Representations of the Derivative Valued by Post-Secondary Teachers

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Introductory calculus classes often serve as prerequisite classes for science and engineering majors. However, some researchers have questioned whether calculus courses, as currently taught, are filtering students appropriately (Black & Hernandez-Martinez, 2016; Williams, 2012; Williams & Choudry, 2016). Central to introductory calculus classes is the derivative (Kidron, 2019). Scholars have discussed the ways in which representations of the derivative differ between disciplines (Dray et al., 2019), but there has been no formal research investigating the representations of the derivative valued by teachers of calculus and teachers of subsequent non-mathematics classes. This study, through a new survey instrument, determined there are significant differences between the representations of the derivative valued by various post-secondary teachers in majors requiring calculus. By analyzing these differences, recommendations can be made to improve calculus as a preparatory course for non-mathematics majors.

Keywords: calculus, STEM education, derivatives, gateway courses

In a popular comedy sketch, a mad scientist reveals his latest depraved invention: junior high school (Studio C, 2014). As part of his invention, the mad scientist forces a student to take algebra and geometry. When asked by his assistant if a student will at least use what he learns later in life, the mad scientist replies, "Yes…if he ever teaches geometry or algebra" (Studio C, 2014, 1:33). In horror, his assistant asks, "You mean they exist only for themselves?" (Studio C, 2014, 1:42). Based on the laughter from the live studio audience in the video, these comments resonated with the audience's experiences.

Research has repeatedly found that students share the belief that mathematics is not valuable to them in their future personal or professional lives (Black & Hernandez-Martinez, 2016; Brown et al., 2008; Di Martino, 2019; Hernandez-Martinez & Vos, 2018; Rellensmann & Schukajlow, 2017; van der Wal et al., 2017; Vos, 2018). This feeling is encapsulated by a study that found a group of graduates from engineering programs "perceived their mathematics courses as islands, with no relation to the rest of their education" (van der Wal et al., 2017, p. S98). Mathematics classes existing only for themselves is concerning because mathematics classes, especially calculus, serve as filters for many science and engineering majors (Bressoud et al., 2013; 2015). As noted by Bressoud et al. (2015),

> At least one term of calculus is required for almost all STEM majors. For too many students, this requirement is either an insurmountable obstacle or—more subtly—a great discourager from the pursuit of fields that build upon the insights of mathematics. (p. v)

There have been calls for calculus to serve as a pump for science and engineering majors, rather than a filter (Bressoud et al., 2015). However, as noted by Bressoud et al., there seems to be little evidence of this change taking place despite multiple efforts to reform calculus classes.

Purpose of the Study

The purpose of this study was to gather quantitative data to determine the representations of the derivative favored by teachers of calculus classes as well as the representations favored by teachers of classes in majors requiring calculus. The goal of this comparison is to make recommendations for improving the current content taught in calculus I. As noted by Dray et al. (2019), students have struggled with representations of the derivative used in upper-level physics classes that are not emphasized in calculus I classes. Since Dray et al.'s report was developed using anecdotal data from the teaching experiences of the researchers, this study aims to generate more general data that can be analyzed to determine which representations need to be emphasized more in general calculus I classes. Additionally, the focus of Dray et al. was on physics students. However, it is estimated that only about 4% of calculus I students are pursuing physical science degrees, which includes both physics and chemistry degrees (Bressoud et al., 2015). This study cast a wider net and surveyed teachers from faculty in multiple disciplines to better determine which representations should be emphasized to better serve the students taking mainstream calculus I.

Research Question

This study sought to answer the following research question: Is there a difference in the value post-secondary teachers of calculus and teachers of non-mathematics classes requiring calculus ascribe to different representations of the derivative?

This survey was distributed to post-secondary faculty at colleges in the state of Wyoming. Since Wyoming has only one public university and seven community colleges, surveying faculty at all institutions was a feasible project. Additionally, Wyoming law empowers the state community college commission to establish common course numbering across undergraduate courses throughout the state (Common College Transcripts, 2018). This made surveying teachers of common classes across the state of Wyoming easy and efficient, as the course numbering and most prerequisites are the same across the state.

Significance of this Study

Based on the work of Dray et al. (2019), I suspected that numerical and experimental representations of the derivative would be more highly valued by teachers of non-mathematics classes than by mathematics faculty. Additionally, it is likely these will be valued more than other representations by teachers of non-mathematics classes. As there have been multiple stated concerns with calculus instruction and curriculum as they currently stand (e.g., Bressoud, 2021; Bressoud et al., 2015, Dray et al., 2019; Tallman et al., 2021; Teague, 2017), this study can help determine how mainstream calculus I curricula can be improved to better prepare students for future non-mathematics classes. Specifically, this study can help determine how calculus I instruction can be made more authentic, to potentially better help prepare professionals to use calculus concepts in their fields.

