Conceptualizing and Representing the Cartesian Connection in Calculus

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Calculus concepts require students to express and interpret distances in the Cartesian plane. Yet previous research shows that many students are not easily able to do so (Parr et al., 2021). To effectively represent distances, students must be able to understand and apply the Cartesian connection, that a point is on graph A if and only if its coordinates (x, y) satisfy the equation of A (Moschkovich et al., 1993). This connection requires students to draw on algebraic and graphical concepts to conceive of coordinate points as pairs of distances rather than simply a location on a graph (Parr et al., 2021). Students should also be able to flexibly move between using a graph and its equation to represent distances both in terms of x and y or determine if points lie on a line (Knuth, 2000). We are interested in why some student may not display this flexibility and what underlying issues are preventing students from making the Cartesian Connection.

In this poster, we explore the following research question: What types of reasoning, successes, and challenges arise when undergraduate students engage in tasks meant to develop their ability to make the Cartesian Connection to represent distances? To answer this question, we presented surveys to 169 undergraduate students at a small private college to assess their ability to represent distance in the Cartesian plane both in terms of x and in terms of y. We selected 9 students who did not correctly complete the tasks to conduct exploratory teaching interviews. During the interviews, students worked through a series of tasks to represent distance in the Cartesian plane while thinking out loud and explaining their thought process and reasoning. We analyzed the students' interviews using theoretical sampling (Corbin & Strauss, 2014). Our results highlighted that whether students can make the Cartesian Connection is not a yes or no question, but rather better represented by a spectrum of student thinking. We broke up the Cartesian Connection into 3 components: (1) the definition, (2) the underlying algebraic meanings, (3) the underlying graphical meanings. As part of our results, we offer a conceptual analysis (Thompson, 2008) identifying the mental operations needed to make the Cartesian Connection in this context.

Many different types of student reasoning emerged, yet we found similar obstacles that students encountered when representing distances in the Cartesian plane. Some common obstacles were students having difficulty understanding the phrase "in terms of" as well as students having difficulty associating distance in terms of y for a horizontal segment. We analyzed students' responses according to two dimensions: 1) categories encapsulating all the mental processes needed to make the Cartesian Connection to represent distances and 2) the extent to which students were reliant on either algebraic expressions or graphical representations. This poster will give an overview of the spectrum of student thinking that highlights the complexity and nuance of the Cartesian Connection. We will also describe the implications of our findings for teaching and research.

References

Corbin, J., & Strauss, A. (2014). *Basics of qualitative research: Techniques and procedures for developing grounded theory.* Newbury Park, CA: Sage publications.

Knuth, E. J. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education*, *31*(4), 500-507.

Moschkovich, J., Schoenfeld, A., and Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representations of functions* (pp. 69-100). Erlbaum.

Parr, E. D., Ely, R., & Piez, C. (2021). Conceptualizing and representing distance on graphs in Calculus: The case of Todd. *2021 Research in Undergraduate Mathematics Education* Reports. (pp. 214-221).

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano and A. Sepulveda (Eds.), *Plenary Paper presented at the Annual Meeting of the International Group for the Psychology of Mathematics Education*, (Vol 1, pp. 31-49). Morelia, Mexico: PME.