A Student's Self-Perceptions and Social Positions in an Undergraduate Precalculus Classroom

Jason Guglielmo
Arizona State University

Mathematics education research has linked students' development of positive mathematics identities to outcomes of academic success and persistence in STEM. I contribute to this research area by adopting a positional approach to identity development, focusing on the students' development of in-the-moment identities through the negotiation of acts and roles. To better understand the connection between the roles adopted by students in class and the students' mathematics identity development, I conducted classroom observations of an undergraduate precalculus classroom and interviewed four focal students on their beliefs about math and experiences in this course early and late in the semester. Student-instructor interactions from the observations were coded to produce positioning profiles for the focal students. Here, I present the positioning profile for one student, alongside a reflection of her transition from not identifying as a "math person" to identifying as one, as developed through narrative analysis of the interview data.

Keywords: positioning, identity, student-instructor interactions

Students' mathematics identity development has been a topic of mathematics education research, with studies linking the development of a positive math identity to students' academic success (e.g., Ma & Kishor, 1997; Lee & Anderson, 2009; Gonzalez et al., 2020) and persistence in STEM trajectories (e.g., Meece et al., 1990; Boaler & Greeno, 2000). This study intends to explore how student-instructor interactions in the classroom support or hinder the development of positive math identities. Identity has been conceived of in many different ways, and here I leverage perspectives with a focus on students' in-the-moment construction of their *positional identity* as they interact with others in math spaces, foregrounding the roles that students are allowed access to and choose to adopt in their math classes (Herbel-Eisenmann et al., 2015; Radovic et al., 2018). This positional take on identity has offered particular insights into the role of communication acts (Andersson & Wagner, 2019) and their associated power dynamics (Esmonde & Langer-Osuna, 2013) in affording (or denying) resources for students' participation in mathematical tasks.

In this contributed report, I utilize longitudinal classroom observation and interview data to describe the ways in which Olivia (pseudonym), a student in an undergraduate precalculus course, (a) adopted particular roles during interactions involving the instructor in the math class, and (b) experienced growth in her sense of self as a "math person" across the semester. The ultimate goal of this research is to better understand how an undergraduate student's positioning during interactions with their math instructor impacts the student's enduring sense of themself as a learner and doer of math. To that end, the next phase of analysis will involve linking repeated positions adopted by multiple students to changes in their self-perceived mathematics identity. A better understanding of the impacts of student positioning during instructor-led interactions on their more stable identification with math will support instructors' ability to consciously reframe their discussions (Louie, 2017) in ways to support their students' mathematics identity development.

Theoretical Framing and Relevant Literature

I define a participant's *positional identity* using Harré and van Langenhove's (1999) positioning theory. This perspective is focused on understanding the meanings behind different communication acts in an interaction through the storylines and positions available to participants. *Storylines* are sociocultural frames for interpreting the available rights and duties of participants in the interaction, and *positions* are the ways that elements of these storylines are taken up by the participants during the interaction to act in particular ways (Andersson & Wagner, 2019). One's identity is constructed through social interactions where (a) the individual acts to position themselves as having particular roles or qualities, and (b) others act to position the individual and accept (or reject) the individual's own acts of positioning (Langer-Osuna & Esmonde, 2017). With this understanding, I define an individual's *mathematics identity* as their personally held beliefs about their ability to participate appropriately in particular mathematics spaces. This framing of mathematics identity assumes a connection between students' acceptance (or rejection) of in-the-moment positions within the mathematics classroom, and their self-understandings about how they are supposed to participate in mathematics spaces.

Significant work has been done in K-12 mathematics education research to better understand students' identity development and the outcomes associated with it using a positional lens (Boaler & Greeno, 2000; Esmonde & Langer-Osuna, 2013; Bishop, 2012; Anderson, 2009; Andersson & Wagner, 2019; Gholson & Martin, 2019; Louie, 2017). In particular, the positional approach to identity has given researchers insights into students' enactment of different mathematics identities in the same local context (Bishop, 2012) and the positive influence of different positioning acts on students' engagement with certain mathematical practices (Esmonde & Langer-Osuna, 2013). However, this positional approach to mathematics identity has been uncommon in RUME (Radovic et al., 2018).

To characterize the varied positions that students could adopt in the classroom, I initially leveraged Andersson and Wagner's (2019) characterizations of high school math students, who highlighted three positions of *leader*, *follower*, and *rebel*. These archetypes focus on characterizing who is making requests and who is fulfilling those requests (or not), but they are somewhat limited. For instance, any interaction involving students and the instructor tends to position the instructor as the *leader* and the students as *followers*, given the nature of their formal relationship. As such, Andersson and Wagner's characterization of positions is more suited for student-student interactions and not student-instructor interactions. Additional approaches to characterizing positions from the literature range from lower inference documentation of the structure and function of an individual's communication actions (Bishop, 2012) to higher inference coding for the emotional content of actions (e.g., Heyd-Metzuyanim & Cooper, 2022). These various approaches to identifying students' positions in the math class collectively influenced the initial analytic coding process.

