Beyond Correctness: Characterizing Authority to Assess in an Abstract Algebra Classroom

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In what ways do students bear mathematical authority in advanced mathematics classrooms? What authority relations arise within inquiry-oriented classrooms? Answering these questions may give insight into the inequities that exist within undergraduate mathematics education. We share findings from our initial investigation of 7 lessons from one inquiry-oriented abstract algebra (IOAA) classroom to determine who gets to have authority to do certain activities. Taking an action-based approach to parsing authority relations, we attended to who bears authority for the activities of authorship, communication of ideas, and assessment of ideas. Our preliminary findings propose five categories of activities for assessment as well as several observations about the nature of authority relations within an IOAA classroom.

Keywords: Abstract algebra, authority, assessment

Increased attention on student-centered instruction eschewing traditional lecture formats in undergraduate mathematics demands better understanding of the complex dynamics by which students participate in mathematics discourse, especially when they are expected to engage in activities like conjecturing, proving and justifying mathematical ideas. Investigating the authority relations created in such courses is one avenue for interpreting both productive and unproductive mathematics discourse. For example, it has been argued that inequitable learning and participation outcomes (Johnson et al., 2020) in inquiry-oriented classrooms arise from students' differing experiences in leveraging their authority in small-group and whole-class discussions, especially in terms of whose ideas are taken up, and how (Hicks, Tucci, Koehne, Melhuish & Bishop, 2021). In this paper, we extend an existing framework for interpreting authority relations by further parsing another relevant activity found within classroom authority dynamics: assessment of mathematical ideas.

Authority is a multi-dimensional construct, and as such, there are multiple approaches to parsing authority structures. Weber's (1947) power-oriented perspectives on authority have been used extensively, often to make observations about the nature of inequitable classroom interactions (e.g., Langer-Osuna, 2017; Esmonde & Langer-Osuna, 2013), or discursive positioning moves (e.g., Wagner & Herbel-Eisenmann, 2009). Despite the prominence of Weber's power-oriented authority in sociological research, concerns have been raised over the negative attention attributed to authority as a necessary evil to be resisted rather than as a positive force for social change. In the context of educational philosophy, Benne (1970) espoused a perspective that characterized authority in education with greater optimism, one which treated *bearers* of authority as a source of assistance to *receivers* of authority who willfully seek help from an authority and may be provided opportunities to bear authority for themselves. This type of authority is known as *anthropogogical* authority. The reform-oriented nature of Benne's contributions established a natural dichotomy between the Weberian and anthropogogical perspectives wherein the first is treated as typical of traditional classrooms and persists in most situations, while the second is thought to be an ideal form of authority to be strived for (Amit & Fried, 2005).

In contrast to the mathematics education literature as a whole, authority relations in undergraduate mathematics education have been understudied. This research has largely focused on *sources* of authority. Gerson and Bateman (2010) conducted a grounded theory investigation of an inquiry-based calculus classroom and developed a framework for identifying the many types of authority that can exist in the university setting, including hierarchal, mathematical, and pedagogical authorities. Other research has focused on authority for specific activities relevant to undergraduate advanced mathematics classrooms. For example, Bleiler-Baxter, Kirby and Reed (2023) investigated ways in which authority manifested in situations related to proof and proving activity, finding that students attended to course syllabi, their peers, and logical structure as sources of authority. In this paper, we focus specifically on investigating *mathematical* authority in the context of an inquiry-oriented abstract algebra class. We pose the following questions:

RQ1. What are the authoritative activities associated with assessment in an inquiry-oriented abstract algebra classroom?

RQ2. What are the mathematical authority relations in this classroom?

Theoretical Framing

We define authority as a dynamic and negotiated relationship between people (or groups, or organizations) in which one party agrees to lead and another party agrees to follow in a given situation. Thus, authority is transient and can shift between parties from one moment to the next rather than remain as a permanent fixture within a single person.

In contrast to Weber's power-oriented and Benne's reform-oriented definitions of authority, we adopt an *action-oriented* approach to parsing mathematical authority (Bishop, Koehne & Hicks, 2022). In particular, we contend that within any given *field* (i.e., the specific context in which bearers and receivers negotiate relevant authority relations), a member of the field becomes a bearer of authority by actively participating in one or more activities. These activities are situated discursively within a community of practice, either by recognizing that the community at large views the activity as pertinent to the field (e.g., a medical doctor bears authority by providing a patient with a diagnosis), or through localized negotiation that might be exclusive to a subset of the community (e.g., students might become bearers of mathematical authority through public *validation* of proof in one class, but never have such an opportunity to publicly validate proof in another.)

