Student's Understanding of the Time Series and State Space Trajectory of the Solution of a Differential Equation

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Over the last decades the study of dynamical systems has become an increasingly important area in biology and the life sciences. High profile national documents call for the reform the mathematics instruction for biology and life sciences students in order to focus on modeling and dynamical systems (e.g., National Research Council, 2003). Related to these calls are time series and trajectories in the state space. The representations are central to the solutions of the differential equations describing a dynamical system as they provide important and complementary information about a system's behavior. It is a critical skill to be able to construct the trajectory associated with a given time series, and vice versa.

In this exploratory study, I analyze to what extent students who have taken an undergraduate course focusing on modeling have developed the skill of navigating between these two types of representations. I conducted think-aloud interviews with eight students who had completed the course. Students were given a time series and asked to construct the associated trajectory. They were then asked to do the opposite task. I conducted a thematic analysis (Braun & Clarke, 2006) of the interviews focusing on what level of covariational reasoning (Thompson & Carlson, 2017) the students exhibited when constructing these graphs. Covariational reasoning has been shown to be a core competency to understand functions and engage in mathematical reasoning.

The analysis of the interviews reveals that students showed a variety of levels of covariational reasoning when sketching these graphs. For example, when given a time series, Stan picked points by "going in units of 5 to make it easy". He then plotted the points for his trajectory and connected them with straight lines, exhibiting what Thompson and Carlson (2017) call chunky continuous covariation. When probed about the precision of his trajectory, Stan said that to make it more precise "you would go in units of one". He then indicated that he would pick more points and still connected them with line segments. In other words, he would use the same process but with a smaller time step, further showing chunky continuous covariational reasoning. When asked how confident he was of his answer he said: "Oh, I am definitely very confident on how to do it". In contrast, Carly picked points where the time series "kind of just change[s] direction in general or […] hit[s] a maximum" denoting that she used important features of the functions to pick her points rather than simply choosing points at regular intervals. She then graphed her trajectory using smooth curves, clearly denoting smooth covariational reasoning.

Both students also completed the "opposite task" consisting in starting with a trajectory and sketching an associated time series. While it could be reasonable to expect students to show the same level of covariational reasoning for this new task, the opposite was actually observed. Stan continued to pick points at regular intervals but this time he connected them with smooth curves, which would suggest smooth covariational reasoning. As for Carly, she picked points at regular interval (not based on the shape of the trajectory) and connected them with line segments, showing chunky continuous covariational reasoning. These results suggest that students do not consistently exhibit the same level of covariational reasoning even if the tasks are related. On the poster, I will share additional results and discuss implications for practice.

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References

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

National Research Council. (2003). *BIO2010: Transforming Undergraduate Education for Future Research Biologists*. The National Academies Press. https://doi.org/10.17226/10497

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.