Online Real Analysis for In-Service Teachers' Meanings for Continuity and Differentiability

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In this poster presentation, we present pilot data that emerged from working with secondary math teachers enrolled in a fully online, Introductory Real Analysis course in the Southeastern United States. This pilot data will inform our second attempt at implementing the recommendations of the Project ULTRA (Wasserman et al., 2022) research group in an online context with in-service teachers in a future academic semester.

Keywords: Secondary Teachers, Real Analysis, Continuity, Differentiability, Online

Calculus, and the related mathematical content, is one of the most important, foundational areas of mathematics for people intending to pursue careers in STEM (Bressoud et al., 2016; Rasmussen et al., 2019). Teachers serve a crucial role in the community by providing students with early and continued education about mathematical ideas which build throughout students' K-12 education, up to and including AP Calculus courses (Ball, 2003; Bressoud et al., 2016; Frank & Thompson, 2021). Understanding how in-service teachers develop their own mathematical understanding of ideas related to Real Analysis topics will therefore provide important insights to researchers about the development of teachers' mathematical meanings (Thompson, 2016), or mathematical knowledge, for teaching calculus (Ball, 1990; Hill et al., 2008). Several researchers have reported on students' understanding of mathematical logic in the context of limit and the Intermediate Value Theorem (Roh, 2010; Sellers et al., 2017), and the format of proof-based mathematics courses as well as secondary teachers' views of the utility if proof in their own teaching (Wasserman et al., 2018; Weber, 2004). This work adds to the current RUME literature by demonstrating a burgeoning relationship between secondary teachers' understanding of continuity and differentiability in the context of a fully online Real Analysis course for teachers and their perception of the impact understanding these mathematical ideas has on their teaching.

Our research team collected pilot data during the Fall 2023 academic semester with the goal of better understanding how in-service teachers develop deeper conceptual understandings of important concepts for understanding introductory Real Analysis. The students are currently employed at secondary schools in the surrounding area teaching secondary mathematics. Our research questions are: *1) What meanings for continuity and differentiability do in-service teachers construct in an online analysis course? How are these meanings related? 2) In what ways can the instructional recommendations of Project ULTRA be implemented in the context of an online course for in-service teachers?*

In this poster, we will present preliminary results from our initial implementation of the instructional recommendations from the Project ULTRA research group in their textbook *Understanding Analysis and its Connections to Secondary Mathematics Teaching* (Wasserman et al., 2022). This work represents an initial attempt to extend this research group's instructional recommendations in a fully online Real Analysis course for in-service teachers in the Southeastern United States. We borrowed and added tasks from Sellers et al. (2017) to better understand the teachers' attention to mathematical logic in addition to their conceptual understanding of the concepts of continuity and differentiability when interpreting alternative statements for the Intermediate Value Theorem and the Mean Value Theorem.

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Understanding the IVT and the MVT does not necessarily mean that **secondary teachers view their students' learning** as **metaphorically "continuous."**

Statement 3 = False Not every c value in (a,b) is equal to a corresponding y value, N, between f(a)=2.5 and f(b)=0.
Statement 4 = False. Each y value between f(a)=2.5 and f(b)=0 do not have the same x value in (a,b). This would make the graph not be a function.

-Student E (Graph 1)

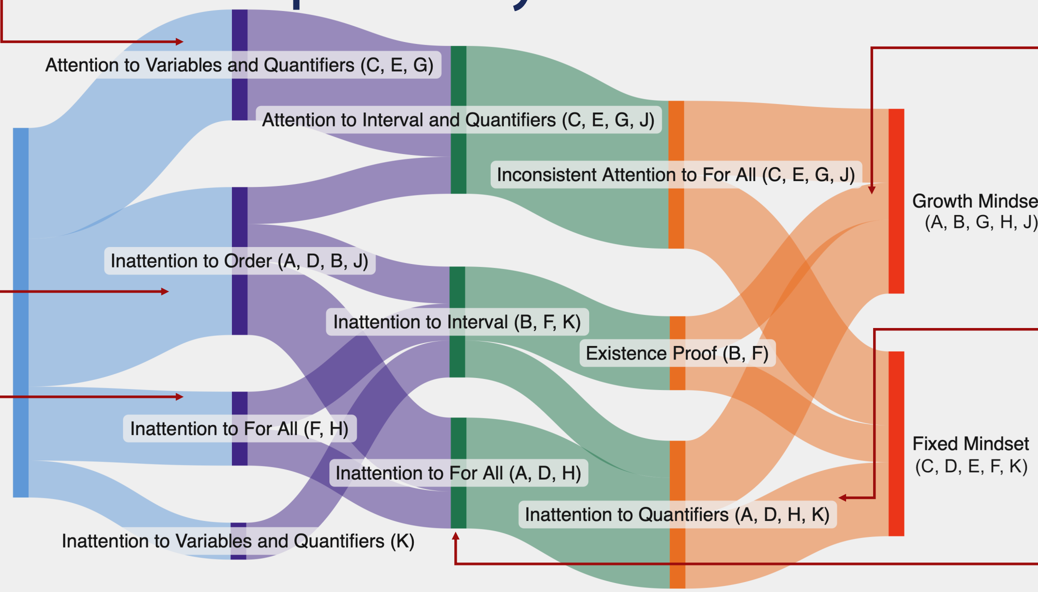
Statement 3 – False
There exist real numbers c in $[a, b]$ such that $f(c) = N$ is not between $f(a)$ and $f(b)$.

