The College Mathematics Beliefs and Belonging Survey: Instrument Development and Validation

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This contributed report describes the development and validation of a new measure—the College Mathematics Beliefs and Belonging (CMBB) survey. The CMBB provides a contemporary measurement of undergraduate students' perceptions of their mathematical proficiency and reasoning, beliefs about mathematics, and sense of belonging in mathematics. Primarily first- and second-year undergraduate students in five courses at a large public university in the United States completed multiple surveys to provide the data used for survey development. Confirmatory factor analysis (N = 935) and a reliability analysis indicate that the CMBB is a survey with fifteen factors that adequately measure various aspects of perceived mathematical proficiency and reasoning, beliefs, and sense of belonging. The CMBB survey is intended for use by both researchers and instructors to assess undergraduate students' perceptions across these three domains with the aim of improving students' experiences in college mathematics courses.

Keywords: proficiency, mathematical reasoning, beliefs, belonging, factor analysis

For decades, mathematicians, mathematics educators, and psychologists have explored individual differences and instructional factors that affect students' achievement and motivation in postsecondary mathematics. There is a long tradition of postsecondary mathematics instructors identifying habits, beliefs, and abilities that are viewed as critical for effective mathematical practice. This collection of habits, beliefs, and abilities is sometimes referred to collectively as "mathematical maturity" (Faulkner et al., 2019; Garrity, 2011; Lew, 2019; Steen, 1983). In a first step toward delineating specific key habits, abilities, and beliefs, mathematicians engaged in scholarly teaching have produced practitioner-focused reports documenting developmental goals for postsecondary mathematics students, often through the work of professional societies such as the Mathematical Association of America (MAA). The MAA Committee for the Undergraduate Program in Mathematics has long been active in this work, producing the series of Curriculum Guides to Majors in the Mathematical Sciences (MAA, 2015) and The Curriculum Foundations Project (Ganter & Barker, 2004), created by the Curriculum Renewal Across the First Two Years subcommittee. This work has been complemented by a growing body of research in undergraduate mathematics education involving a range of themes including student thinking, student-instructor interaction, and effective pedagogy, among others (e.g., Bressoud et al., 2015; Carlson & Rasmussen, 2008; Laursen & Rasmussen, 2019).

Psychologists have also pursued parallel lines of investigation. Psychological research has revealed that students' persistence in STEM courses is highly influenced by their beliefs about math, perceptions of their own learning, and their academic motivation more broadly (e.g., Hulleman & Harackiewicz, 2009; see Eccles & Wigfield, 2020, for a review). However, many existing measures of students' beliefs, motivation, and sense of belonging in mathematics based on psychological research have paid insufficient attention to how mathematicians and mathematics educators conceptualize student learning in the context of college mathematics.

The goal of the current investigation was to develop an instrument to measure college students' beliefs about mathematics, and beliefs about their own mathematics learning, that have been identified as effective for mathematics practice by scholarship, research, and theory from multiple communities of practice. The *College Mathematics Beliefs and Belonging* (CMBB) survey can be used by researchers and instructors to assess student attitudes and beliefs in undergraduate mathematics courses, including students' views about their own mathematical proficiency, their beliefs about the nature of mathematics, aspects of what motivates them in mathematics learning, and multiple facets of their sense of belonging in mathematics. By considering the contributions of multiple communities of practice, we believe that the CMBB survey is representative of a wide range of perspectives that reflect the increasing diversity of undergraduate students taking introductory mathematics courses at the postsecondary level.

Theoretical Frameworks

Mathematical Proficiency and Reasoning

First, we sought to capture student individual differences in theoretically meaningful aspects of their mathematical proficiency and reasoning. One influential framework is the 5-strand model from the National Research Council's (NRC) "Adding It Up: Helping Children Learn Mathematics" report (Kilpatrick et al., 2001). The 5-strand model is comprised of five theoretically unique, but related, aspects of students' knowledge: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. The NRC Committee proposes that these five strands of mathematical proficiency and reasoning are separable yet interwoven constructs, with each supporting the others and productive disposition bringing together all strands. This view is also aligned by psychological research on conceptual and procedural understanding in mathematics (e.g., Rittle-Johnson & Siegler, 2021) and academic motivation (e.g., Eccles & Wigfield, 2020).

