Tasks for Exploring Fraction Understandings of Undergraduate Developmental Mathematics Students

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Many students experience difficulties with fractions (Mesa et al., 2014; Ngo, 2019). Thus, it is unsurprising that fractions act as a gatekeeper for college-level mathematics courses (Ngo, 2019). Research on how undergraduate developmental mathematics students understand fractions is scarce (Alexander, 2013; Mesa et al., 2014). Moreover, existing studies in this area seem to take a deficit perspective, focusing more on what students in this demographic do not understand about fractions rather than reporting what is productive in their thinking (Alexander, 2013; Baker et al., 2012; Doyle et al., 2015).

In this poster, I will report on tasks that I developed to examine the fraction thinking that undergraduate developmental mathematics students can utilize. I implemented these tasks during clinical interviews (Ginsberg, 1997), which come from a larger dissertation study that was designed to (a) examine how undergraduate developmental mathematics students understand fractions and (b) explore pathways to support fraction understandings that will help the student to be successful in future mathematics courses. In designing the tasks, I drew from existing K–12 fraction literature, including Lamon's (2020) fraction understandings and Steffe & Olive's (2010) fraction schemes, which I will explain in the poster. In short, Lamon's fraction understandings include part-whole, operators, quotients, measures, and ratio understandings. Fraction schemes I attend to include part-whole, partitive, and iterative fraction schemes (Steffe & Olive, 2010).

Twelve fraction tasks are spread over two 45-minute sessions for this clinical interview (Ginsberg, 1997). The interview tasks were designed to help me answer the following research question: How do undergraduate developmental mathematics students understand fractions? Specifically, (a) which of the five fraction understandings outlined by Lamon (2020) do the students operate with, and how do they support their fraction understandings; and (b) which of the fraction schemes outlined by Steffe & Olive (2010) do the students operate with and how do they support their fraction understandings?

In this poster, first, I will present an overview of Lamon's (2020) five fraction understandings, three of Steffe & Olive's (2010) fraction schemes, and how I correlated these views of fractions. Second, I will present examples of the tasks that I developed. I will identify which of Lamon's fraction understandings or Steffe & Olive's fraction schemes were intended to be elicited from each task. I will also provide the goals and design principles, as well as follow-up questions for these tasks. Finally, I will discuss some of the affordances and limitations of each task after implementing them in three clinical interviews (Ginsberg, 1997). For example, I designed a number line task that targets the measure understanding of fractions (Lamon, 2020). This task provides insight into the participants' fraction thinking beyond measures. However, due to the complexity of the measure understanding, the task may not reveal enough information to definitively say that the participant has constructed the measure fraction understanding (Lamon, 2020). I hope to provide examples of productive mathematical thinking that undergraduate developmental mathematics students have about fractions. This study aims to counter the deficit narrative prevalent in developmental mathematics.

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Background and Goals

- Many students experience difficulties with fractions (Mesa et al., 2014; Ngo, 2019).
- Fractions act as a gatekeeper for college-level mathematics courses (Ngo, 2019).
- Research on the way that undergraduate developmental mathematics students understand fractions is scarce (Alexander, 2013; Mesa et al., 2014).
- Existing studies in this area focus more on what these students do not understand about fractions (Alexander, 2013; Baker et al., 2012; Doyle et al., 2015).
- Goals: (1) Build models of developmental math students' fraction understandings (Lamon, 2020; Steffe & Olive, 2010). (2) Explore pathways to support fraction understandings that help the student to be successful in future mathematics courses.

Research Questions

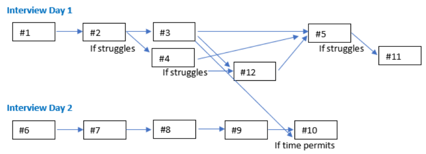
1. How do undergraduate developmental mathematics students understand fractions?
   a. Specifically, which of the five fraction understandings outlined by Lamon (2020) do the students operate with and how do they support their fraction understandings?
   b. Which of the fraction schemes outlined by Steffe & Olive (2010) do the students operate with and how do they support their fraction understandings?
2. What ways of fraction thinking are present (from RQ1) that could specifically help to further the participant's measure fraction understanding?

