Argument Mapping for Proof Comprehension

Andrew Baas
Texas State University

Keywords: argument mapping; proofs; proof comprehension

Proof comprehension has long been recognized as a critical skill for mathematics students (Conradie & Frith, 2000; Cowen, 1991). Cowen specifically states that, while most students will not be proving theorems after graduation, many will need to be able to read and understand mathematical writing. While undergraduate proof comprehension has historically received little attention from researchers (Mejía-Ramos & Inglis, 2009), this has begun to change in the last decade due to theoretical-methodological advancements like the assessment model developed by Mejía-Ramos et al. (2012). These models have overcome challenges inherent to measuring students' proof comprehension, enabling researchers to evaluate efficacy of methods developed to improve students' proof comprehension. For example, two empirical studies have shown that proof presentations which have been assumed to improve proof comprehension (Alcock & Inglis, 2008; Rowland, 2002) actually fail to improve student proof comprehension compared to traditional proof presentations (Lew et al., 2020; Roy et al., 2017).

Borrowing from research in philosophy education, I present argument mapping as a promising instructional technique for improving students' proof comprehension. Argument mapping (also called argument diagramming) is the practice of converting a written argument into a graph-like representation which depicts the logical relationships among statements within the argument. One form of argument mapping, Toulmin diagrams, has been extensively studied both as an analytic tool to investigate proof production (e.g. Aberdein, 2006; Corneli et al., 2019; Pease et al., 2009) and as structured representation to facilitate collaborative argumentation (Wagner et al., 2014; Zambak & Magiera, 2020). Researchers in philosophy education have investigated how argument mapping techniques can improve students' critical thinking (Harrell, 2012; Twardy, 2004). Twardy (2004) demonstrated that first year philosophy students who were taught argument mapping had significantly higher gains on the California Critical Thinking Skills Test (CCTST) than those taught traditionally. Similarly, Harrell (2012) showed that students taking an introduction to philosophy course which taught argument mapping had significantly higher gains on a critical thinking-focused pre/post-test than those who had traditional instruction. While critical thinking in philosophy is not identical to proof comprehension, the skills measured by the CCTST and by Harrell's critical thinking test seem to align with some of the dimensions of proof comprehension laid out by Mejia-Ramos et al. (2012). For example, one goal of Harrell's test was to test whether students were able to "determine how the premises are supposed to support the conclusion" (2012, p. 34), which aligns with the dimension of comprehension "justification of claims" (Mejía-Ramos et al., 2012, p. 9).

Based on this research in the field of philosophy education, argument mapping has the potential to improve student comprehension of proofs. For this poster, I will report a literature review of the current use of argument diagramming in mathematics education and in adjacent fields (like philosophy and logic) and present a study design which seeks to address the question: how does diagramming the argument of a proof impact a student's comprehension of that proof?

References

Aberdein, A. (2006). Managing Informal Mathematical Knowledge: Techniques from Informal Logic. In J. M. Borwein & W. M. Farmer (Eds.), *Mathematical Knowledge Management* (pp. 208–221). Springer. https://doi.org/10.1007/11812289_17

Alcock, L., & Inglis, M. (2008). Doctoral students' use of examples in evaluating and proving conjectures. *Educational Studies in Mathematics*, *69*(2), 111–129. https://doi.org/10.1007/s10649-008-9149-x

Conradie, J., & Frith, J. (2000). Comprehension tests in mathematics. *Educational Studies in Mathematics*, *42*(3), 225–235. https://doi.org/10.1023/A:1017502919000

Corneli, J., Martin, U., Murray-Rust, D., Rino Nesin, G., & Pease, A. (2019). Argumentation Theory for Mathematical Argument. *Argumentation*, *33*(2), 173–214. https://doi.org/10.1007/s10503-018-9474-x

Cowen, C. C. (1991). Teaching and Testing Mathematics Reading. *American Mathematics Monthly*, *98*, 50–53.

Harrell, M. (2012). Assessing the Efficacy of Argument Diagramming to Teach Critical Thinking Skills in Introduction to Philosophy: *Inquiry: Critical Thinking Across the Disciplines*, *27*(2), 31–39. https://doi.org/10.5840/inquiryct201227210

Lew, K., Weber, K., & Mejía Ramos, J. (2020). Do Generic Proofs Improve Proof Comprehension? *Journal of Educational Research in Mathematics*, *30*, 229–248. https://doi.org/10.29275/jerm.2020.08.sp.1.229

Mejía-Ramos, J. P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, *79*(1), 3–18. https://doi.org/10.1007/s10649-011-9349-7

Mejía-Ramos, J. P., & Inglis, M. (2009). Argumentative and proving activities in mathematics education research. *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education*, *2*, 88–93.

Pease, A., Smaill, A., Colton, S., & Lee, J. (2009). Bridging the Gap Between Argumentation Theory and the Philosophy of Mathematics. *Foundations of Science*, *14*(1–2), 111–135. https://doi.org/10.1007/s10699-008-9150-y

Rowland, T. (2002). Generic proofs in number theory. In *Learning and teaching number theory: Research in cognition and instruction* (pp. 157–183). Ablex Publishing.

Roy, S., Inglis, M., & Alcock, L. (2017). Multimedia resources designed to support learning from written proofs: An eye-movement study. *Educational Studies in Mathematics*, *96*(2), 249–266. JSTOR Journals. https://doi.org/10.2307/45185371

Twardy, C. (2004). Argument Maps Improve Critical Thinking. *Teaching Philosophy*, *27*(2), 95–116. https://doi.org/10.5840/teachphil200427213

Wagner, P. A., Smith, R. C., Conner, A., Singletary, L. M., & Francisco, R. T. (2014). Using Toulmin's Model to Develop Prospective Secondary Mathematics Teachers' Conceptions of Collective Argumentation. *Mathematics Teacher Educator*, *3*(1), 8–26. https://doi.org/10.5951/mathteaceduc.3.1.0008

Zambak, V. S., & Magiera, M. T. (2020). Supporting grades 1–8 pre-service teachers' argumentation skills: Constructing mathematical arguments in situations that facilitate analyzing cases. *International Journal of Mathematical Education in Science and Technology*, *51*(8), 1196–1223. https://doi.org/10.1080/0020739X.2020.1762938