In addition to these five strands, we reintroduce Gray and Tall's (1994) conceptualization of proceptual thinking, a separate construct that focuses on the fundamental integration between understanding a procedural process and understanding conceptual mathematical relationships. Proceptual thinking reflects a key aspect of how mathematicians perceive mathematical symbols. Gray and Tall argue that mathematicians' perception of symbols includes understanding processes that a symbol represents, the mathematical object a process produces, and an understanding of the symbol flexible enough to include both the processes and object.

Beliefs About Mathematics

Postsecondary students enter college with a range of beliefs about the nature of mathematics (see Philipp, 2007). For example, some students think of mathematics as a collection of relatively independent rules and procedures (Schifter, 1990). Another common belief is that mathematics is fundamentally a form of logical thinking (Dossey, 1992). Finally, some students consider efficiency to be a key characteristic of doing mathematics (Boaler et al., 2015). Many of these beliefs held by students do not accurately reflect mathematicians' conceptions of mathematics (Gold et al., 2017; Hersh & John-Steiner, 2010). Thus, we aimed for our contemporary scale to reliably assess how undergraduate students conceptualize the nature of mathematics.

Students also possess motivational beliefs about mathematics (Kloosterman, 2002; Royster et al., 1999), including the usefulness of mathematics (utility value; Eccles & Wigfield, 2020), confidence in their own mathematical proficiency (self-efficacy; Bandura, 1997), and their own and others' abilities to learn and understand mathematics (growth mindset; Yeager et al., 2019). Importantly, motivational beliefs often uniquely predict academic outcomes, over and above mere content knowledge. For example, classroom-based experimental studies reveal that when students can see the utility of their college STEM classes, they are more likely to persist in STEM majors years later (Asher et al., 2023). Additionally, mathematical confidence is a driving factor in persistence in college mathematics, especially among women (Ellis et al., 2016). Finally, students who understand that mistakes are a part of learning are more likely to persist in advanced mathematics and earn higher grades in mathematics courses (Yeager et al., 2019).

Given these considerations, our view is that understanding the multidimensional nature of student beliefs of mathematics is pivotal to fostering student learning, understanding differences between groups of students (e.g., gender differences) within a course, and assessing the impact of different pedagogical approaches. Thus, we aimed to include a broad range of belief-oriented items—encompassing both beliefs about the nature of mathematics and motivational beliefs about mathematics—in our survey.

Centering Equity and Inclusion through Sense of Belonging

A growing body of work has adopted sociocultural frameworks that acknowledge the importance of the broader learning context—such as belonging—in shaping student learning and engagement (Allen et al., 2016). Sense of belonging is defined as "the feeling that a member fits in, belongs to, or is a member of the academic community in question" (Good et al., 2012, p. 700). Among college students, sense of belonging affects mathematics motivation differently for different students. For example, Good and colleagues (2012) found that when women experienced a decline in sense of belonging across the academic year, they were less likely to intend to enroll in additional courses in mathematics.

There is also substantial overlap between sense of belonging and research regarding inclusive and equitable teaching practices and the development of supportive learning communities. Leyva et al. (2022) analyzed Black and Latine students' perceptions of practices in calculus instruction intended to be supportive for all students, finding that these were generally perceived as necessary but insufficient to create equitable opportunities and content access. Following reflection and re-evaluation of data from the MAA's National Studies of College Calculus, Hagman (2019) articulated that diversity, inclusion, and equity practices constituted a critical factor for student success; this factor was not considered explicitly in the original findings of the MAA studies. Further, many mathematicians and mathematics educators have reported on their efforts to build inclusive and welcoming communities within mathematical contexts (Cunningham et al., 2021; Hardin & Shahriari, 2022; Karaali, 2022).

