Students' Attitudes and Perseverance in Problem-solving in Undergraduate Precalculus

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Keywords: precalculus, curriculum enactment, attitudes, perseverance in problem-solving

This exploratory study investigated the relationships between professors' enactments of a research-based precalculus curriculum and changes observed in students' attitudes towards mathematics and perseverance in problem-solving. While much research focuses on improving student achievement in undergraduate STEM courses, we must also support them in developing the positive dispositions and practices needed to sustain them through years of mathematics-based STEM coursework (Bressoud et al., 2015). When students hold positive attitudes towards mathematics, they are more likely to both succeed in calculus (Sonnert et al., 2020) and persist in completing a STEM degree (Wu et al., 2022). When students persevere in their problem-solving amidst challenges, they make meaning of mathematics and develop productive dispositional factors for STEM majors, including building resilience for overcoming setbacks (Middleton et al., 2015). The Pathways precalculus curriculum (Carlson et al., 2021) leverages research on student thinking to support them in developing problem-solving skills and in making connections across concepts (Moore & Carlson, 2012), and has been tied to improvement in students' covariational reasoning and subsequent success in calculus (McNicholl et al., 2021). However, since curriculum and its enactment can transform students' opportunities to learn in various ways (Stein et al., 2007), including outcomes related to disposition and mathematical practices (DiNapoli & Morales, Jr., 2021; Ruthven, 2011), we chose to focus on the relationships between different enactments of Pathways and students' attitudes and perseverance measures.

Three precalculus professors (Profs. A, B, and C) and 33 of their students participated in this study. Two classroom observations for each professor were video recorded and transcribed. The professors' pedagogical choices were scored on a 0-3 scale along three Pathways-aligned dimensions: support of student problem-solving, understanding and advancing student thinking, and making connections (rubric adapted from Schoenfeld et al., 2014). Student participants completed an Attitudes Towards Mathematics Inventory (ATMI; Tapia & Marsh, 2004) pre- and post-survey, which measured students' mathematical self-confidence, value, enjoyment, and motivation. Student participants also engaged in 12 video-recorded problem-solving sessions. Students' perseverance was measured using the Three-Phase Perseverance framework (3PP; DiNapoli & Miller, 2022), which considered the extent to which students initiated and sustained, and re-initiated and re-sustained upon impasse, productive struggle on a challenging task.

Profs. A, B, and C implemented Pathways in different ways along the dimensions analyzed; over the two observations, Prof. A's scores were almost uniformly low, Prof. B's consistently high, and Prof. C's scores were medium-to-high. Each professor's students exhibited changes in attitudes: Prof A's students shifted towards more negative attitudes across the semester, whereas Profs. B and C's students exhibited positive attitude shifts. Finally, students in all three classes averaged positive 3PP slope scores, indicating some perseverance growth for all, yet Prof. B and C's students experienced about 5 and 4 times the growth, respectively, compared to Prof. A's students. More detailed qualitative and quantitative evidence of our findings will be shared in our presentation. These findings suggest that although the Pathways precalculus curriculum may support the development of positive attitudes toward mathematics and improved perseverance in problem-solving, this potential is influenced by professors' pedagogical choices.

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This project was supported by a grant from the State of New Jersey

Introduction and Rationale

- Student achievement in precalculus is necessary, but not sufficient
  - **Positive attitudes towards mathematics** help students succeed in calculus and persist in completing a STEM degree (Sonnert et al., 2020; Wu et al., 2022).
  - **Persevering in problem-solving** helps students make meaning of mathematics and build resilience for overcoming setbacks (Middleton et al., 2015).
- The **Precalculus: Pathways to Calculus** curriculum has the **potential** to support students in developing positive attitudes and improving their perseverance
  - Content development embedded in **rich, real-world contexts**
    - Reduces **math anxiety** (Karunakaran, 2020)
    - Supports students' value of mathematics (Aikens et al., 2021)
  - Focus on developing students' **problem-solving skills** and **covariational reasoning**
    - Increases **enjoyment** (Schettino, 2016) and **motivation** (Peterson, 2019)
    - Supports **perseverance** development (Barnes, 2019; DiNapoli & Miller, 2022)
  - **Scaffolded questioning** based on research on how students make sense of precalculus concepts (e.g., Frank, 2017; Kuper, 2019; Moore, 2010)
    - Improve students' mathematical **self-efficacy** (Jatisunda, 2020)
    - Helps students initiate and sustain **productive struggle** (Reiser & Tabak, 2014), and thus, their **perseverance** at times of impasse (DiNapoli & Miller, 2022)
- Curriculum enactments can differentially impact
  - Students' **opportunities to learn** (Stein et al., 2007)
  - Students' **attitudes** towards mathematics (Ruthven, 2011)
  - Students' improvement in **perseverance** in problem-solving (DiNapoli & Miller, 2022)

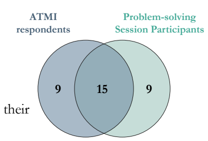
Research Question

What are the relationships between professors' enactments of a research-based precalculus curriculum and changes observed in students' attitudes towards mathematics and perseverance in problem-solving?