Table 1. The AAA Authority Framework for Authority Relations

Authorship	Refers to the significant contribution to the mathematical ideas under consideration.	Who is the primary source, or author, of the mathematical ideas?
Animation	Refers to who is publicly communicating or extending mathematical ideas.	Who is communicating mathematical ideas in the classroom?
Assessment	Refers to judgements/evaluations made about an idea.	Who is assessing mathematical ideas in the classroom and who is being assessed?

While several researchers have investigated authority relations between individuals, we examine the relations between students (as a collective party) and their teacher in this study. Thus, when one student partakes in an activity and bears authority, they bear authority for the students as a whole. This allows for a broadscale analysis of authority relations across several days of class, a scale of analysis that is absent from current literature. To characterize mathematical authority, we attend to the activities of *authoring* ideas, *animating* (or communicating) ideas, and *assessing* ideas. See Table 1 for a description of these broad activities.

Methods

The preliminary data consist of seven 1.25-hour long recordings of whole class activity from an introductory abstract algebra class at a large, land-grant university in the eastern United States, taught using the Inquiry-Oriented Abstract Algebra (IOAA) curriculum materials (Larsen et al., 2013). The class was composed of 15-20 students and covered the group isomorphism units from the curriculum materials in the 7 classes that were analyzed. Analysis began with segmenting classroom video, which indicated a stretch of classroom activity associated with a focal mathematical idea or task, or consistent participation structure, ranging from 30 seconds to 5 minutes in length. Four types of segments were identified within each video: whole class (WC) activity, small group activity (SG), independent activity (i.e., students worked by themselves rather than in groups), and segments that were not considered mathematical in nature (i.e., task set-up or returning graded work). Each segment that was labelled as WC was assigned codes using a version of the AAA framework modified to capture assessment authority.

Because the AAA framework was originally developed to capture the authority of whole-class interactions, the authority of students (versus the authority of the teacher) was documented collectively, rather than attending to the authority of individual students. In other words, if a student authors an idea in a segment, then students are credited collectively for that authorship. Codes were assigned based on which group (teacher, students, or both) held authority for authorship, animate-speak, animate-represent, and several types of assessment. In our initial analysis, we viewed assessment as any explicit statement that indicates judgement or evaluation of a mathematical idea. That is, utterances that could be implicit assessments were not coded.

The first two classroom videos were coded as a team to establish an initial coding scheme. The third lesson was coded independently by the first two authors, and then disagreements were resolved as a group. After that, the first author coded the fourth and sixth lessons, and the second coded the fifth and seventh lessons, with each author bringing difficult segments to the group to get consensus. The codebook was then refined over the course of analysis resulting in five types of mathematical assessment. These types are exemplified below in our findings, along with some general trends in authority relations captured by preliminary counts of the current coding.

Findings

In order to parse the authoritative activity of assessing, we attended to five different categories related to how students or teachers assess mathematical ideas within public classroom discourse: (1) correctness focused assessments, (2) assessments focused on comparison of ideas, (3) assessments focused on a particular quality of an idea, (4) justification of assessments, and finally (5) reflective or metacognitive assessments. Below we share examples of a subset of these activities identified within two interactions.

We view correctness focused assessments as largely related to the traditional discursive structure of initiate-respond-evaluate (IRE) (Mehan, 1979), although correctness focused assessments can appear more productively as well. Consider the following excerpt from one lesson in which the instructor sets up the class for a new phase of investigation into subgroups, beginning first by establishing collective understanding of some possibly unfamiliar notation:

Teacher:	We're going to go into our next reinvention phase now. And to get there, we need to know what 5Z is. Do you guys know what 5Z is?
Student A:	Z like integers?
Teacher:	Yes, Z as in integers (*points and nods at student while speaking.)*

The above interaction showcases an example of a correctness-focused assessment where the teacher was a bearer of authority for assessing a mathematical contribution. To capture the wide range of productive assessment activity within an inquiry-oriented course, attention to other forms of assessment was required. Consider the following excerpt:

Teacher:	*(Revoices student's idea at the board proving an implication about abelian groups.)*
Student B:	Is it not simpler than that?
Teacher:	Can I make it even simpler than that?
Student B:	*(Describes a proposed alternative proof.)*

In this interaction, Student B claims to have a simpler alternative proof than the previous one, and thus bears authority for comparing two mathematical ideas. In the discussion that followed, the teacher commented, "I think they are equally slick," giving her authority for comparison in addition to the students. Because each of students and the teacher was a bearer of authority for comparing ideas, we assigned a code of Both to this segment. Such interactions are evidence that students are engaging in mathematical practices valued by the field as a whole. Here, the student is attending not only to the correctness of a given proof, but also to the comparison of multiple ideas in an effort to find a more efficient approach. This is just one of five types of assessment indicative of mathematical community practices that arose, the remainder of which are summarized in Table 2 below.

