# Bridging the Mathematical and Social Dimensions of Undergraduate Calculus: Students' Perspectives on a Program of Weekly Guided Collaborative Problems Solving

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*This study explores undergraduate Calculus students' perspectives on the Course Assistants (CA) program at a southern US university, where upper-class undergraduates guide weekly Math Lab sessions of collaborative problem-solving. Using sociocultural theories and qualitative methods the research investigates what aspects of Math Lab students and CAs found useful or not. Students' interviews and CAs' weekly reflections indicate that, on the one hand, students by and large appreciate the experiences of collaborating over the weekly problems with their peers and the CAs, the social networking it provided, and the mathematical confidence it helped build. In our own words, students appreciated how the academic and social dimensions of math learning came together. However, many students and CAs are skeptical about the connections between the weekly tasks and success in the course exams. Relatedly, students often complained about Math Lab being another structured hour in an already time-consuming course and about varying student engagement.*

*Keywords:* undergraduate calculus; collaborative problem solving; course assistants

Research and professional reports of undergraduate mathematics education often state the promises and challenges of shifting learning towards more active approaches (where *active learning* is conceived broadly and inclusively). For example, research reports in the proceedings of the 2022 Research of Undergraduate Mathematics Education conference (RUME 2022) addressed a variety of issues around active learning including instructors' workshops (Archie et al., 2022), instructors' dispositions (Ireland et al., 2022), online active learning (Kerrigan et al., 2022), linguistic diversity (Rios, 2022), and more. Such RUME research reports often cite as a starting point The Common Vision (Saxe & Braddy, 2015) and empirical evidence about the impact of active learning in undergraduate STEM courses (Freeman et al., 2014; Laursen, 2019; Laursen et al., 2014; Theobald et al., 2020). Side by side with these encouraging outcomes, we also know that K-16 efforts to implement collaborative math learning approaches (the specific type of active learning which is the focus here) are complex to incorporate and facilitate, sometimes fall short, and might even amplify exclusionary dynamics (Ehrenfeld & Horn, 2020; Louie, 2017; Reinholz et al., 2022; Rios, 2022; Shah & Lewis, 2019). In this paper we explore such tensions in the context of a Course Assistants (CA) program, in which upper class undergraduate students facilitated weekly collaborative problem solving sessions (known as Math Lab) in Calculus courses at a private southern US university. First, we provide a general overview of the program. Then, we share our sociocultural theoretical perspective, an overview of the data and methods of analysis we used, and finally, the main themes that emerged from the data. The overall question we address in this report are *What aspects of Math Lab students and CAs found more or less useful, how, and why?*

## Research Context: The CA Program

The overarching goal of the CA program was to support undergraduate Calculus students mathematically and socially during this transition to college. Students worked in small groups of 3-9 with a CA in weekly Math Lab sessions on calculus problems which were longer and more conceptual in nature than typical homework and exam questions. Those problems were written collaboratively by the course instructors, with direct relation to the weekly materials. Socially and mathematically, we hoped that students' experiences in the Math Lab would support them in developing collaborative study skills, conceptual understanding, and a sense of community and belonging in the course. After a pilot semester in Spring 2022, the Fall 2022 program started in one course with 20 course assistants (CAs) who facilitated 40 Math Lab groups which served ~250 students. The Spring 2023 program expanded to three courses with 20 CAs (17 returning and 3 new) who facilitated 40 Math Lab groups which served ~250 students. Math Lab was required only for students in the first Calculus course of their sequence. The rationale behind not requiring it in the following course was that by the time students are taking a second semester course, they have more practice in figuring out what kind of support they need and should have more agency to decide that for themselves.

## Theoretical Framework: Sociocultural Theories of Mathematics Learning

Sociocultural theories of mathematics learning extend a cognitive focus on individual learners, instead focusing on the participation of learners in social practices within a particular context (Danish & Gresalfi, 2018; Sfard, 1998). They inform our work by providing core assumptions about students' mathematical competencies and identities as constructed through interactions within context rather than predetermined, acontextual, and act as individuals' static traits (Ehrenfeld & Heyd-Metzuyanim, 2019; Gresalfi et al., 2009; Gresalfi & Hand, 2019). These theoretical assumptions often imply a tendency towards qualitative methodologies that allow a deeper look at how participants engage within their learning environments, navigating and reconciling the mathematical and social aspects of learning.

## Methodology

In this section we describe the data we collected along the year, including students' interviews and recordings of the CA training across two semesters (see Table 1 for a summary). We then describe how we reviewed and interpreted the data, with students' interviews being the primary data for this report.