Task Design Overview

Methods

- Clinical interviews (Ginsberg, 1997)
- 12 questions over two 45-minute sessions
- Video recorded from multiple angles
- 8-12 participants
- Ongoing and retrospective analysis (Steffe & Thompson, 2000)

Potential Paths According to Students' Responses



Analysis Framework

Lamon's Five Understandings	Steffe & Olive's Fraction Schemes
Part-Whole: Relating the number of equal parts of a unit to the unit	**Part-Whole Fraction Scheme: Simultaneously partitioning the whole into equal distant, connected, lengths**
Partitioning: Splitting the unit into equal parts **Iterating: Creating copies of the unit fraction**	**Partitive Fraction Scheme: The first true fraction scheme involves comparing the part to the whole. Note: The part-whole fraction scheme is a pre-requisite for the partitive fraction scheme.**
Partitioning and Iterating Beyond the Whole: Creating copies of the unit fraction beyond what is necessary to construct a figure that is equivalent to the whole	**Iterative Fraction Scheme (beyond the whole): Taking a unit fraction or a complex unit and iterating it to create a composite unit. Note 1: The partitive fraction scheme is a pre-requisite for the iterative fraction scheme. Note 2: This generally requires coordinating 3 levels of units.**
Operator: Viewing a fraction as a calculation(s) to be done (e.g., $\frac{a}{b} \cdot k$ means $a \cdot k \div b$)	
Quotient: Not only viewing a fraction as a set of instructions for calculation but assigning meaning to the result	
Measure: This is a geometric meaning of a fraction that involves magnitudes, or lengths, of intervals. **Magnitude recognition: Recognizing that relative size needs to be taken into account when working with the fraction**	Note: the above fraction schemes are part of measurement thinking (Steffe & Olive, 2010, p. 121).
Ratio: This fraction meaning can be comparative of the numerator and denominator as ordered pairs. Note: Not all ratios are fractions.	

This table shows how I correlate Lamon's (2020) fraction understandings with Steffe & Olive's (2010) fraction schemes.

Iterating Beyond the Whole

This figure represents a whole. Draw [ ]

[Goal] To examine whether the student has a partitive unit fraction scheme with a whole greater than one.

a. If student has difficulty, with #2, start with 4/6.
b. **Can you show me 1/6 in your figure?**
c. Round 2: **Let's try another one. Our figure still represents a whole. Draw 15/6**.

- Affordance: Fractions beyond the whole in action
- Limitation: 15/6 may not provide further information if the participant is still constructing an understanding of improper fractions.

Opening Tasks

What does 2/3 mean to you?

[Goal] To examine students' initial immediate response regarding their fraction conception without further prompting that targets specific fraction understandings, as in the following tasks. The student is being asked to read the task prompt to gather more information about how they read/write a fraction.

a. **Can you draw me a picture to show me the 2/3?** (Make sure to follow the student's language.)
b. ***Are there any other ways you can think about this fraction (2/3)?***
c. **Keep going until you exhaust this line of thinking.**

What does 7/3 mean to you?

[Goal] To examine whether they iterate the 1/3 beyond the whole. How they would make sense of an improper fraction in relation to their fraction conception.

a. ***Can you draw a picture of 7/3*** (if not done already-use their language)**?**
b. ***How do you know that's 7/3 and not 7/9?***

- Affordance: Initial reaction before targeted tasks
- Limitation: Not a full story of their understanding

Quotient

Jade shared 5 candies (cherry, chocolate, cinnamon, lime, and peppermint) among 3 friends. What amount of candy did each friend get?

[Goal] To examine whether the student has a quotient understanding of fractions.

a. Have index cards labeled with the candy flavors for the participant to demonstrate the sharing process.
b. If student can do the mental calculations and say that each person gets 5/3 candies each, ask student to use the index cards to show their thinking.
c. If student struggles, ask **Can you show me how to share these candies using the index cards?**

- Affordances:
  - Fair sharing (whole number not divisible by the number of people).
  - Partitioning an object into thirds.
  - Operator vs. quotient understanding
- Limitation: Consideration of number choices

Measure

Directions: Give the participant four index cards marked with 1/2, 2/3, 3/8, and 7/6, respectively.

Without using decimals, put these fractions in order from least to greatest.

[Goal] To examine whether the student has a measurement understanding of fractions.

a. If student indicates they do not know how to do this without fractions, ask the student to give me their best guess.
b. **What were you thinking as you ordered the fractions?**
c. Give the participant a piece of aper with a line across it lengthwise. **If this is a number line, where would you place the fractions as accurately as you can?**
d. *Offer sticky flags for placing numbers (they are moveable).*
e. Pay attention to whether the participant marks 0, 1, etc. and/or benchmarks on the number line.

- Affordances: Operator understanding, commensurate fractions, percentage thinking, and benchmarks
- Limitations: May need further questioning due to the complexities of the measurement understanding

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