Methods

Context and Participants

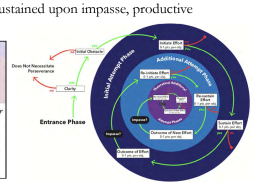
- Public research university in the Northeast U. S.
  - ~23,000 students, HSI, R2: Doctoral University
- Coordinated, entry-level precalculus course
  - ~40 sections and ~1200 students per year, optional PD
- Three precalculus professors (Profs. A, B, and C) and 33 of their students participated.



Data Collection and Analysis

- Two **classroom observations** for each professor were video recorded and transcribed.
  - The professors' pedagogical choices were scored on a 0-3 scale along three Pathways-aligned dimensions: support of student problem-solving, understanding and advancing student thinking, and making connections (Schoenfeld et al., 2014).
- Student participants completed an **Attitudes Towards Mathematics Inventory** (ATMI; Tapia & Marsh, 2004) pre- and post-survey, which measured students' mathematical self-confidence, value, enjoyment, and motivation.
- Student participants also engaged in 12 video-recorded **perseverance in problem-solving sessions**.
  - Students' perseverance was measured using the Three-Phase Perseverance framework (3PP; DiNapoli & Miller, 2022), which considered the extent to which students initiated and sustained, and re-initiated and re-sustained upon impasse, productive struggle on a challenging task.

Pathways Implementation Observation Rubric		
Engaging and Supporting Students in Problem-solving	Understanding and Advancing Students' Thinking	Providing Opportunities for Students to Make Connections
To what extent do classroom activities actively engage students in problem-solving? To what extent does the professor discuss, model, and encourage productive problem-solving practices?	*To what extent does the professor endeavor to understand students' thinking about content and respond in ways that help them to construct more productive understandings?*	*To what extent does the professor support students in making connections: to real-world contexts, across course content, and between functional representations?*



Results

Classroom Observations

		Pathways Implementation (0 to 3 point scale)		
		Engaging and Supporting Students in Problem-solving	Understanding and Advancing Students' Thinking	Providing Opportunities for Students to Make Connections
Prof A	Obs 1	1.00	1.40	2.10
	Obs 2	1.00	1.14	1.70
Prof B	Obs 1	2.25	2.17	2.38
	Obs 2	2.00	2.31	2.40
Prof C	Obs 1	2.20	2.14	2.40
	Obs 2	1.70	1.97	1.40

Classroom Observation Findings

- Prof A had somewhat lower scores across the board compared to Profs B and C, particularly in the Problem-Solving category.
- Prof B's scores were consistent across all categories and were generally highest.
- Prof C's scores were mixed, a bit higher for their first observation than their second.

Note: Maintaining a level 3 in all three categories would be impossible.

Examples of Understanding and Advancing Student Thinking

Prof A's Level 1 Interaction

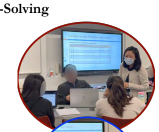
Prof A: As my angle gets bigger, if I measured this, what's this distance relative to the one down here, bigger or smaller?
Student: Smaller.
Prof A: Smaller. … And now as I move further, between pi over two and pi, this distance here is getting bigger because it's moving away. But because it's on the left-hand side, is it positive or negative?
Student: Negative.
Prof A: Negative…

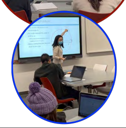
Prof B's Level 3 Interaction

Prof B: So, I know the 10-hour decay factor [is .2722], how can I use [this] to find the five-hour decay factor?
Student 1: You could try and divide it by half…
Student 2: Square root of .2722?
Prof B: Ah. We should explore these options … [Student 1], would you mind telling me your reasoning … for this?
Student 1: Well, since it's a 10-hour decay, and they ask you for like the five-hour growth decay. I just thought if we just divided by two.

Prof B: [Student 2], what's your reasoning for this?
Student 2: Because the five-hour times the five- hour would be the 10 hour.
Prof B: [Takes a class poll, which is split]
So, I want to talk about how we can use our intuition to maybe rule this out a little bit. …if [.2722 divided by 2] was the five-hour growth or decay factor, this is … .1372… in five hours, I only have about 14% [of the caffeine] left in my system. But in 10 hours, I have more than that left in my system. That feels weird, right?

Examples of Engaging and Supporting Students in Problem-Solving

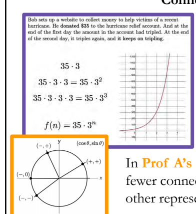




In **Prof C's first observation**, students spent the majority of class working in small groups with Prof B circulating around the room in a supportive role.

