An Exploration of Precalculus Students' Reasoning about Exponential Functions

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In recent years both the teaching and learning of introductory mathematics courses, such as Precalculus, have made immense strides to improve students' understanding of foundational skills such as covariational reasoning (e.g., Thompson & Carlson, 2017). Despite the emphasis being placed on teaching from a covariational lens, exponential functions continue to be taught from a correspondence lens (Ellis et al., 2016). In other words, the focus is on solving an algebraic expression that defines a static relationship between two variables instead of the underlying covariational relationship between two quantities. This work is a subset of a larger project to support students' covariational reasoning and specifically focuses on exponential functions. We are guided by the following questions: What type of covariational reasoning is present in students' responses on exponential growth/decay questions? How do the students' responses align (or not) with the mathematical goals of each lab?

Methods

This study is situated at a large R1 university in the American Southeast. In Summer 2022, the project began by developing five precalculus lab units based on the findings of small-scale teaching studies and other relevant literature on student thinking (e.g., Cobb et al., 2003; Ellis et al., 2017; Oehrtman et al., 2008). Each lab consists of a brief pre-lab assignment, a lab activity, and an exit ticket. The purpose of this presentation is to detail the ongoing development of two labs which were designed to support students' conceptualization of exponential functions from a covariational reasoning lens. Students' anonymized written work from the two focal labs were used as the primary source of data analyzed for this study. There were approximately 480 students across twenty classes from Fall 2022 and Spring 2023 included in the data set. Data analysis began by identifying which questions from each lab most closely aligned with the learning goals of the corresponding lab. An a priori codebook was created based on the Exponential Growth Learning Trajectory (EGLT) (Ellis et al., 2016). Student responses were coded for correctness and salient reasoning in several iterative rounds (Miles et al., 2020).

Findings and Discussion

One learning goal which we focused on was based on the ninth covariational reasoning ability from Ellis et al.'s (2016) EGLT which states that "any constant change in x results in a proportional multiplicative constant change in y" (p. 160). The main finding was that the majority of students were able to correctly answer questions despite their reasoning not always accurately reflecting the exponential relationships. For example, students agreed that halving the x value does not halve the y value; however, their explanations did not reference proportional multiplicative changes in the y value. Instead, students tended to rely on linear reasoning (i.e., additive changes in both x and y values. The misalignment between students' reasoning and their ability to get a "correct" answer implies that there is a disconnect between how the labs are currently designed and their intended learning goals.

References

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Background

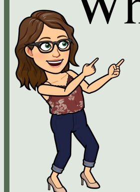
Alice Shrinking is the first of two 50-minute lessons that were designed using the work of **Ellis et al.'s (2016) Exponential Growth Learning trajectory (EGLT)** as a guide. The two labs are designed to support a key shift in student reasoning from **coordinating constant ratios for x-value changes larger than 1 to constant rations for fractional changes** (Ellis et al., 2016).

This poster highlights the findings from an analysis of Desmos responses from **120 students across 5 classes** during a lesson on exponential functions from **Fall 2022** titled **Alice Shrinking**. Student responses were assigned codes deductively based on the salient reasoning in the students' response using the categorizations from **Ellis et al.'s (2016) EGLT** as an a priori codebook.

Research Question

What types of covariational reasoning are salient in undergraduate precalculus students' responses to an exponential decay question with successive changes of an x-value that are greater than 1?

Scan to read our proposal!



When presented with successive additive and multiplicative changes in an exponential decay context, most students reasoned using repeated multiplication or exponentiation of the scale factor.

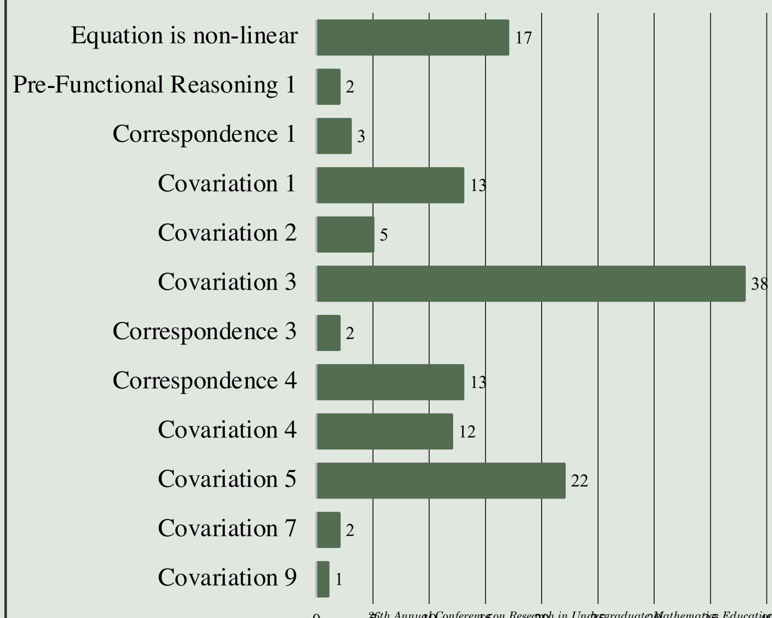
Drink Me: After each sip of potion, the drinker shall shrink to 2/3 their current size. (1 sip = 0.5 oz.)

Your roommate claims that if Alice drinks 1 oz., she would shrink twice as much as if she'd only drank 0.5 oz.

Do you agree or disagree?

Scan for current lesson on Desmos

Frequency of Codes for Student Reasoning



Examples of Most Common Student Reasoning
(Explicit Coordination for Multiple-Unit Changes)

Covariation 3: Repeated Multiplication defined as "coordinating the change in y-values for multiple–unit changes in x-values, but [the student's] mental imagery is grounded in the actions of repeated multiplication" (Ellis et al., 2016, p. 164).	**Covariation 5: Exponentiation Imagery** defined as the "coordination of the ratio of y-values for any $\Delta x > 1$; students no longer rely on repeated multiplication imagery" (Ellis et al., 2016, p. 169).
"After drinking 1 ounce of potion Alice would shrink 0.44 times her current height because .5 oz = 2/3 h (height). meaning 1 oz = 2/3 * 2/3 h, equaling 4/9."	"After drinking 1 ounce of potion Alice would shrink to 0.44 times her current height because she would be taking 2 sips; Therefore, she would shrink by (2/3)^2."