Initial Instrument Development

Our instrument development process unfolded in two distinct stages. First, we developed a set of preliminary survey items based on an extensive literature review, piloted the survey with a small sample of students, and conducted a preliminary analysis to assess the factor structure of these survey items. Second, based on results of our preliminary analysis, we revised the survey items and collected data to assess the validity and reliability of the measures.

Item Development and Pilot Data Analysis

The 5-strand model combined with proceptual reasoning combine to provide six strands of proficiency: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, Productive Disposition, and Proceptual Thinking. For each of these strands, the two lead authors wrote specific items guided by both the construct definitions and the specific content knowledge that students in undergraduate mathematics courses should master, with a focus on courses taken by first-year students. All items were designed as Likert-type items on a 1–6 scale (strongly disagree to strongly agree, with no neutral option). All items in the initial version of the survey are available on our Open Science Framework (OSF) page https://osf.io/683ek/?view_only=cadf80e3935d41caa88699d9f1f3e4a8.

We also sought to characterize students' feelings of belonging, membership, and acceptance in the mathematics community (Good et al., 2012). Items in this section were modeled after Good et al.'s (2012) scale of sense of belonging in mathematics, questions asked by Piatek-Jimenez (2008) in their interviews of women in mathematical careers, and themes described in Oppland-Cordell and Martin (2015). The remaining items were written by the two lead authors to capture students' feelings of belonging and community when working with peers in the classroom and students' own identification with mathematics.

In Summer 2022, we conducted an Exploratory Factor Analysis (EFA) on the initial survey data. The EFA showed that we should remove most reverse-coded items and clarify the item questions regarding beliefs about mathematics. The EFA for the first version of our survey did show that there were two factors clearly loading on proficiency/beliefs and belonging; thus, we determined it was appropriate to revise the survey and collect new data. The final survey items are available on our OSF page.

Reliability and Validity Study on Final Survey Items

To establish reliability and validity, we adopted the approach set forth in the Standards for Educational and Psychological Measurement in Education (AERA, APA, & NCME, 2014) in which the importance is placed on gathering multiple sources of validity evidence relevant to the interpretation of an instrument. The Standards recommend considering five types of validity evidence: test content, response processes, internal structure, relation to other variables, and consequences of testing. Validity evidence from item content was addressed by drawing on literature in the content domains of interest and by the composition of a multidisciplinary team of content experts. Our multidisciplinary team of mathematicians, cognitive developmental psychologists, mathematics educators, and an engineer required several discussions about test content as we aimed to have a shared understanding of how the items informed each scale and the overall purpose of the survey from our multiple perspectives.

Validity Evidence: Response Process and Test Content

Six undergraduate college students were recruited for 30- to 45-minute cognitive interviews in Fall 2022. A cognitive interview is an interview procedure meant to explore a participant's comprehension of an item or task (Leighton, 2017). When participants articulate their thoughts in response to an item survey (i.e., a "talk-aloud" concurrent verbal report; see Ericsson & Simon, 1993), researchers can determine if their interpretation of an item matches the construct it was designed to measure. The interviews indicated that college students interpreted most of the items as they were intentionally designed.

Reliability and Validity Evidence: Internal Structure

To provide validity evidence for internal structure, we conducted independent analyses on the data we collected at the beginning and at the end of the Fall 2022 semester. In one dataset, we conducted an exploratory factor analysis (EFA). The EFA was used to guide a confirmatory factor analysis (CFA) on a second dataset.

Revised survey administration. We administered the CMBB survey to students in five university mathematics courses (Contemporary Mathematics, College Algebra, Precalculus, Calculus I, Calculus III) that primarily serve first- and second-year undergraduates. The surveys were administered using an online survey software (Qualtrics); they were advertised and distributed by course instructors on our behalf.