Calculus: A Problematic Filter

There are many potential reasons for this failure to reform calculus classes. One concern postulated by researchers is that mathematics classes privilege those who know the "rules of the game" of math education rather than prepare students to use math outside of math class (Black & Hernandez-Martinez, 2016; Williams, 2012; Williams & Choudry, 2016). This suggests that proficiency in mathematics classes, as evidenced by good grades, is not of value for use in future classes or careers, but as a form of exchange that can be leveraged to enter competitive fields or prestigious post-secondary institutions (Williams, 2012). This form of exchange has been dubbed "mathematics capital" (Williams, 2012).

This notion of mathematics classes acting more as producers of mathematical capital rather than transferable skill is concerning considering results found by Tallman et al. (2021). In their study, they reviewed 254 calculus final exams to gain a better understanding of how calculus students were assessed. Tallman et al. found that the calculus final exams they reviewed did not generally assess students' understanding of calculus concepts, but their abilities to use mathematical procedures to answer particular types of calculus questions. Additionally, few questions in the final exams allowed students to engage meaningfully with real-world contexts.

The results found by Tallman et al. (2021) along with Williams' (2012) notion of mathematical capital suggest that calculus classes are not acting as a pump but as a filter that does not filter for attributes that help students succeed in science and engineering degrees. Some suggest this may be a problem as the number of engineering graduates has not seen real gains over the last three decades despite calls from political and advocacy groups for more students to obtain such degrees (Bressoud et al., 2017). Additionally, there are concerns that mathematics classes as they currently stand reinforce existing inequities rather than make it possible for all students, including those from marginalized populations, to succeed in science and engineering careers (Williams, 2012; Williams & Choudry, 2016). As students from marginalized populations are generally more likely to struggle in calculus classes (Bressoud et al., 2015), the filtering done by calculus classes, in their current form, presents serious concerns for educational equity.

These issues show that mainstream calculus is in need of improvement. For such improvement to happen, the content of calculus classes must be better examined. Some science education researchers, in conjunction with math education researchers, have pointed out that mathematicians and scientists often utilize different representations and understandings of important calculus concepts (Dray & Manogue, 2005; Dray et al., 2019). As noted by Dray and Manogue (2005), "The way mathematicians view and teach mathematics, and the way mathematics is used by physicists and other scientists, are completely different; we speak different languages, or at least different dialects" (p. 2). This suggests that calculus content should focus on future applications of calculus. However, more information is needed to determine which applications should be included in an introductory calculus class. This study aimed to begin the process of determining which applications should be included through a quantitative survey.

Theoretical Framework: Multiple Representations of the Derivative

Since mainstream calculus focuses mostly on using the derivative, this study will examine how the derivative is represented by calculus teachers, as well as by teachers of STEM classes. As noted previously, researchers have observed differences in the ways scientists and mathematicians use and discuss derivatives (Dray et al., 2019; Dray & Manogue, 2005). Although the formal definition of the derivative is given using limits, this definition is often not directly used when determining a derivative from real world data. Roundy et al. (2015) saw this when they asked groups of mathematicians, physicists, and engineers to find the derivative of a certain rate of change in a mechanical machine created for the purpose of the case study. The physicists and engineers found approximations of the derivative by looking at ratios of small changes in the input and output variables and were thus able to find "good enough" approximations of the derivative to the point they described their answers as "the derivative" rather than as an approximation of the derivative. However, the mathematicians were unable to find a derivative and were unsure of the approximation they were able to generate.

In later work, Dray et al. (2019) described this issue seen by Roundy et al. (2015) as an overreliance, by the mathematicians, on the limit definition of the derivative. Along with their firsthand experiences in working with derivatives in their research and in upper-level physics classes, Dray et al. (2019) used this difference between the results of the mathematicians and the physicists and engineers in Roundy et al. (2015) to develop the notion of a "thick derivative." A "thick derivative" is a derivative function or value developed using numerical data to approximate the formal limit process. Dray et al. (2019) postulated that this notion of a thick derivative is used by scientists and engineers in their work as an approximation of the limit definition of the derivative.

To operationalize these differences, Dray et al. (2019) created a framework describing the multiple representations of the derivative they encountered in their professional work. This framework was an extension of a framework developed by Zandieh (2000). The framework of Dray et al. (2019) lists five main representations of the derivative: (a) graphical, (b) verbal, (c) symbolic, (d) numerical, and (e) physical. These representations are all considered process-objects (Sfard, 1991) that begin as ratios that are then reified into limits and functions. This framework was used to help determine which of these five primary representations of the derivative are favored by post-secondary teachers in majors that require students to take calculus.