### Data Collection

In Fall 2022, the study was approved by the IRB around week 7 of the semester. At this point we started to video record the last 7 weekly CA training sessions. In Spring 2023, we video recorded 13 of the 14 weekly CA training meetings in the Department of Mathematics. Finally, we interviewed nine students about their Math Lab experiences. The students were not chosen by us individually but rather answered our general call for interviews. Some of the students participated in the program in Fall 2022 and decided to register or not in Spring 2023 as well. Some of the students repeated their course with Math Lab at their second round and spoke about their experience in the same course with and without Math Lab. One student simply took both the course and the Math Lab for the first time in Spring 2023

*Table 1. Summary of data collected for this study.*

| Data for this study | | |
|---|---|---|
| Semester | Data | Including… |
| Fall 2022 | 7 (of 14) weekly CA meetings | Video + groups audio |
| Spring 2023 | 13 (of 14) weekly CA meetings | Video + groups audio |
| Spring 2023 | | Zoom interviews |

| | | |
|---|---|---|
| | End of semester interviews with 9 students | (primary data for this paper) |

**Data Analysis**

For the purposes of this paper we primarily reviewed the end of Spring 2023 semester interviews with the nine students. As secondary data we also used recordings of the weekly CA meetings where students reflected on their Math Lab facilitation. We collected codes and then themes (e.g., Charmaz, 2006; Strauss & Corbin, 1990), allowing our reading of the data to suggest the themes, rather than searching for themes identified a priori. In practice, this meant assigning initial codes to episodes where students and CAs were talking about aspects of Math Lab as more or less useful for the students. Note that after reviewing all interviews and about half of the CA training data we reached a point of saturation where codes repeated, and we almost did not encounter new ones. At this point we had an initial set of 10 codes under Strengths describing participant-identified dimensions of utility of the MathLab, and a set of 10 codes under Weaknesses describing less useful participant-identified aspects of Math Lab. In the final phase we merged and reduced each category of codes into three themes that were common in the data, significant for the students, and provoked generative points for this discussion.

## Themes Emerging From the Data

In this section we first describe and illustrate three strengths of the program from the students' and CAs' perspectives: the social support and network that Math Lab provided, the opportunities to develop mathematical competence and confidence through conversations about math problems, and the affordances of near-peer math support. These repeating themes sketch a collective image of what was found conducive to success in Calculus by the interviewed participants. All three themes illustrate how students found useful bringing together the social and academic dimension of mathematics learning. Some also contrast this interrelatedness with typical mathematics lectures and university life. Then, we describe and illustrate three problems, areas for revision, or less useful aspects of the program, from the students' and CAs' perspectives: The fact that it requires extra time in courses that are already dense and demanding, that students and CAs are not seeing the weekly Math Lab assignment as directly connected to the tests, and that for some, fully participating within the group was a challenge. Finally, we briefly discuss the implications of these themes, how they add to a collective image of needs, pressures, and valued support for success.

**Strengths of the Program and Things Students and CAs Found More Useful**

**Social support and networks.** Many students and CAs described their Math Lab groups as a friendly and fun environment, where positive relationships were established. For some, these relationships stayed within the scope of the Math Lab meetings. For example, Student 3 said that "our group like even though we didn't really talk outside of Math Lab we always had the really like friendly funny conversations during math lab we were always cracking jokes and everything." For others, those relationships extended onto the general course, or more broadly to social life on campus. For example, when asked what the highlight of Math Lab was for her, Student 4 said:

> [The highlight is] the people that you meet. Especially first semester just because like you know during that time everything is so stressful and especially if you don't know people in the class. And like maybe you are someone that struggles, [...] or like just like have

questions sometimes. I think that like being in the Math Lab group is important because you have like just a few- handful people you know you can reach out to like if you need help. And I feel like that is better because sometimes being like in a class with 30 students is little intimidating when you like want to speak to other people and ask where they are at, but the Math Lab group kind of makes it a little bit more like a social- like add a social aspect which I think is important especially when you are a first semester freshman...

All in all, often students and CAs saw their Math Lab groups as a friendly and welcoming environment, where positive bonds were created (sometimes contrasting it with other courses without Math Lab). These relations were of different type and scope: with some being more local in nature, just within Math Lab (as Student 1 described), some extended to the course classroom (as Student 6 described), and for some (like Student 4) those relations continued to students' ongoing academic and social campus life. Importantly, we know from the data and the literature that improvements in sense of belonging and deeper social connections in turn translate to better learning outcomes. The next theme further illustrates how students experienced the convergence of the social and the mathematical.