Table 2. Five Authoritative Activities for Assessment

Correctness Focused	Assessments that indicate correctness/incorrectness of a mathematical contribution.
Comparison Focused	Refers to the comparison of two or more mathematical contributions.
Quality Focused	Assessments that are focused on a particular quality of a contribution, such as efficiency or clarity.
Justification Focused	Providing evidence or rationale to support a previously made assessment. Often paired with another assessment type.
Reflective/Metacognitive Focused	Assessments that are reflective of one's own thinking, and typically appear as an assessment of one's own contributions.

General Trends and Observations on Authority Relations

Authority was shared by teacher and students across the activities of authoring, animating, and assessing. Out of 150 total segments, students had or shared authority to author ideas in 101 segments. This preliminary finding count may be an indicator of the increased attention to student thinking within the inquiry-oriented setting. Inquiry-oriented classrooms often leverage tasks designed for students to inquire into mathematical ideas with their peers in small group settings, but allowing students to work in small groups generates a certain level of risk. Pimm (2019) identified these risks as *gambits.* One trend we have observed within this IOAA classroom is that by allowing students to engage in group work, students may bear either more authority for themselves or share more authority with the teacher. Small group work was followed by student authorship in nearly all transitional segments across all seven lessons.

Furthermore, authority for speaking was nearly always shared between teacher and students, with speech coded as both in 131 segments. Taken together with the high level of student and shared authorship, we view this as an indicator that the class did not engage in teacher-driven lecture very often. However, students were also provided little opportunity to completely control the spoken public discourse without teacher intervention. In contrast to speaking, students rarely wrote on the board during whole class discussion, with examples of students bearing authority for written representations occurring in only 17 segments. Thus, the teacher had significantly more authority to animate ideas through representation.

Students bore authority to make assessments in all 7 classes, most often focused on correctness, justification, or metacognitive aspects. Strikingly, students engaged in justification assessment nearly as often as in correctness assessment, and they made assessments that reflected on their own thinking far more than the teacher (see Table 3). Furthermore, students also always had authority to assess when assessment was the focus of a segment. In terms of the teacher's authority for assessment, the teacher bore authority for corrective assessment most often, while authority for justification was the second highest. Note that due to varying segment lengths, these percentages are proportions of segments containing assessment activity, of which there were 143. Because segments could receive multiple codes, the totals do not add to 100%.

Table 3. Number of Segments Containing Assessment by Type

Type	Student	Teacher	Both
Correctness	11 (~8%)	69 (~48%)	7 (~5%)
Comparison	6 (~4%)	11 (~8%)	3 (~2%)
Quality	2 (~1%)	19 (~13%)	2 (~1%)
Justification	10 (~7%)	21 (~15%)	9 (~6%)
Reflective	8 (~5%)	2 (~1%)	0 (0%)

Discussion

As this work is preliminary, more analysis is needed to establish claims about trends throughout the course of the semester. In particular, as the content within abstract algebra increases in difficulty (e.g., quotient groups are commonly identified as a difficult topic), authority relations may shift more toward one of student, shared, or teacher. In particular, we wonder whether teachers are more likely to assess as content difficulty increases. In addition to the success of groupwork gambits at promoting student authority in whole class discussion, the very act of engaging in groupwork yields students some private authority to do mathematics, free of assessment from the teacher. While prior research indicates inequities in small group interactions due to social factors, such as influence (Engle et al. 2014), examining assessments on a small group level within an inquiry-oriented course could explore inequities related to mathematical authority, such as parsing whose ideas are assessed and how, as well as who bears authority to make such assessments. Future work may consider the following questions: Are marginalized students' ideas assessed only for correctness, as opposed to more mathematically demanding assessments for justification or quality? Do students of higher social status assess more often? More generally, what other activities can lead to students bearing authority?

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