Exploring Mathematical Intuition and its Role in Physics Problem-Solving

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A large body of work in both mathematics and physics education research has explored mathematical reasoning in physics contexts (Redish & Kuo, 2015; Van den Eynde et al., 2020; Serbin & Wawro, 2022; Zimmerman et al. 2023b). Studies show that expert reasoning in physics problem-solving is not purely physical or mathematical, but a blended way of thinking (Redish & Kuo, 2015; Schermerhorn & Thompson, 2023). In this study, we build on the recent work of Zimmerman et al. who observed that one way physics experts reason while modeling is by "mathematical riffing"–manipulating familiar physical models, having a feel for when one might be useful, and quickly rejecting those that are not (Zimmerman et al., 2023a). Just as jazz might seem random and unpredictable to the untrained ear, mathematical riffing is hard to pinpoint and practice for physicists-in-training, even at the graduate level. There is no direct instruction to develop this kind of mathematical intuition on which experts rely to propel their thinking.

While the physics community values mathematical intuition as a mark of expertise, little work has been done to understand what it looks like, or what role it plays in generating quantitative physics models. To better understand mathematical intuition and its role in physics problem-solving, we conducted a pilot study in which we interviewed three physics graduate students after completion of their first year PhD coursework. We asked the interviewees to think aloud while solving unfamiliar graduate-level physics problems that relied on familiar mathematics. The data were coded using a grounded theory approach (Glaser & Strauss, 1967). This poster describes the study and its preliminary findings.

We identified three ways that mathematical ideas drive physics reasoning that, collectively, we call *mathematical impetus*. We see mathematical impetus as part of the set of behaviors associated with riffing. Each feature relies on *interactions with symbols* and is associated with having an *expectation of an outcome* without feeling the need to prove it mathematically. We also developed a description of a few distinct ways that graduate students act on mathematical impetus in tandem with other problem-solving skills. These findings help characterize problem-solving approaches that are typical in graduate classrooms, and can help inform instruction, making expert mathematical intuition more transparent to the learner.

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Exploring "Mathematical Intuition" and its Role in Physics Problem-Solving

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Background

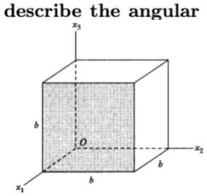
- **This study:** We explore the *"mathematical intuition"* experts rely on to *propel their thinking,* and the role it plays in *generating quantitative physics models.*

- **Analysis framework:** Zimmerman et al. physics expert way of reasoning–*"mathematical riffing"*
  - manipulate familiar physical models, recognize when one might be useful, and quickly reject those that are not [1].
  - analogous to jazz: seems random and unpredictable to physicists-in-training

Methodology

- Interviewed 3 first year graduate students at University of Washington in July 2023.
- Think-aloud interviews that included solving the following classical mechanics problem [2]:

> You wish to study this object's angular momentum as it rotates with the origin fixed, given its moment of inertia tensor, the equation of motion for rigid body rotations, and the initial conditions
>
> - $\theta_1(0) = \theta_2(0) = \theta_3(0) = 0$
> - $\omega_1(0) = \omega_2(0) = 0\ ,\quad \omega_3(0) = \omega_0,\ \omega_0 > 0$
>
> **What would you do next to describe the angular velocity of this cube?**
>
> $$I_{ij} = mb^2 \begin{pmatrix} \frac{2}{3} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{2}{3} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{2}{3} \end{pmatrix}$$
>
> $$0 = \frac{d\boldsymbol{L}}{dt} = \frac{dL_i}{dt}\boldsymbol{e}_i + L_i\boldsymbol{\omega} \times \boldsymbol{e}_i$$



- Unfamiliar physics topic that relied on familiar mathematical tools.
- Data coded using a grounded theory; results are emergent from the data.

Some Emergent Ways of Symbolic Reasoning

Defining quantities	providing a working definition or understanding of what a symbol represents
Analogous symbols	using familiar quantities to understand new or unfamiliar ones
Symbols-guided math doing	doing algebraic manipulations, guided by symbols' meanings, interpretations, implications
Symbols-motivated process choosing	choosing a process to move forward in problem solving, cued by symbols' meanings, interpretations, implications
Extracting physical meaning	coming up with a physical interpretation for a mathematical result
Expected outcome	having an expectation for the mathematical/physics implications based on symbols

Mathematical Impetus

Examples of "mathematical impetus" from the transcripts

Symbols-motivated process choosing	Extracting physical meaning	Expected outcome
"But then usually, when I encounter [...] a differential equation like this, [...], where I'm like, fairly certain that there's no closed form solution--- um, we should just linearize? I think?"	"And then we know that DL dt is going to be zero. So this is going to be, um, constant in time."	"yes, that like decouples the set of equations very nicely. Like with this, you get, instead of the nasty i j omega j within the, um, in the differenti---in the differential equations I got, I'll just get a bunch of like [...]decoupled equations."

Preliminary Results

- 3 emergent ways of symbolic reasoning drive physics reasoning; we call this driver ***mathematical impetus***:
  - associated with mathematical riffing.
  - ***interactions with symbols*** with an ***expectation of an outcome*** without feeling the need to prove it mathematically.

- We are embarking on a case study analysis of the individual graduate students to characterize their ***unique ways*** of blending mathematical impetus with other problem-solving skills.

Further Work

- "Mathematical intuition" as a way of knowing is poorly understood and extremely important in physics. There is *no direct instruction to develop it in our classrooms.*
- Toward a characterization of mathematical impetus we will:
  1. continue the analysis of the transcripts to uncover the diverse ways that graduate students use mathematical impetus when solving new physics.
  2. use the codebook developed in this project to study how experts engage with mathematical impetus in generating new physics.

For more materials:

http://tinyurl.com/y8vkvm9x

[1] Zimmerman, C. & Olsho, A. & Loverude, M. & Brahmia, S. W. (2023). Empirical evidence of the inseparability of mathematics and physics in expert reasoning about novel graphing tasks. arXiv:2308.01465v1.
[2] problem modified from chapter 11 of Thornton, S. & Marion, J. (1988). Classical Dynamics of Particles and Systems (3rd edition). Saunders College Publishing.