**Communicating about math and developing competence & confidence.** When we asked students what their Math Lab looked like, they often described positive experiences of doing math together, communicating and reasoning collaboratively about the content. Sometimes they also connected this type of math to their sense of confidence and competence, or to the advantages of explaining math out loud or being exposed to the ways other students are thinking about the weekly problems. For example, Student 2, who described herself as someone who "hates math" and wished she wouldn't have to take any math courses, told us that the best thing about Math Lab was "I think the most beneficial thing for me in Math Lab was like knowing that I can do these sort of problems, this is a good confidence thing..." In another example, Student 6 shared:

we would just go problem by problem and a lot of- we would be like "hmm: what did people like get for solutions on problem 1." And if there was like difference in opinions we would hmmm like go to the whiteboard and work out our work or show our work on paper or ipad and discuss and like discuss collaboratively about different methods for this solutions and what do we think we should put on the group doc we submitted.

Similarly, Student 8 shared how they were "talking it through" and her insight that "going through your line of reasoning could help you better understand how you look at math and how others look at math, and that can inform a more solid foundation…" as well as how sharing her thinking with the group and being validated by the CA was gave her a "confidence boost." Finally, Student 9 concluded his Math Lab experience with the insight that it is really good to have a dedicated time for groupwork because "sometimes you can learn more from your fellow students in a way that you can't from your instructor or the TA."

**Near-peer support.** While many students appreciated the opportunities to do math with their peers, the Math Lab experiences were different from a regular student learning group at least in one significant way: having their near-peer support. Near-peer support was discussed both in terms of the CAs, and sometimes also in terms of their upper-level Math Lab friends (e.g., freshmen and sophomores in the same groups). In addition, near-peer support was mostly discussed in terms of math support during Math Lab, but it also extended to other aspects of students' life like choosing courses and applying for summer jobs. Near-peers were also mentioned in contrast to TAs and to instructors, as being more approachable and as allowing to form closer mentoring relationships. The CAs reflections pointed out similar narratives. Capturing nicely the near-peer theme, Student 6 articulated the experience of being with a near-peer CA as being both “formal and informal,” She said: “My CA did a really good job about making it like formal but also informal at the same time. Kind of like just a different setting than the actual class which I think was helpful.” Describing the work with the same CA, student 8 explained the importance of the CA being someone who took this class recently:

> [The CA] was awesome she was really [unclear] and she understood the importance of the emotional support aspects, and like, someone who took the course and she is sophomore so she is like close to our age actually one of the kids in our lab group is older than her, and it felt like a peer relationship and she was very open if you guys need anything at all i'm always here for advice.

Others often mentioned their CAs for their math support (Student 1), their ability to get to know students and how they feel (Student 4), and their support with academic issues beyond math content (Student 3). Student 3, who is a student of color who had in the Fall semester a CA of color, described how the two are still in touch one semester after the course ended. She particularly appreciated how beyond the math assignments, the CA “loved to like… she was always talking about, like, doing internships and encouraging us to, like, apply ourselves outside the math lab and do different programs and stuff and so, um, yeah. She was awesome and we all realized that very early on.”

In sum, the near-peer support was a recurring motif in students’ and CAs’ descriptions of Math Lab, and this position seemed to offer some affordances that are different from the support typically provided by same-level peers, by TAs, and by instructors.

**Weaknesses of the Program: Things Students and CAs Found Problematic, Less Useful, or Requiring Revision**

**Math Lab takes extra time in already time-demanding courses.** With the Math Lab (and its weekly problems), the TA discussions, and the daily homework, Calculus students at the university have a relatively extended set of obligations. For example, the first thing that Student 1 told us about, was that she was surprised by the time demands of the calculus sequence. In the context of an already-demanding course, the Math Lab could seem like an added burden to students having an experience like that of Student 1.  Similarly, Student 6 explained that “for some students it can be hard to justify like showing up somewhere for another extra hour a week especially when we are all kind of like busy.” For Student 3, who was generally happy with her Math Lab experience in the Fall, time constraints were the main reason she decided not to continue with Math Lab for the Spring when it was not required. Just as Student 1, she explained at the very start of the interview that “math is a lot of work here, it's a lot of homework and I feel like I have math every single day with the math lab and lecture and discussion....” Then, when we asked why she decided on not going to Math Lab on MATH 1201 she added:

> Math is basically like everyday here with the lectures and discussions so I thought like if I didn't have to sign up for Math Lab to begin with, so maybe it would help my schedule just a little bit. Because I just remember last semester like trying to balance Math Lab with like clubs and everything was a lot of work so I thought like it would be better to allocate that time to something else, some organization or something else on campus.