Methods

Data Collection and Context

Data for this preliminary report consist of (a) classroom observations of an undergraduate precalculus classroom, and (b) early- and late-semester interviews with one of four focal students and the instructor. The course was taught in Spring 2023 at a large research university in the Southwestern U.S. Class sessions were 75 minutes long, and students typically engaged in small group work at their tables while the instructor circulated the room, interspersed with whole class discussions (WCD). During WCD, the instructor frequently used an Initiation-Response-

Evaluation model for questioning (Mehan, 1979), where she initiated the interaction by asking a question to the class, a student responded to the instructor's question, and the instructor assessed and evaluated the student's response.

Three preliminary classroom visits occurred at the start of the semester, to identify and recruit focal students and to refine the video and audio recording setup. Following this, seven observations were conducted for study: three earlier in the semester and four later in the semester. After the preliminary classroom visits, students who had interacted with the instructor at least once were invited to participate in the study, and four students were chosen who had interacted with the instructor with varying frequency. Small group work was recorded visually using two tripod-mounted iPads and audibly using two recording devices placed on tables with the focal students; both visual and audio recording devices were adjusted throughout the class session to best capture student-instructor interactions in both small group and whole class discussions. While observing and recording class sessions, field notes were taken to provide context and to indicate times when there were interactions between the instructor and one of the focal students. The instructor was interviewed at the beginning and end of the semester to provide additional context regarding instructional practices and how she interacted with students.

The focal students were interviewed at the beginning and end of the semester, immediately following the early-term and late-term observations. All interviews were video- and audio-recorded using Zoom conferencing software; some interviews were conducted in-person while others were conducted over Zoom. Interviews were semi-structured, with questions about the students' (a) academic status and past experiences in math courses, (b) general interests in and beliefs about mathematics, (c) performance and participation in this particular course, and (d) perceptions of how they and significant others in their life and course (family, friends, instructor, classmates) viewed them as a "math person." At the end of each interview, participants were asked to reflect on particular moments from a recent class session, supported by clips from the class recording in which the participant was interacting with the instructor.

Analysis of Observation Data

Analysis of the observation data began with the identification of notable moments from class recordings. A *notable moment* is one where (a) an interaction occurred involving the instructor and a focal student, and (b) the interaction was *not* focused on course logistics (e.g., asking about exam dates). Although this report focuses on the notable moments involving one particular student, a total of 17 notable moments were identified between all four focal students, ranging from one to three minutes in length. These clips were transcribed using Descript software and manual cleaning, with notes added to indicate associated physical movements and gestures. Analysis of these notable moments was done in accordance with thematic analysis (Braun & Clarke, 2012) to develop roles for these focal students and the instructor. Following familiarization with the data, initial codes were assigned to each talk-turn, with multiple codes sometimes associated with the same turn. Codes were developed in a predominantly deductive fashion, where prior positioning studies were leveraged to construct (a) *structural codes* (Andersson & Wagner, 2019; Bishop, 2012) that capture the discursive function of the talk-turn in the interaction, and (b) *subjective codes* (Heyd-Metzuyanim & Cooper, 2022) that incorporate inferences about turn-taker's emotional hue. Though informed by the literature, these codes were refined through an iterative reflection with the data to more accurately capture the roles that were common in the class. Once refined, the position codes were grouped by student, and *positioning profiles* were created for each student and the instructor to reflect any patterns noted in the individual's positions throughout the class. In the next section, the developed codes are presented

alongside a checklist to indicate if Olivia or her instructor adopted any of those roles during notable moments involving Olivia.

Analysis of Interview Data

Interview recordings were automatically transcribed by Zoom and manually checked. Analysis of interview data was conducted in accordance with narrative analysis (Polkinghorne, 1995) to configure the elements and events of Olivia's interview into a developmental account of her feelings as a "math person" throughout the semester. The use of "math person" here is due, in part, to prior findings that the term is associated with mathematics identity for some undergraduate students (Guglielmo et al., 2023). Initially, events relevant to Olivia's perceptions of herself as a learner and doer of math were identified and summarized. These events were then organized to construct an initial recounting of Olivia's math identity development from beginning to end of the semester. This narrative was refined through a recursive movement between the data and the emerging thematic plot, to ensure it succinctly brought meaning to the interview data while maintaining accuracy. In the next section, a condensed version of the narrative along with relevant excerpts from the interview transcript are presented.