Participants. After removing participants who did not pass an attention check item or had less than 30% survey response completion, our analytic sample included 1,135 students who completed the pre-survey and 935 students who completed the post-survey. Approximately half of the students in the pre-survey (49%) and the post-survey (53%) reported currently being in a mathematics-intensive major. More women than men completed the survey: 56–57% women, 34% men, 0.2–0.4% transgender, 1% nonbinary, and 5–6% other. In both the pre-survey and post-survey, the racial and ethnic distribution of the data was highly similar with 80–82% reporting as White, 8–10% Black/African American, 1% American Indian or Alaskan Native, 5% Asian, 0.5% Native Hawaiian and Other Pacific Islander, and 5% reporting as other. In both the pre-survey and post-survey, the first-generation student composition is representative of the university at which the data were collected, with 26–27% of the students reporting that their parents do not hold at least a 4-year college degree.

Exploratory factor analysis. First, we carried out a parallel analysis leading to an EFA on the pre-survey data. Computations were done in RStudio version 2022.07.1+554 using the base R package. The parallel analysis suggested an 18-factor model, where this large number of factors aligned with the design of the instrument, therefore orthogonal (varimax) rotation was used in the EFA to minimize correlations between factors. In the resulting EFA with 18 factors, two of the factors had no items load above .3, and one of the factors had only one item load at .31. Combining the EFA results with validity evidence considerations, we proceeded with a 15-factor model on 56 items. The loading matrix and item correlations for the EFA analysis are available on our OSF page.

Confirmatory factor analysis. Next, to confirm that our hypothesized 15-factor model fit the data, we conducted a CFA on the post-data using maximum likelihood estimation with the *lavaan* package version 0.6-12. The CFA model is represented in Figure 1. To interpret our goodness-of-fit indicators, CFI and RMSEA, we used equivalence testing to establish comparison and cutoff values (Marcoulides & Yuan, 2016). Using equivalence-testing based cutoffs, we find that an RMSEA of .047 is identified as a “Close” fit while a CFI of .92 is identified as a “Fair” fit (our T-size RMSEA and CFI values were .048 and .91, respectively). Thus, we conclude from our equivalence testing that our 15-factor model is a good fit for the data. Note that the subscales within each factor are also aligned with the constructs in our literature review.

Reliability and relationships among factor scores. To assess reliability, we examined Cronbach’s alpha for items within each subscale (factor). Alpha scores ranged from .74–.95 with one scale having an alpha of .66. Thus, most subscales had high internal consistency. To provide additional validity evidence, we calculated correlations between the item subscales associated with each factor (see Figure 2).

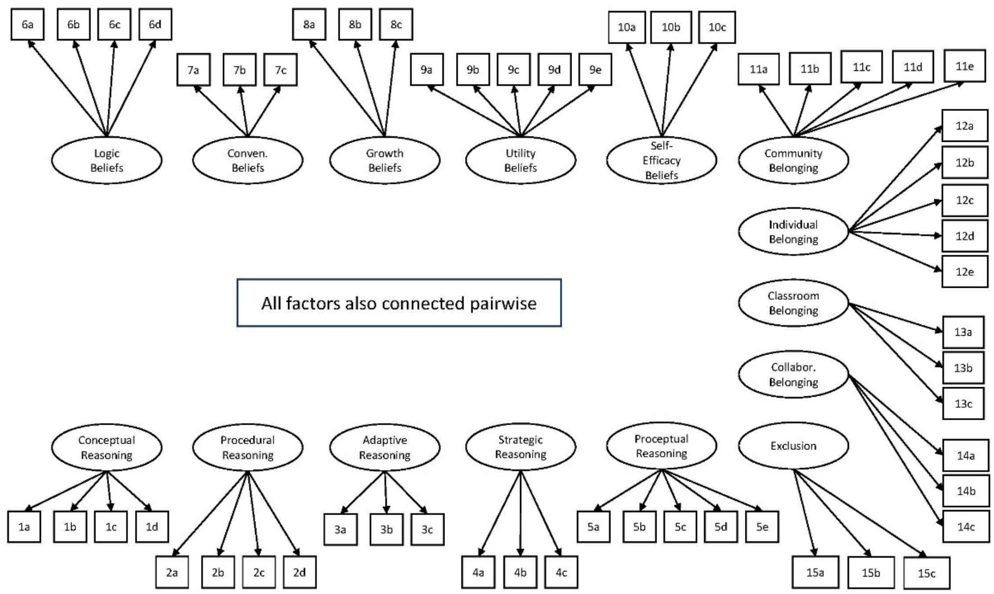

Figure 1. This graphic presents our CFA model, including the 15 factors and the items that load onto them.