In **Prof C's second observation**, they spent more time at the board delivering content.

Examples of Providing Students with Opportunities to Make Connections



In **Prof A's first observation**, they developed the exponential function formula using a real-world context, connected symbolic and graphical representations, and contrasted exponential and linear growth.

In **Prof A's second observation**, they made fewer connections to real-world contexts, other representations, and other content.

Attitudes Towards Mathematics and Perseverance in Problem-Solving

	Average ATMI Changes (Post – Pre, 5-point Likert Scale)					3PP Slope (Avg increase per session)
	Self-Confidence	Value	Enjoyment	Motivation	Overall Shift	
Prof A	0.00	-0.31	-0.17	-0.25	-0.15	0.09
Prof B	0.17	0.37	0.10	0.20	0.21	0.44
Prof C	0.72	0.16	0.30	0.20	0.41	0.34

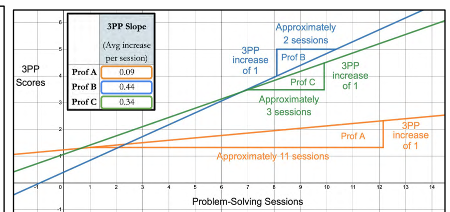
Attitudes Findings

Small shifts in ATMI outcomes:

- Prof A's students: mostly negative shifts in attitudes
- Profs B's students: mostly positive, particularly in value
- Prof C's students: mostly positive, particularly in self-confidence.

Perseverance Findings

- All students showed perseverance growth
  - Prof B & C's 3PP growth ~4-5 times greater than Prof A's
- To show 1 point of 3PP growth:
  - Prof A: ~11 problem-solving sessions
  - Prof B: ~2 problem-solving sessions
  - Prof C: ~3 problem-solving sessions
- Interpreting 1 point of 3PP growth, e.g.:
  - 0→1: none → some effort
  - 2→3: sustaining → productive effort
  - 3→4: first → second effort upon impasse



Discussion

In this early exploratory study, we were able to see some potential connections between different ways of implementing Pathways and students' changes in attitudes towards mathematics and perseverance in problem solving, and many of these perceived connections are consistent with the literature.

- We saw evidence of the **links between actively engaging students in problem-solving and improvements in attitudes and perseverance in problem-solving** (Barnes, 2019; DiNapoli & Miller, 2022; LeSage et al., 2021; Peterson, 2019; Schettino, 2016; Schoenfeld, 1992).

	Problem Solving		ATMI Changes	3PP Slope
Prof A	Obs 1	1.00	-0.15	0.09
	Obs 2	1.00		
Prof B	Obs 1	2.25	0.21	0.44
	Obs 2	2.00		
Prof C	Obs 1	2.20	0.41	0.34
	Obs 2	1.70		

  - Prof A's lack of problem-solving pedagogies were due, in part, to COVID concerns.
  - Prof B's best problem-solving pedagogies involved leading interactive, whole-class discussions.
  - Prof C's best problem-solving pedagogies involved small group work.
- We saw evidence of the **links between understanding and advancing student thinking to improve students' self-confidence, motivation, and perseverance in problem-solving** (DiNapoli & Miller, 2022; Harper et al., 2019; Jatisunda, 2020; Reiser & Tabak, 2014).
  - Valuing student thinking in class can help them develop their mathematical agency.
  - Students seeing how their in-progress thinking can lead to mathematical productivity in class can reinforce the value of productive struggle.

	Student Thinking		Self-Confidence	Motivation	3PP Slope
Prof A	Obs 1	1.40	0.00	-0.25	0.09
	Obs 2	1.14			
Prof B	Obs 1	2.17	0.17	0.20	0.44
	Obs 2	2.31			
Prof C	Obs 1	2.14	0.72	0.20	0.34
	Obs 2	1.97			

- We saw evidence of the **links between providing students opportunities to make connections between content and real-world contexts to their perception of the value of mathematics** (Aikens et al., 2021).

	Making Connections		Value of Math
Prof A	Obs 1	2.10	-0.31
	Obs 2	1.70	
Prof B	Obs 1	2.38	0.37
	Obs 2	2.40	
Prof C	Obs 1	2.40	0.16
	Obs 2	1.40	

  - Whether the professor or the students make connections may influence students' value of math.
  - Prof A and Prof C scored similarly, but Prof C more often provided students with opportunities to make these connections for themselves.
  - Prof B often used real-world scenarios suggested by students to create contextualized problems for in-class discussion.

Main Takeaway of Research #betterposter

This research suggests that although the Pathways precalculus curriculum may support the development of positive attitudes toward mathematics and improved perseverance in problem-solving, this potential is influenced by professors' pedagogical choices.

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