Student Perceptions of Individual and Group Creativity in Proving

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Student described experiences in creativity can provide insight into instruction to foster creativity. In this report, I describe the emergent themes among student narratives describing when and why they felt creative during a collaborative proving activity. Continued work to refine these themes and their prevalence in student reflections can provide implications for how to best structure collaborative work to foster creativity in proof-based courses.

Keywords: Creativity, proof, collaboration

Both collaboration and creativity are central to the work of mathematicians (Karakok et al., 2015; Sriraman, 2004). Professional mathematicians have indicated collaboration is an important feature of their work (Sriraman, 2004), and the Conference Board of Mathematical Sciences (2016) has called for university mathematics classrooms to incorporate more active, collaborative learning. Therefore, there is a substantial need for K-16 education to focus on developing creative mathematicians and problem solvers with strong communication and collaboration abilities. In mathematics, students follow a journey in which there is an eventual transition from computational to proof-based mathematics (Civian & Schley, 1996). During this transition to proof, point students are often expected to become producers of mathematical ideas on their own for perhaps the first time (Boyle et al., 2015). For this reason, it is critical to study mathematical creativity and collaboration within the context of undergraduate mathematics courses and proof in order to cultivate the expertise needed by future mathematicians.

The purpose of this study is to investigate the creativity at work within an individual during creative collaborative proving. Although this study is being conducted in the larger context of a dissertation study, for this report I focus on the following research question: What actions or moments of the collaborative proving process do individuals report fostered their creativity? In this report, I first provide background literature on undergraduate mathematical creativity. I then describe the definitions and theoretical framings adopted in this study, outline the context and methods of the study, describe some preliminary results, and discuss the potential implications and future directions of this research.

Definitions and Theoretical Framing

Mathematical Creativity

Creativity has been notoriously difficult to define. Mann (2005) claimed there are over 100 existing definitions of creativity in the mathematics education literature, and this number has only grown with the volume of research on creativity published in the last 17 years (Savic et al., 2022). In this study, I adopt a perspective on mathematical creativity that describes creativity as *domain-specific*, meaning specific to the context of mathematics and mathematical proof, *a process*, meaning I will consider the motivation, perception, learning, thinking, and communicating involved in creativity, and *relative*, meaning considered within the context of the knowledge, abilities, and experiences of an individual. Put succinctly, ***mathematical creativity*** is defined to be the processes of creating, constructing, or implementing mathematical ideas, strategies, or processes, which are perceived as non-routine by the individual.

In this study I use the terms ***collaborative creativity in proving****,* or creative collaborative proving*,* to describe the process of approaching a shared goal in proving with significant contributions from two or more people, in which the proving task is either (a) assumed to be non-routine for all group members based on context or (b) established to be non-routine for all group members by asking them for their perspective.

Proof

In this study, situated within a university Introduction-to-Proof course, I adopt the definition of ***proof*** given by A. J. Stylianides, (2007):

> Proof is a *mathematical argument*, a connected sequence of assertions for or against a mathematical claim with the following characteristics:
> 1. It uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification.
> 2. It employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; and
> 3. It is communicated with forms of expression (*modes of argument representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community. (p. 291, emphasis in original)

This definition allows for flexibility in the growth of a classroom community and the tools (both accepted statements and modes of argumentation) valid within this community.

Collaboration

One way to highlight the inner workings of students' approaches to proof initiation, proof construction, and proof validation is to engage students in collaborative proving. Collaboration is a tool for revealing the deeper processes behind proving and reflects the work of mathematicians (Grossman, 2002). To define collaboration beyond social interaction (Sriraman, 2004), I emphasize two elements of collaboration as a process: (1) contribution from two or more parties and (2) a common goal. This definition of collaboration can be applied to a classroom setting to describe collaborative learning, wherein typically student groups of two or more gain mutual understanding or create a product (Smith & MacGregor, 1992)**.**

Residue

Taking mathematical creativity as a relative construct requires asking students what they recall as fostering creativity for them, and these recollections will describe "what actually comes to the fore of [students'] attention" (Marton et al., 2004) regarding their creativity during collaborative proving as well as other activities in the Introduction-to-Proof course. Hiebert et al. (1996) described the importance of remembered experiences and influences as residue. Residue provides a way for thinking about what students take with them from classroom experiences. Therefore, residue for students with respect to creativity can give insight into the experiences, activities, and interactions that were meaningful for the student and inform how instructors can best shape classroom environments and activities to foster collaborative creativity in proving.