As we illustrate in the next section, others questioned the usefulness of Math Lab because they did not see the direct connections between the weekly problems and the exams.

**Students and CAs are not seeing connectedness to exams.** The concern that the weekly Math Lab problems are not useful for students because they are not similar in nature to the daily homework question nor to the exam (e.g., longer, more conceptual, open ended, use tech like Desmos), came up across students' interviews and CAs' discussions. For example, when we discussed metacognition in the CA training, we asked the CAs when and how they were noticing students' frustration with the weekly problems. One CAs answered as follows.

> I feel like where I see the peak of frustration so like- my group they work on their homework as well, so like they go through the problem set and the homework because it is due the same day. And so their frustration is often times the weekly [Math Lab problem] is different than the [non Math Lab] homework, like significantly different. So in their minds it's like how is this benefiting me because I have to worry about my homework right now which is like my grade that has like a higher grade [weight], I see more of the correlation from this to my exam vs. like some of the... [...] they wouldn't be writing something like that [on the exam]. I guess it's gonna be more you're given a function and then you write the derivative something like that. So they get frustrated because they are comparing both of them [non Math Lab homework and weekly Math Lab problem] because they are doing it at the same time. I don't know how to explain like why the dynamic should be different because you don't do the same thing. That's not gonna benefit you.

These conversations were typical in the CA training where CAs described their students' perspective about the tasks, their own perspectives, and their concerns with regards to responding to students complaints. And while in some interviews with students we did hear some appreciation to the conceptual nature of the weekly assignments, students mostly expressed dismay with not seeing connectedness to exams. For example, when asked to elaborate on different aspects of Math Lab, Student 1 said:

> I feel like the actual Math Lab tasks were very- they felt very like disconnected at times from what we were actually learning. Because none of the stuff we did in Math Lab was never on any test like I feel like it always required some outside knowledge [...]

In sum, many MATH 1200, 1201 students and their CAs identified and struggled with the incoherence between the Math Lab and other course activities, including those who appreciated the opportunity to do math with peers and the CA and generally enjoyed their Math Lab. It is worthwhile to note that MATH 1300 students generally responded better to the open and conceptual nature of the problems, and more easily saw the connections between the Math Lab assignments and the course.

**Limited student engagement within the Math Lab group.** While we aspired to provide students with productive and inclusive mathematical work in small groups, this was not always exactly what played out in practice. Students and CAs often described less effective ways of working together (and not together) and encountered different forms of disengagement. Interpretations of disengagement ranged from deficit-oriented (e.g., a student described some students as "piggybacking" the group submission) to acknowledging that it is challenging to learn how to do math together, and it might require time and support for students to "put themselves out there."

We start by illustrating how students themselves pointed out certain group dynamics as troubling. Some students described them as a constant concern across the semester, and others as growth pains at the beginning of the semester. For example, when Student 2 was asked what makes a good Math Lab, she mentioned having regular attendance and that it should be clear that "if you are in a group you participate in doing the work." She then elaborated, expressing frustration that some students were “piggybacking” on the submission, showing up but not really participating. Student 8 shared a similar feeling. However in her case group dynamics shifted after the first 2-3 weeks:

> I was worried in the beginning like not everyone putting in effort but as we went on I think everyone started to see how valuable the assignments were and how like talking it through and going through your line of reasoning could help you [...] The first like two or three meetings it was kind of mostly 2 other girls and me sort of carrying stuff and everyone else were kind of silent. I don't know exactly why it changed but it did. And then everyone started contributing very equally and I felt more comfortable coming in with questions rather than answers.

The narrative of group growth across the semester towards more effective and inclusive participation repeated in interviews with the CAs at the end of Fall semester and in their end of year feedback. These accounts were typical in the CA training, and we encouraged CAs not to share just “success stories” but also everyday problems. Our hope was that they will develop sensitivity to notice non-productive and exclusionary patterns of student engagement, and will improve their capacity to respond to them.