Statement 4 – True
Since f is continuous, every N between $f(a)$ and $f(b)$ corresponds to a c in domain (a, b).

-Student A (Graph 1)

- Statement 3 is true because for all the c values, the N value is always between f(a) and f(b), so we can definitely find at least one case.
- Statement 4 is true because all the N values between f(a) and f(b) correspond to at least one c value, so we can definitely find at least one case.

-Student F (Graph 4)



"I think this statement is a **good preliminary definition** for the graph of a continuous function, but it is **not at the caliber that is necessary** for analysis."
-Student B (Reflection 1)

"**This book is almost antagonistic with its problems** that keep bringing up things to consider, (I know a good textbook should and we are grad students) But **with our kids** we are at the same time trying to **teach then skills to use that work**."

"... it seems to me **such a huge gap** between general Algebra 2 and what the text is saying it is **almost not useful at all**, it has to be **translated several times into a language that they can understand**, and with that translating come the inferences and ambiguity that has caused the problems all along."
-Student D (Reflection 1)

Since Statement 1 says that there exists a point c in [a,b] where $f'(c) = \frac{f(b)-f(a)}{b-a}$. There exists means that there just has to be one c in (a,b) where this is true.

-Student K (Graph 3)

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LITERATURE & PROBLEM STATEMENT

Students' attention to logical quantifiers in the context of theorems related to continuity and differentiability impacts their understanding of theorems in Real Analysis courses (e.g., Roh et al., 2010; Wasserman et al., 2018).

There need to be clear connections between content and teaching, and those connections need to be authentic from a pre-service teacher's perspective (Wasserman et al., 2017).

But what does this look like for in-service teachers?

CONTEXT

In-service secondary teachers enrolled in a graduate-level, **online** Real Analysis course at a regional comprehensive University in the Southeastern United States.

2 out of 10 students had completed an undergraduate Real Analysis course prior to

TASKS & REFLECTIONS

Borrowed/modified tasks from Sellers et al. (2017) to better understand the teachers' attention to mathematical logic in addition to their conceptual understanding of the concepts of continuity and differentiability when interpreting alternative statements about the IVT and MVT.

They read Project ULTRA book and reflected on the relationship between their understanding of analysis and the high school mathematics that they teach.

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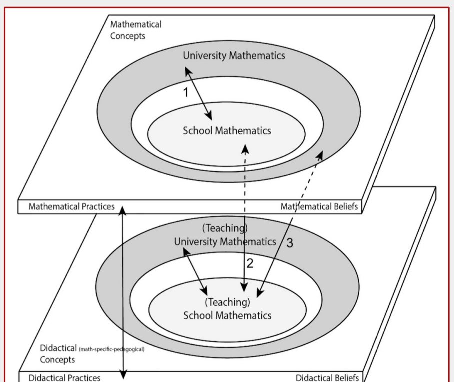

First published by Springer Nature in 2023 in Wasserman, Buchbinder, and Buchholtz's article, "Making university mathematics matter for secondary teacher preparation", in ZDM–Mathematics Education, 55(4), pp. 719-736." *1, 2, and 3 added by authors.*

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RESEARCH QUESTIONS

Our work adds to the current RUME literature by demonstrating a burgeoning relationship between **in-service secondary teachers'** understanding of continuity and differentiability in the context of a fully **online** Real Analysis course and their perception of the impact understanding these mathematical ideas has on their teaching.

1) What meanings for continuity and differentiability do in-service teachers construct in an online analysis course? How are these meanings related?

2) In what ways can the instructional recommendations of Project ULTRA be implemented in the context of an online course for in-service teachers?

DATA ANALYSIS

The teachers' responses to the IVT and MVT tasks were coded using Roh et al. (2010) codes in addition to emergent codes.

Teachers' reflection data was coded using open coding with an attention to their meanings for continuity, differentiability, quantifier order in the IVT and MVT statements, and perceptions of teaching or teacher change.