths students may take when progressing through courses. To begin, we loaded the enrollment data into a relational database and cleaned entries to ensure that the three-tuple of pseudo-ID, course name, and term of enrollment was unique. This cleaning step made it possible to accurately account for students repeating courses by computing the instance number of each student's enrollment in any given course. That is, for a student with a given pseudo-id enrolling into course X in term Y, we counted the number of times that this same student enrolled in course X in a term chronologically before term Y, then added 1. This yielded a natural way to find students taking a given course for the first, second, or *n*th time by searching for instance numbers of 1, 2, or *n*, respectively within those course enrollments and forms the basis for the analysis necessary to answer RQ2.

We then used these data to create a course transition table to track the history of what mathematics courses a student had taken prior to any given enrollment by identifying pairs of prior and subsequent courses. This transition table forms the basis of the analysis of student grades in subsequent courses necessary to answer RQ1.

The DFW rates we reference in our Results section are obtained by dividing the number of grades of D, F, W, and NC (No Credit) out of the total number of grades consisting of A, B, C, D, F, W, NC (No Credit) and CR (Credit). This excludes grades such as Incomplete or Audit which have no clear interpretation. Our analysis in this paper focuses on a sequence of courses typically taken by STEM-intending students within their first year or two of entering college: Intermediate Algebra, College Algebra, Pre-Calculus Algebra, Trigonometry, Precalculus, Calculus I, and Calculus II.

Results

Effect of Grades in Prior Courses

To analyze pass rates in subsequent courses, we first searched within the transition table for pairs of prior/subsequent courses associated with the sequence of STEM-intending courses described above. The data was then filtered to look only for cases where the student had passed (C- or above) the previous course within 24 months and was taking the subsequent course for the first time. Prior courses from the Fall 2018 semester onward were considered. The resulting DFW rates in subsequent courses are reported in the table below grouped by the grade the student achieved in the prior course. We group together +/- grades (i.e., earning an A means that a student earned an A+, A, or A- in the prior course). As an example, of the 2144 students who took Intermediate Algebra followed by College Algebra, students who earned an A in Intermediate Algebra went on to earn a grade of DFW or NC in College Algebra at a rate of 13.29%. Row and column totals refer to specific course parings and grades, respectively. That is, 31.11% of the 2144 students who took Intermediate Algebra followed by College Algebra received a grade of DFW or NC in College Algebra, whereas 12.64% represents the aggregate DFW rate of enrollments with a grade of A in the prior course.

Table 1. DFW Rates in Subsequent Courses

		Grade in Prior Course			
Sequence	*N*	A	B	C	Total
Int. Alg. → College Alg.	2144	13.29%	43.72%	66.47%	31.11%
College Alg. → Precalc. Alg.	679	3.30%	18.57%	37.14%	13.25%
Precalc. Alg. → Trig.	416	14.40%	36.67%	66.67%	25.72%
Trig. → Calculus I	339	22.06%	50.74%	61.19%	41.30%
Precalc. → Calculus I	435	25.37%	58.22%	72.62%	45.52%
Calculus I → Calculus II	936	7.99%	32.36%	43.18%	25.96%
Totals	4949	12.64%	39.61%	56.29%	29.20%

Looking over longer timescales, we can also see how DFW rates in courses compare with student grades in a prerequisite course several courses prior in the usual sequence. Table 2 provides a few examples of these course sequences with intermediate courses omitted. It is evident that far fewer students progress through the entire sequence of these courses – especially in the groups with lower grades in prerequisite courses. As such, data entries for groups with fewer than 15 students are left blank.

Table 2. DFW Rates for Sequential Courses with Further Separation.

		Grade in Prior Course			
Sequence	*N*	A	B	C	Total
Int. Alg. → Precalc. Alg.	285	5.53%	30.65%	45.83%	14.39%
Precalc. Alg. → Calculus I	261	34.24%	54.84%	66.67%	41.00%
Trig. → Calculus II	118	27.42%	36.59%	40.00%	32.20%
Precalc. → Calculus II	159	22.34%	48.89%	55.00%	33.96%
College Alg. → Calculus II	83	29.85%	46.67%		32.53%
Totals	906	21.78%	43.11%	50.67%	29.47%

Analysis of DFW Rates by Course Repetition

Students taking courses from Fall 2018 through the Spring 2023 semester were grouped by their instance taking (or re-taking) given courses. The values reported in the Table 3 are presented in the form "DFW Rate (*N*)," where *N* is the total number of students taking the course for the *n*th time. As with the prior analysis, entries with fewer than 15 students are left blank to avoid reporting on exceptionally small populations.

