Is it “fast enough?”: Two undergraduate students’ guided reinvention of the comparison test

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Series convergence is a key part of the calculus curriculum. Much of the related education research has focused on students’ understandings of sequences and series (Earls, 2017; Eckman & Roh, 2022; Askgun & Duru, 2007; Kung & Speer, 2013; Martínez-Planell et al., 2012), the notation (Larsen et al., 2022), or textbook analysis (González-Martín, 2010). However, there is limited research on ways to support students’ conjecturing about series convergence.

Our data comes from a teaching experiment (Steffe & Thompson, 2000) with two post-Calculus I students, Lara and Stella. Instruction during this experiment was informed by Realistic Mathematics Education’s guided reinvention heuristic (Freudenthal, 2005; Gravemeijer & Doorman, 1999). Here, we focus on the last four (of 11) sessions in which the students were guided to reinvent statements about series convergence. During these sessions, the students played a game in which they imagined moving an object a certain number of feet each day. A sequence $\{x_n\}$ indicated how many feet x_n to move the object on any given day n. Winning the game required predicting the object’s location if the experiment continued indefinitely (i.e., $\sum x_n$ converged and the students reported the value that it converged to). After playing the game with various sequences, the students wrote a “cheat sheet” with tips for which sequences win/lose the game and some warnings for future players. We interpreted much of their cheat sheet as common tests for series convergence (i.e., p-series test, comparison test). For this study, we investigated: *How did two undergraduate students reinvent the comparison test?* We explored how their version of the comparison test emerged from their previous informal ideas by rewatching videos and re-reading transcripts to identify relevant key moments.

Stella and Lara’s reinvention was tied to their understanding of the sequences n^{-1} and n^{-2} and their corresponding series. Using speed as a metaphor, they identified that a winning sequence must converge to zero at a certain “rate” using n^{-2} and n^{-1} as their benchmarks for a sequence which decreased “fast enough” or “too slowly,” respectively. The teacher-researcher then graphed the sequence n^{-1} and a generic sequence that was greater than n^{-1} for all n, asking the students if the generic sequence would win. Lara quickly inferred that it would lose, and Stella explained it converges to zero “at a slower rate” than n^{-1}. They formalized this on their cheat sheet as “if a sequence is always above the sequence n^{-1} it will lose the game” (which we interpret as a specific case of the comparison test). They compared other sequences to n^{-1} and were unsure whether sequences in between n^{-1} and n^{-2} won the game since they decreased “faster” than n^{-1} but “slower” than n^{-2}. To find clarity, they played (and lost) the game with the sequence $(2n)^{-1}$. They generalized this with two warnings: “sequences that are below n^{-1} will not necessarily win the game” and “sequences above n^{-2} might lose,” which we interpret as the comparison test’s possible inconclusiveness. Finally, with the teacher-researcher’s guidance, the two students considered their benchmark sequences (n^{-1}, n^{-2}) as hypothetical losing and winning sequences, and this encouraged them to write the conjecture which we interpret as (part of) the comparison test: “any sequence that is below a known winning sequence but above 0 will win.”

Our study is an existence proof of two students being guided to reinvent the comparison test, showing that students can be supported to conjecture tests for series convergence. In the poster session, we will elaborate on our data and discuss our future work.

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