Opportunities for Reasoning-and-proving with Trigonometric Identities in Precalculus Textbooks

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Outside of geometry, United States high school students have few opportunities to engage with proof (Thompson et al., 2012; Otten et al., 2014). It is therefore likely that for most precalculus students, proving trigonometric identities represents a significant departure from familiar modes of reasoning. As both a thorough understanding of the relationships between trigonometric functions and the ability to reason deductively are critical in undergraduate mathematics, learning trigonometric identities may be an opportune time for students to develop skills at all levels of Stylianides' (2009) taxonomy of reasoning-and-proving (RP) activities.

This poster will summarize a study that examined five precalculus textbooks for the opportunities they afford students to engage in RP activity in the context of trigonometric identities. One textbook was selected from each decade from the 1970s to the 2010s, and each represents a distinct instructional approach (Stylianides, 2008). The research questions were:

1. Across the five textbooks, what opportunities do students have to derive or prove essential trigonometric identities?
2. Across the five textbooks, what types of RP activity, and in what amounts, are expected of students in the context of learning to apply and verify trigonometric identities?

Twenty trigonometric identities were identified as *essential* (defined as necessary in future problems and/or in proofs of other identities). Each textbook's presentation of each identity was assigned one of six author-created codes, capturing the extent to which the text solicited student involvement in deriving it. A modified version of Otten et al.'s (2014) framework for coding RP opportunities in geometry textbooks was used to code justification-based problems in lessons about trigonometric identities. Percentages of items marked with each code were tabulated and trends identified. The poster will report on those trends, of which two were most salient:

1. In nearly all instances of an opportunity to derive an essential identity, the text provided enough guiding detail that a student would not bear the cognitive load of creating a proof.
2. The two 21st century textbooks afforded noticeably more frequent opportunities to make graphical arguments than the three 20th century textbooks.

The second finding is particularly significant in light of Stylianides' (2008) comments on the potential curricular effects of the 2000 NCTM Standards' unified view of reasoning and proof. Because of the small sample of textbooks under analysis, a natural next step would be to repeat the analysis with the precalculus and trigonometry textbooks that have been most popular over time. Significance tests of the differences in proportions of codes could show with greater clarity how responsive trigonometry curricula have been to long-term trends in instructional approaches.

References

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