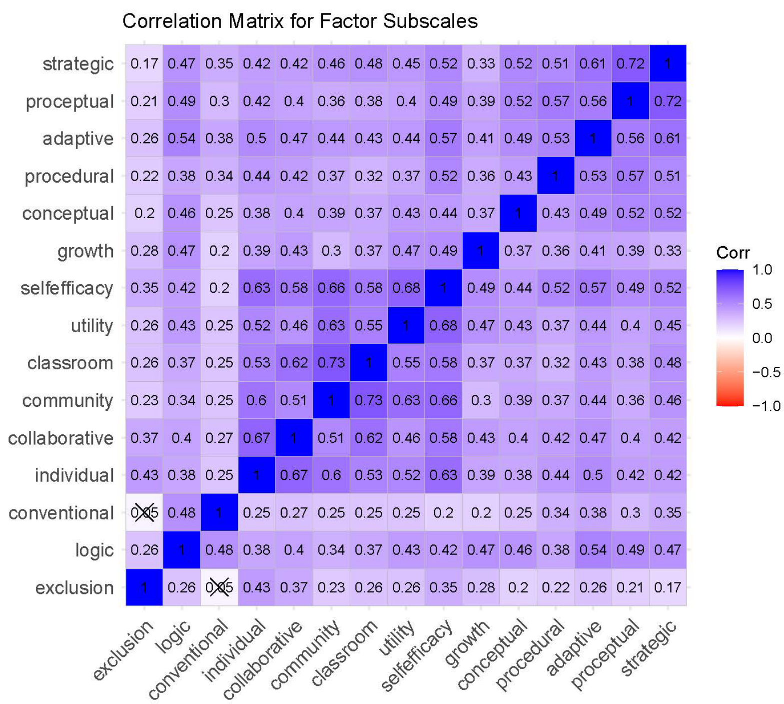

Figure 2. Correlation matrix for factor subscales. All correlations are significant at the alpha = .05 level except those indicated with an X.

Importantly, our analysis reveals strong correlations among items tapping components of belonging (Community, Individual, Classroom, Collaborative), suggesting that these subscales do, indeed, measure interrelated aspects of students' sense of belonging in mathematics. In contrast, belonging subscales are less strongly correlated with students' beliefs about the nature of mathematics and their self-perceived proficiency. Likewise, subscales that measure students' self-perceived proficiency (Conceptual, Procedural, Strategic, Adaptive, Proceptual) are strongly related to each other as well as the Self-Efficacy subscale and the Logic beliefs subscale. In sum, the general pattern of correlations across subscales reflects our expected theoretical structure.

Discussion and Conclusions

Researchers and educators interested in understanding how instructional factors in postsecondary mathematics education shape students' beliefs about mathematics and their own approaches to mathematics need a valid and reliable tool for measuring these beliefs. Our team successfully developed a contemporary instrument, the College Mathematics Beliefs and Belonging (CMBB) survey, to assess a variety of related theoretically meaningful constructs that capture various students' experiences in college mathematics courses. The survey assesses students' self-perceived proficiency in a way that is aligned with theory and prior research delineating the multiple facets of student reasoning that are crucial for engaging in college mathematics. The survey also measures a variety of beliefs, including students' beliefs about the nature of mathematics, the utility of the mathematics they are learning, their own abilities to learn mathematics, and the possibilities for growth in mathematical knowledge. In sum, the College Mathematics Beliefs and Belonging (CMBB) survey captures meaningful variability in students' beliefs and is a valid tool for gathering information about college students' beliefs and sense of belonging in mathematics. It is our hope that postsecondary mathematics instructors and education researchers can use this tool to examine and improve students' experiences in mathematics courses.

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