Background

It is undisputed that mathematical creativity has grown immensely as a research domain in the last ten years (Savic et al., 2022; Sriraman, 2017), yet research on mathematical creativity has been slow to extend to the tertiary setting (Savic et al., 2022). In this report, I aim to describe the actions or moments students experienced during in-class collaborative proving that fostered creativity for either themselves or their group. It is generally accepted that mathematical creativity can be developed and enhanced in students (Sriraman & Haavold, 2017; Zazkis & Holton, 2009), and mathematicians believe that mathematical creativity can and should be fostered in undergraduate mathematics courses (Karakok et al., 2015). Despite this consensus, there is little empirical research on what teaching strategies develop mathematical creativity among undergraduate students (Savic et al., 2022). No extant research has explicitly considered the impact collaborative work or collaborative proving may have on creativity. Investigating student perceptions of how collaborating with their peers on proving tasks fosters (or does not foster) creativity for them can inform how to best facilitate collaborative activities to promote creativity in proving.

In extant literature, the teacher actions most supported as effectively promoting mathematical creativity are choosing appropriate tasks (Satyam et al., 2022; Savic, El Turkey, et al., 2017a; Tang et al., 2022), allowing time for incubation (e.g., Savic, El Turkey, et al., 2017b), demonstrating different ways to solve problems and illustrating the mathematical process for students (Satyam et al., 2022; Tang et al., 2022), making the classroom a safe place to take risks (Satyam et al., 2022; Savic, El Turkey, et al., 2017b; Tang et al., 2022), attending to the emotions of students (Satyam et al., 2022; Tang et al., 2022), and providing space for discussion and disagreement (Satyam et al., 2022; Savic, El Turkey, et al., 2017a, 2017b; Tang et al., 2022).

Both Cilli-Turner et al. (2023) and Satyam et al. (2022) investigated how to foster creativity in undergraduate settings by considering student perceptions of creativity. Both of these studies, conducted in the context of Calculus I, reported six themes of student views of creativity: Actions and Attitudes, Application, Different Ways, Originality, Against Authority, and Understanding. Although these themes can help describe the ways in which students in an Introduction-to-Proof setting may view creativity, I seek to understand the specific ways in which students perceive themselves (or their group) as creative in a collaborative proving setting.

Methods

Context and Participants

Data were collected from an undergraduate Introduction-to-Proof course at a large public southeastern university in the United States. This course was facilitated in a collaborative, inquiry-based learning environment in which small groups of 3-4 students worked together to prove instructor-provided mathematical conjectures. The study discussed in this report is a small portion of a dissertation study on collaboration and creativity in mathematical proving. This report focuses on undergraduate students' impressions of their individual and group creativity after working in groups in class to collaboratively prove the statement: The product of consecutive twin primes is one less than a perfect square. In this report, I draw upon available written narrative data from eight participants.

Data Collection and Analysis

Students were instructed to remain seated at their group tables for five minutes of individual think time and then move to a whiteboard space to share their ideas with their peers and construct a proof. Students were each given a different color whiteboard marker to track their ideas on the collective dry erase board space. After the class session, students completed a retrospective writing assignment in which they were tasked with reflecting upon their experience working collaboratively in class and describing the times in which they felt (or did not feel) creative during the activity. These assignments were due at 11:59pm the day of the collaborative proving activity to mitigate delayed recall (Gass & Macky, 2000), in which a person creates a new mental process instead of recalling their original ideas (Lyle, 2003). Students were asked to (I) provide a sequential narrative of their group proving experience and (II) answer the question "When during the collaboration did you feel like you or your group were creative? If you did not feel creative, explain why not. Include as many details as you can."