Table 3. DFW Rates for Students Retaking Courses

Course	1st Instance	2nd Instance	3rd Instance	4th Instance
Intermediate Algebra	32.06% (3656)	52.40% (521)	46.67% (60)	
College Algebra	30.32% (5301)	53.54% (693)	50.00% (148)	23.33% (30)
Precalculus Algebra	16.81% (2046)	42.64% (197)	52.27% (44)	64.71% (17)
Trigonometry	27.58% (689)	50.00% (106)	68.57% (35)	66.67% (18)
Precalculus	19.58% (715)	41.67% (72)		
Calculus I	34.10% (2217)	46.51% (458)	53.77% (106)	62.86% (35)
Total	28.78% (14624)	50.02% (2047)	52.22% (406)	52.83% (106)

As is the case for the preceding analysis of moving from one course to the next, one limitation of the method employed here it that it is not possible to determine if a student took any other prerequisite courses between instances of retaking the course in question.

Conclusions and Future Work

Our findings have implications for instructors at all levels as well as for course conveners and departmental administration. Our results seem to confirm that grades in previous courses are a strong predictor of whether a student will pass a subsequent course. To an instructor of a subsequent course, our results suggest that particular attention should be paid to students who passed their previous course with a final letter grade of B or C, as these students are at roughly three- and four-times higher risk, respectively, of not passing their present course as compared with students who earned an A in their previous course on the basis of Table 1. To the instructor of a student's prior course, our results underscore the importance of supporting and encouraging students to gain a deep understanding of what they are learning – and doing so the first time to avoid failing and repeating a course. It also brings into focus the reality that students leaving a given course with a grade in the C range have less than a 50/50 chance of passing their next math class on the first attempt. Preliminary results from future work confirm the fact that student grades decrease on average when students move between any of the course pairs detailed in our results section and emphasize the need to focus especially on STEM-intending students who begin the sequence in courses below Calculus. Instructors should consider how to incorporate specific interventions to better support these students to help them persist in their STEM majors.

Our preliminary results also provide a method whereby particularly troublesome pairs of course transitions could be systematically identified and investigated further. In future work, we hope to strengthen this case for efficacy of grades in predicting outcomes with a more rigorous statistical analysis and expound further on some of the other aspects of student demographics which may factor into these results. It is further worth mentioning that some of the most "difficult" pairs of course transitions (viewed either in terms of DFW rate or an average decrease in grade) may not be responsible for the greatest numbers of DFW grades issued within a given department. Future work will aim to further quantify the numbers of students transitioning

between various course pairs with the aim of suggesting where limited resources to help students could be most effectively distributed.

The results for students who end up having to repeat a class are stark where, in aggregate, students are 1.75 times less likely to pass a course when repeating for the second, third, or fourth time as compared to taking it for the first time. Some courses (e.g. Calculus I) exhibit a nearly linear increase in DFW rates with each subsequent retake ($R^2 = 0.9883$) whereby a student is nearly 10% less likely to pass the course on each subsequent retake.

This somewhat surprising statistic raises questions about why students who repeat a course may be less likely to pass it. On one hand, one might expect that retaking a course would confer certain advantages to students such as familiarity with the content of assessments and insight into what aspects of the material are most relevant, in addition to having recently seen the material. On the other hand, if the student was not successful on a first attempt, then it may be unlikely that they will succeed on a second attempt if the instruction is not significantly different. Furthermore, students who repeat a course may feel more confident than their peers due to having seen material before, and as a result, they may be less likely to regularly attend class or study as much as their classmates who are taking the course for the first time.

Moreover, viewing DFW rates grouped by the instance in which a student is taking the course can provide a much more detailed picture than simply considering the aggregate DFW rate alone. That is, it allows the assessment of how well a given course is performing for students who are seeing it for the first time as opposed to those that are repeating it.

Another area of future work is investigating how students are distributed across sections of multi-section courses. Coordinators of larger courses should be aware that the mathematical background of students may not be comparable between different sections, and that the DFW rates alone may not give a complete picture of the success or failure of an instructor's or team's teaching; one must consider factors such as total enrollment across all sections, class sizes within individual sections, the mathematical backgrounds of the students enrolled, and other factors. This is especially important at any institution where faculty may have DFW rates included as part of their evaluation process.

Some technical limitations of this analysis were expounded upon in the methodology and results section: most notably the inability to exclude intervening classes in many instances. While there is reason to believe that such events may be comparatively rare, we plan to undertake a more thorough study of this in future work and switch to a more robust method of recording student progress through courses that could account for these factors if warranted.

Questions for the Audience

- Are there researchers or practitioners at other institutions who are performing similar analyses? Have you identified similar groups of students who might need additional supports and what interventions have you studied or tried?
- What are reasonable metrics to "assess" a course by? Is it useful to partition DFW rates by metrics like "instance re-taking a course" to better reflect course outcomes? What are the most representative DFW rates to consider?
- What do our results say about prerequisite knowledge and coursework? How might we use our results to ensure students are on a successful trajectory of courses?
- To help refine the range of factors which might affect students' grades, we are interested in looking at other data such as ACT scores, high school courses, placement exams, etc. What other factors would you recommend investigating more thoroughly?

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