## Conclusions and Implications

Students and CAs found several aspects in this program useful, including the development of social support networks, enhanced mathematical competence and confidence through collaborative problem-solving, and the value of near-peer support from CAs. These strengths illustrate that students found important the convergence of the social and academic dimensions of mathematics learning, which they often noted as missing in their undergraduate math journey. With regards to areas of revision, students felt that Math Lab added to the time demands of an already challenging course, and they struggled to see direct connections between Math Lab assignments and course exams. These themes could be interpreted either from the CA program perspective or from a general Calculus program perspective. From the CA program perspective, one attempt we made in the Spring to bridge for students the Math Lab weekly tasks with other course activities for students was to include some related homework/exam-like exercises with each conceptual lab problem that are rooted in the same mathematical concepts but are not for submission. From a general program perspective, if conceptual understanding is a goal of Calculus programs overall, building coherence across component of the course might also imply broader changes in the course. In that sense, the themes we presented are not only addressing students’ perspective on this specific CA program, but they also direct us to the general needs of undergraduate Calculus students. These findings highlight the value and complexities of implementing undergraduate Calculus collaborative math learning approaches that bridge the academic and social dimensions of mathematics learning.

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## References

Archie, T., Daly, D., Laursen, S. L., Hayward, C. N., & Yoshinobu, S. (2022). Investigating the effectiveness of short-duration workshops on uptake of inquiry-based learning. In Karunakaran, S. S., & Higgins, A. (Eds.). *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education*. Boston, MA.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis.* Thousand Oaks, CA: Sage.
Danish, J. A., & Gresalfi, M. (2018). Cognitive and Sociocultural Perspectives on Learning. In F. Fischer, C. E. Hmelo-Silver, S. R. Goldman, & P. Reimann (Eds.), *International Handbook of the Learning Sciences* (1st ed., pp. 34–43). Routledge.
Ehrenfeld, N. & Heyd-Metzuyanim, E. (2019). Intellective identities and the construction of a hybrid discourse: the case of an ultra-Orthodox Jewish mathematics classroom. *International Journal of Science and Mathematics Education, 17*, 739–757.
Ehrenfeld, N., & Horn, I. S. (2020). Initiation-entry-focus-exit and participation: a framework for understanding teacher groupwork monitoring routines. *Educational Studies in Mathematics, 103*, 251–272.
Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences, 111*(23), 8410-8415.
Gresalfi, M., Martin, T., Hand, V., & Greeno, J. (2009). Constructing competence: An analysis of student participation in the activity systems of mathematics classrooms. *Educational studies in mathematics, 70*, 49-70.
Ireland, J., Dorko, A., Cook, J. P., Tallman, M., Oehrtman, M., & Jaco, W. (2022). Conceptions of active learning held by undergraduate mathematics instructors participating in a statewide faculty development project. In Karunakaran, S. S., & Higgins, A. (Eds.). *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education*. Boston, MA.
Laursen, S. (2019*). Levers for change: An assessment of progress on changing STEM instruction.* Washington, DC: American Association for the Advancement of Science. https://www.aaas.org/resources/levers-change-assessment-progress-changing-stem-instruction
Laursen, S. L., Hassi, M.-L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry based learning in college mathematics: a multi-institution study. *Journal for Research in Mathematics Education, 45*(4), 406–418.
Rios, J. (2022). Active learning in the language diverse undergraduate math classroom. In Karunakaran, S. S., & Higgins, A. (Eds.). *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education. Boston*, MA.
Kerrigan, J., Tabanli, S., Cochran, G., Charnley, M., Mulvey, S. (2022). Transitioning active learning STEM recitations online. In Karunakaran, S. S., & Higgins, A. (Eds.). *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education*. Boston, MA.
Reinholz, D., Johnson, E., Andrews-Larson, C., Stone-Johnstone, A., Smith, J., Mullins, B., Fortune, N., Keene, K., & Shah, N. (2022). When active learning Is inequitable: Women's participation predicts gender inequities in mathematical performance. *Journal for Research in Mathematics Education, 53*(3), 204–226. https://doi.org/10.5951/jresematheduc-2020-0143.
Saxe, K., & Braddy, L. (2015). *A common vision for undergraduate mathematical sciences programs in 2025.* The Mathematical Association of America.
Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational researcher, 27*(2), 4-13.

Shah, N., & Lewis, C. M. (2019). Amplifying and attenuating inequity in collaborative learning: Toward an analytical framework. *Cognition and Instruction, 37*(4), 423-452.
Strauss, A., & Corbin, J. (1990). *Basics of qualitative research (Vol. 15).* Newbury Park, CA: Sage.
Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., ... & Grummer, J. A. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences, 117*(12), 6476-6483.