To determine the actions or moments students perceived as influential on their individual and group creativity during the in-class activity, I narrowed my data source to only include written responses to task (II) of the writing prompt. First, I conducted an inductive, in-vivo coding strategy (Saldaña, 2016). The in-vivo codes were grouped into themes, and I conducted a second round of coding using this list of themes to identify the most frequently cited moments and incidents that initiated creativity for students. All sentences of responses to (II) containing reasons for feeling (or not feeling) creative were given at least one thematic code.

Preliminary Results

The inductive analysis of participant retrospective writing assignment submissions revealed 11 themes in their reflection on their individual creativity: Brainstorming, mistakes, noticing patterns/making conjectures, task novelty, suspense/pressure, teamwork, trying different ways/seeing things in different ways, writing an equation/formula, connecting mathematics and words, uncreative, and other. The most common themes were teamwork/collaboration, noticing patterns/making conjectures, uncreative, and writing an equation. Table 1 provides the frequency of sentences, proportion of participant responses, and a participant quotation illustrating the code theme for the themes reported by at least three participants, excluding the "other" category.

Table 1. Themes to Answer RQ1: What actions or moments of the collaborative proving process do individuals report stimulated creativity for them?

Theme	Frequency	Proportion of Students	Participant Quotation
Teamwork	13	5/8	"I believe that my group was even more creative than we could have been individually."
Noticing Patterns/ Conjectures	10	6/8	"We also felt creative in noticing the pattern with every pair of numbers we used to verify the proof."
Uncreative	10	5/8	"I did not feel very creative at the beginning of the group exercise, as I was sure that every group was thinking about writing out examples for the first step."
Writing an Equation/ Formula	9	6/8	"In our group work I felt that we were able to be the most creative when we were figuring out our initial equation."

Trying Different Ways/ Seeing Things in Different Ways	7	5/8	"Thinking outside of the box and using a variety of methods to solve a proof is what creativity in mathematics is all about."
Brainstorming	4	3/8	"We all began throwing out ideas on how we could solve this problem."

Discussion and Conclusion

In this report, I have characterized the types of actions or moments students reported as causing them to feel creative during an in-class collaborative proving exercise. A first observation is that students rarely emphasized themselves as feeling individually creative in their narratives (e.g., "I felt creative when…"), but rather emphasized the creative actions of the group as a whole (e.g., "my group was creative when…"). This distinction, in part, may be attributed to feeling as though the group's capacity for acting creatively was greater than capacity of the sum of the constituent members, as one participant indicated, "I believe that my group was even more creative than we could have been individually." Further, students may feel uncomfortable taking individual ownership in their narrative writing reflecting upon a collaborative task.

Regarding the themes present in student narratives, many align with the teacher actions previously indicated to foster creativity in different undergraduate mathematical contexts (e.g., Tang et al., 2022). For example, five students recognized themselves or their groups as creative when they tried to approach the proof using a different strategy or look at the problem in a new way, which corresponds to Tang et al.'s "encourage mathematical behavior" (p. 541) teaching action to foster creativity. Moreover, the most frequently mentioned moment, engaging in teamwork (13 mentions), corresponds to "allow for discussions" (Tang et al., 2022, p. 541), yet participants in this study attributed their feeling creative to positive teamwork, rather than simply the opportunity to engage with their peers. One student said,

> I think a vital part of collaboration is the group's overall willingness to actively collaborate. In this way uncooperative group members stifle the creativity of the group. I really appreciated that no one in my group ignored the ideas of others.

These preliminary findings indicate students may recognize opportunities to be creative in mathematical proving by collaborating with their peers in specific ways. Data used in this report is limited to eight student narratives following a single in-class collaborative proving activity; however, data collection and analysis will continue throughout the Fall 2023 semester to provide a more holistic image of the ways in which students perceive themselves and their teams to be creative in proving. The results of this study may be able to both verify the results from previous research on student perceptions of creativity in Calculus I (Satyam et al., 2022; Cilli-Turner et al., 2023) as applicable also to an Introduction-to-Proof setting, but also provide a guide for facilitating creativity-fostering collaborative work in undergraduate proof-based mathematics courses.

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