Methods

For this study, a quantitative, correlational study design was used (Field, 2018). Correlational studies are used to determine if there is a relationship between sets of independent and dependent variables. In this study, I specifically examined the relationship between the subject domain of a teacher and the value they ascribe to different representations of the derivative. As such, I considered the subject domain of collegiate teachers surveyed as the independent variable for this study. This independent variable is categorical. There are six dependent variables, each representing the value a teacher places on a certain representation of the derivative. These variables follow from the representations of the derivative described by Dray et al. (2019). Although Dray et al. describe only five representations of the derivative, the graphical, verbal, symbolic, numerical, and experimental representations, they do divide their symbolic representation into two parts. There is the symbolic representation of using a limit to determine a derivative and the symbolic representation of using derivative rules to find derivatives of known functions. Since derivative rules play a key role in calculus I classes (Tallman et al., 2021), I considered the limit representation of the derivative and derivative rules used to find derivative functions separately within this study. This resulted in a survey examining six representations, the graphical, verbal, symbolic, rules, numerical, and experimental representations. As the value respondents ascribe to each representation will be determined using Likert-scale questions, the value respondents assign to each variable can be considered scale variables (Field, 2018).

Data Analysis

Since the data collected consisted of one categorical independent variable and multiple dependent scale variables, data was analyzed using a multivariable analysis of variance (MANOVA) (Fields, 2018; Huck, 2012). This analysis is appropriate as it prevents possible type I errors that may result from repeated uses of an analysis of variance procedure for each separate dependent variable (Fields, 2018). Additionally, MANOVA can be used to reveal possible relationships between the dependent variables. To further analyze differences for the representations between individual subject areas, I used Tukey's post-hoc test.

To help measure the reliability of the items used to assess the value ascribed to each representation of the derivative, Cronbach's alpha was calculated for each set of items used to create the normalized representation scores (Fields, 2018; Huck, 2012). All analyses were carried out using SPSS.

Participants

For this study, I surveyed Wyoming public community college and university faculty who teach in a major that includes calculus as a requirement. Of those who were sent the survey, seventy responded and completed the survey.

Although surveying teachers within the state of Wyoming does not strictly represent a random sample, this sample is still representative of post-secondary teachers across the country. All colleges in the state of Wyoming are accredited by a national accrediting body (Higher Learning Commission, 2019), and thus educational requirements for these positions are the same as at other institutions in the United States. These colleges recruit faculty from all around the country, with some faculty even coming from international sources.

Participant anonymity was protected by collecting no personally identifying information. Furthermore, only I had access to the data on a password-protected cloud storage system. This research was also protected by having the study approved by the institutional review board of the University of Wyoming.

Data Sources

Respondents participated in this study by completing a survey I created. This survey was created by designing Likert-scale items based on the framework of Dray and colleagues (2019) previously described. Multiple items were created for each conception described in the framework. These questions were items to elicit respondents' values of each process-object layer for each conception. For each item, respondents were asked to share how important they thought various concepts were for students to understand at the end of an introductory calculus class. For example, respondents were asked how important it was for students to memorize basic derivative rules. For each statement, respondents could rank the statement, "Not at all important," "Slightly important," "Moderately important," "Very important," or "Extremely important."

I created this survey based on Dray and colleagues (2019) framework due to its suitability as a framework of the mathematical knowledge to teach derivatives, as discussed in the literature review. Since an overwhelming majority of calculus students have been found to declare non-mathematics majors (Bressoud, 2015), this framework is especially well-suited to assess the value non-mathematics teachers place on different conceptions of the derivative.

Results

As noted earlier, this survey had 70 responses. Participant breakdown by subject area is as follows: (a) Mathematics: 22 respondents; (b) Biology: 6 respondents; (c) Chemistry and physics: 8 respondents; (d) Engineering and computer science: 9 respondents; and (e) other: 25 respondents.

Reliability Results

Each subscale of this survey, the graphical, verbal, symbolic, rules, numerical, and experimental subscales, had high reliability scores. Additionally, to meet the assumptions of MANOVA, the graphical and verbal subscales were combined. This new, larger subscale also had high reliability scores. This resulted in five representations analyzed as part of the MANOVA: the graphical & verbal representation, the symbolic representation, the rules representation, the numerical representation, and the experimental representation. Each of these representations had a reliability score of 0.891 or better.

Test Results

Based on Pillai's trace, there was a significant association of subject area with the value ascribed to different representations of the derivative, $V = 0.758, F(20,256) = 2.690, p < 0.001$. For each representation, the mathematicians yielded the highest value for their mean score. In other words, mathematicians in this study valued each representation more than their counterparts. Separate univariate tests for each representation yielded significant associations within each representation as well. Tukey's Post-Hoc test was used to consider subject areas pairwise and determine which had significantly different values ascribed to the representations. Most notably, mathematics and biology teachers differed significantly on the graphical & verbal and rules representation ($p = 0.018$ and $p = 0.042$ respectively), with the mathematicians valuing each representation more than their biologist counterparts.