Results

Olivia's Positioning Profile

Olivia was involved in six notable moments over four class periods (two early-class, two late-class). Five of these moments occurred while students were working in small groups, the sixth occurred during whole class discussion. Table 1 describes all nine position codes that were developed during analysis. In this table, both the position code and its description are provided, alongside a checklist that acknowledges if the instructor or Olivia exhibited that role during *any* of Olivia's six notable moments. The first six codes are structural codes and the last three are subjective codes.

Table 1. Codes developed for Olivia and the instructor's roles during Olivia's six notable moments, with a checklist for which individuals exhibited these roles in at least one notable moment.

Code: Description	**Instructor**	**Olivia**
Requester (general): Making a request expecting a response from members of a work group or the entire class.	x	
Requester (specific): Making a request expecting a response from a specific individual.	x	x
Requester (support): Making a request looking for approval or support for their prior responses or actions.	x	
Responder: Responding to a previous request by another individual, with or without justification for response.	x	x
Subject: Being the explicit or implicit subject of another individual's request or response.	x	x
Revoicer: Rephrasing or restating the response of another person out loud.	x	x

Approver: Responding to another individual's request or response, with explicit or implicit approval of some part of their contribution.	x	x
Critiquer: Responding to another individual's request or response, with explicit or implicit critique of some part of their contribution.	x	
Apologizer: Expressing regret for a prior response or action.	x	x

The following excerpt illustrates how these codes were applied, using a small group interaction between Olivia, the instructor, and StudentA (a non-focal student in Olivia's small group), discussing the domain of the sine function:

		Code [*Subject*]
Instructor	So, do we have any restriction on our angle measure?	Requester [*General*]
Olivia	Well, I was thinking, wouldn't, wouldn't it be like zero to two pi?	Responder [Instructor]
Instructor	Can we go (*upward vocal inflection*) above two pi?	Critiquer [Olivia] Requester [*General*]
StudentA	Yeah, yeah, if you do multiple rotations.	Responder [Instructor]
Instructor	Okay. Can we go the (*upward vocal inflection*) other direction?	Approver [StudentA] Critiquer [Olivia] Requester [*General*]
Olivia	Yeah.	Responder [Inst.]
Instructor	So (*expectant pause*)	Requester [*General*]
StudentA	Kind of (*pause*) infinity?	Responder [Instructor] Requester [Instructor]
Olivia	Okay.	Approver [StudentA]
Instructor	Yeah, we can go in both directions. We'll still get out something, right, from our function. Yeah, we haven't been working too much in terms of, like, more than one full rotation. But, um, we definitely can.	Approver [StudentA] Revoicer [*General*]

Initially, Olivia responds to the instructor's request for the domain of the sine function (talked about as an angle measure) as zero to two pi. The instructor, instead of approving Olivia's contribution, makes a request for further information from the whole group, implicitly critiquing Olivia's response. StudentA responds to the instructor's first request with support, and then Olivia responds to the instructor's second request without support. After the instructor's final request, StudentA provides a hesitant response that also functions as a request for support, to which both Olivia and the instructor approve. This was one of three notable moments where Olivia made a mathematical error in her response, and the instructor adopted a Critiquer role by questioning the student's response. In the two other instances, one similarly involved an implicit critique of the student's response mediated by another student, while the other was an explicit critique of the student's algebra.

Olivia's Narrative Reflection on Mathematics Identity

Olivia's mathematics identity narrative was constructed relative to three phases of Olivia's participation in her precalculus course: the pre-precalculus narrative, the initial reflections on precalculus, and the final reflections on precalculus. Before enrolling in her precalculus course, Olivia had some positive associations with mathematics, based on prior experiences in math courses and her own interests in the sciences. In high school, Olivia found particular math subjects interesting and "easy" (trigonometry, statistics), while also developing a preference for math classrooms with an active instructor who provides step-by-step explanations and examples. In her transition to university, mathematics played a peripheral role in her choice to major in biological sciences, with Olivia desiring a biology-focused major that was not "fully set on just math." In relation to her biology identity, Olivia views math as a useful tool for interpreting graphs, analyzing data, and making predictions, with its application to many areas and its integral role in the advancement of our society. In sum, Olivia has an appreciation for mathematics, an interest in learning (at least some areas) of mathematics, and a view of mathematics as significant (in some regards) to her future academic and career motivations.

