Exploring Students' Mathematical Convictions Through a Script-writing Task

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Some empirical studies that report on mathematical convictions include proof comprehension (Stylianides et al., 2007) or evaluation (Inglis & Mejia-Ramos, 2008). Some focus on students' convictions (Lockwood et al., 2020; Zaslavsky, 2005). One underreported method in the literature for this topic of research is s*cript-writing*—the creation of a dialogue, between oneself and an imaginary student or classmate, focused on issues that may be encountered and resolved (Zazkis & Zazkis, 2014). This method is often used with teachers as a form of lesson-play (Koichu & Zazkis, 2018; Zazkis et al., 2013) while others have used it to investigate proof comprehension (Brown, 2018; Zazkis, 2014).

The purpose for this study was to explore this method's utility when investigating students' mathematical convictions. The guiding research question was: how do students' responses to a script-writing task describe their mathematical convictions? Take-home script-writing tasks, initial interview recordings, and one follow-up interview recording were collected. The statement and argument presented to students for their task were:

> *Statement*: For $n \in \mathbb{N}$, the sum of the first n consecutive odd numbers is n^2: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
>
> *Argument*: Let n be a natural number and consider $n = 1$. We have $2(1) - 1 = 2 - 1 = 1$ which is 1^2. Assume that $1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1) = n^2$ is true for the natural numbers: $1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1) + (2n + 1) = [1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1)] + (2n + 1) = n^2 + (2n + 1) = n^2 + 2n + 1 = (n + 1)^2$. Therefore, $1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1) = n^2$ is true for the natural numbers.

The task had instructions on creating a dialog between them and a classmate named Gamma. These instructions asked to introduce and explain the statement and argument to Gamma and to identify problematic points (see Brown (2018, p. 66) for similar instructions).

Taylor, in his final semester, was completely convinced that the statement was true and the argument proved the statement in the initial interview. For his script, he used an example showing the statement to be true and invited Gamma to try examples. In the follow-up interview, Taylor explained he tried to help Gamma "understand intuitively" because it's easier to do that with examples. When asked how he would resolve the issue if Gamma was still not convinced that the statement was true, he responded with "like if I lay down a whole proof?... I guess they're just not convinced…." This was interpreted that Taylor had an assumption that proofs should be convincing. I asked Taylor if he agreed that if Gamma understood the statement and argument more, then Gamma would be more convinced. He responded with "definitely."

Stewie was a fifth-year student who recently completed a transition-to-proof course. In the initial interview, he was 99.99% convinced by the statement and argument saying, "it's hard to be 100% certain on anything." He cited possible human errors for this judgment. In his script, he identified issues regarding the formatting and clarity of the proof. Stewie appeared to explain the components of the proof, like the purpose of the base case, to help Gamma.

In this study, I hoped that the scripting task would influence participants' convictions. However, their convictions remained the same throughout the study. The script-writing task revealed that they viewed ideas of helping a classmate in: 1) comprehension and clarify of the method of proof and 2) conviction in the statement's truth. Future research may include more complex statements and arguments to explore the script-writing task's influence on convictions.

References

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