Discussion

Assuming the differences found in the values ascribed to different representations found in this survey are accurate, there are two main ways to address these differences to improve calculus I classes; curricular adjustments and creating calculus classes specifically for certain majors, often termed "siloed classes" (Ellis et al., 2021; Luque et al., 2022; Voigt et al., 2020).

Curricular Adjustments

A call to adjust the curriculum of calculus I is not new. There have been attempts to reduce the calculus curriculum in the past to focus less on rote skills and more on relational understanding (Douglas, 1986). More recent research, however, indicate that these reductions have not yet occurred (Jones & Watson, 2018; Tallman et al., 2021). Despite this lack of reduction, researchers involved in more recent studies still expressed a desire to reduce and focus the calculus curriculum. Jones and Watson (2018) suggested focusing on Zandieh's (2000) version of the derivative representation framework to help focus instruction, while Tallman et al. (2021) suggested that calculus exams should assess relational understanding of the derivative rather than solving specific types of calculus problems.

The results from this study lend additional credence to the suggestions of Jones and Watson (2018) and Tallman et al. (2021). Scholars often suggest lessening the emphasis on complex uses of rote procedures, especially with the advent of computer technology that can complete these procedures more accurately and efficiently than humans (Douglas, 1986; Gravemeijer et al., 2017; Tallman et al., 2021).

As mentioned before, non-mathematicians do not view such procedures as unimportant; they merely view them as less important. These data do not suggest a complete removal of rote procedures from calculus I curricula; instead, they suggest a partial removal of such material only. Furthermore, research literature suggests that building fluency with these procedures through relational understanding of these procedures, rather than through extensive practice of complex combinations of such procedures, can help ensure future professionals can effectively use technology to assist them with such procedures in the future (Gravemeijer et al., 2017; Hoyle et al., 2013; Tallman et al., 2021; van der Wal et al., 2017).

Siloed Classes

The significant difference in the rules category between mathematicians and biologists is both striking and practically important. As a reminder, the mathematicians reported valuing the rules category more than the biologists, with a difference of 1.5076. These differences between the biology and mathematics teachers is made practically important by the number of students from these majors in mainstream calculus. Based on a large national survey, about 30% of calculus I students are biology or life science majors (Bressoud et al., 2015). This group is second only to engineering majors, who make up about 31% of calculus I students. The group with the next highest percentage is business majors, with 7% of calculus I students. Thus, this disparity where biologists value the Rules category much less than mathematicians, while the engineers answer quite similarly to the mathematicians in that category, leads to a conundrum that is difficult to solve in a class serving both populations.

One possible solution to this conundrum is to create siloed calculus classes (Ellis et al., 2021; Klingbeil & Bourne, 2013; Luque et al., 2022; Voigt et al., 2020). These calculus classes are variations on the traditional calculus curriculum that focus on using calculus in a particular subject area. There are many types of siloed classes, but one that has received significant attention is a calculus class for future biologists and other life scientists. These commonly referenced siloed classes parallel the large numbers of such students in calculus I classes as well as the results of this study.

Research into siloed classes has shown such classes can lead to lower failure rates in calculus classes, especially those for biology and life science majors (Luque et al., 2022; Voigt et al., 2020). Siloed classes overall seem to have helped students who were less prepared for college math (Klingbeil & Bourne, 2013; Voigt et al., 2020). In one study, the drop, failure, and withdrawal rate (DFW) for such students was equal to that of their colleagues who entered college with better math skills, suggesting that siloed classes "levelled the playing field for less prepared students" (Voigt et al., 2020, p. 868). Siloed classes also tend to lead to lower failure rates as well as increased enrollment in such classes and majors (Klingbeil & Bourne, 2013; Luque et al., 2022). These findings suggest that siloed classes have the potential to help students proceed past calculus into major coursework.

This study adds additional evidence to the need for siloed classes especially for biology majors. The significant difference found between the value ascribed to the Graphical and Verbal representation and Rules representation between the mathematicians and biologists suggests mainstream calculus classes may not be properly serving biology majors. Further research, including targeted interviews with course coordinators, is needed to validate this notion.

Future Research

This study opens the door to several avenues of future research. In many ways, this study can be considered a pilot study for the derivative representations survey used herein. As mentioned before, the framework by Dray and colleagues (2019) used to design this survey has been used in the past to analyze differences in the ways mathematicians and non-mathematicians approach differentiation tasks in task-based settings. As piloted in this study, such differences can continue to be analyzed among larger groups of professionals using the derivative representations survey. However, before such research could be conducted, improvements would need to be made to the survey as it currently stands.

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