Despite these positive associations with mathematics, Olivia explicitly did *not* describe herself as a math person during the initial interview, in which she said:

> **I want to be a math person**. I associate a lot of like math people, **they're really smart**. Like they're really smart. They think really well, like the way their minds work, they're very calculated […] whereas, like, with English, it's okay, there's more leniency in it […] I feel good knowing that, like, my family would probably see me as [a math person], but **I just don't see myself as it probably due to some, like, I see, I'm like, I'm hard on myself. I'm a pretty, like, when it comes to that, I have a pretty high standard**.

Olivia often contrasted her sense of being an "English person" with being a "math person" during this first interview, as noted in the excerpt above. She viewed herself as an English person, in part because it "just comes more naturally," while not viewing herself as a math person due to her own "high standard." During this first interview, Olivia thought of "math person" as a title reserved for "really smart" individuals, of which she did not view herself.

While reflecting on her classmates' perceptions of her as a math person during the first interview, Olivia noted the mixed perceptions they likely have. She mentioned some students in the class that likely *don't* view her as a math person due to her getting multiple answers wrong, but also one particular student who likely *does* view her as a math person, because Olivia opens up more around her and explains her process. This focus on opening up and reaching out to others was brought up multiple times by the student, mostly expressing a desire to contribute her ideas more during discussions. Specifically in previous math courses, Olivia acknowledged that she often would not speak out during class due to her "low self-esteem when it comes to math related subjects." However, with her precalculus course, Olivia stated that "[the instructor] makes it to where you wanna talk to the tables," and that "this is the perfect moment for me, in my mind, to like, try and branch out more." Though she still has to "fight the feeling" of being afraid to be wrong, Olivia acknowledges that she has started sharing her ideas more openly with others in her precalculus class, due in part to the interactive nature encouraged in the class.

During the second interview, and after asking Olivia "How has this class affected your level of interest in math as a whole?" Olivia immediately responded, "Okay, this is crazy, but I've actually, I went from thinking that I wasn't a math person to actually thinking I am a math

person now." Throughout the interview, the reasons behind this change in the student's self-perception as a math person were explored. One particular reason, highlighted in the excerpt below, had to do with the students' shift in thinking about being wrong as necessary in the process of learning mathematics:

> I'm, I'm feeling a lot more comfortable with being wrong. Just like being able to discuss in groups, if I'm wrong, I kind of, I, I used to just care about it so much. **It was all I would think about. I, I was afraid of like speaking.** But like now it's, I don't know. I, I'm feeling a lot more comfortable in doing that, because **I've seen everyone get something wrong before and I'm like, I'm not the only one** […] It's not like I care that they got that wrong, so why would others care about me getting stuff wrong, you know? It's all part of the process. **It's all part of learning. That's why we're learning it**.

Olivia more explicitly notes her instructor's influence on this shift, through her "sweet and genuine" approach to teaching and her ability to "understand where [Olivia's] coming from." In contrast, Olivia states that with her current English instructor, "I'm afraid that if I say this, I'm gonna be wrong, and she's gonna scream at me." Unlike other classes, Olivia consistently looks forward to going to her precalculus lessons, sees the importance in them, and feels "happy" and "motivated" when she learns something new. Overall, Olivia now predominantly sees a math person as one who enjoys doing math and participating in math contexts. As such, she now views herself as a math person.

Discussion and Next Steps

During early- and late-semester class observations, Olivia adopted relatively consistent positions during interactions involving the instructor. In five of the six notable moments, Olivia made a mathematical error in her response to the instructor's request, and another individual would implicitly or explicitly critique her response. In these interactions across the semester, Olivia took on the role of a quiet yet persistent member of her precalculus class, whose contributions were open to critique by others. This prevalence of critiquing contributions was mirrored in Olivia's reflections during both interviews, where she often mentioned a fear she had of speaking out and being wrong in previous math classes. By the end of her precalculus course, Olivia saw the process of trying and getting things wrong as a necessary part of learning mathematics. This shift coincided with her coming to claim the title "math person," as Olivia found enjoyment and motivation in her precalculus class. In accordance with positioning theory (Harré & van Lagenhove, 1999) and mathematics identity, I conjecture a link between Olivia's frequent in-class role as the subject of others' critique and her development of understanding wrong answers as necessary for learning.

Additional positioning profiles and math identity narratives for the other focal members of this class will contribute to a broader understanding of ways in which students' mathematics identities may shift in relation to the positions they adopt during interactions with their instructor. These will inform future research intended to better understand the ways in which undergraduate students' positioning in their math class affects their mathematics identity development, including exploring the storylines from which they draw their various roles and classroom norms. Ultimately, this will lead to a more comprehensive understanding of the factors that influence the students' positioning and mathematics identity development.

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