Uncovering Mathematics Capital: Using Structural Equation Modeling to Investigate Interrelations with Traditional Measures of Mathematics Assets

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Students enter undergraduate mathematics classrooms with a variety of mathematical backgrounds. We view these past mathematical experiences as assets that support students in their future mathematical learning. In this paper, we seek to characterize these assets by introducing a new concept in mathematics education: mathematics capital. This concept emerges from a framework connecting Archer et al.'s (2015) conceptualization of science capital and Bourdieu's seminal work on the forms of capital (1986). Survey data were gathered in Spring 2023 from a total of 219 students in undergraduate mathematics courses at an urban midwestern university. Path analyses in structural equation modeling showed a strong association between the variables of mathematics capital and self-efficacy. Other associations are also discussed. Our results indicate that mathematics capital is a measurable, quantifiable variable, independent from others. Future research is ongoing to better understand the nature of mathematics capital and how it is related to other variables.

Keywords: Mathematics capital, Self-efficacy, Mathematics mindset, Structural equation modeling, Path analysis

The current state of undergraduate mathematics education involves a multitude of variables related to student demographics and success. An important variable to student success is equity, which addresses purposeful accommodations and adaptations regarding student socioeconomic status, gender, and cultural differences (National Council of Teachers of Mathematics [NCTM], 2014). Students bring their individual characteristics and identities into the classroom, which ultimately impacts their learning experiences and success in mathematics (Gutiérrez, 2009, NCTM, 2014). In this study, we seek to identify student backgrounds, history, and characteristics that potentially impact their success in mathematics education. We view these student characteristics through a Bourdieusian style lens, parallel to the previously researched theory of science capital espoused by Archer and colleagues (2015). Herein, we describe our approach of using structural equation modeling to investigate a novel theoretical construct, which we define as *mathematics capital.* We posit that mathematics capital is a variable that can be measured and is related to other important variables in mathematics education.

Conceptual Framing

Bourdieu (1986) rationalized that each field has inherent capital veiled in three ways: economic, cultural, and social. He fashioned the idea that patterns are not coincidental but rather are researchable and employable trends in the social world. Previous research from Archer et al. (2015) widened the Bourdieusian lens from art-based concepts to include the theory of science capital in society. Through their work, Archer et al. developed a three-part model in order to quantify students' science capital. Our research extends this work to the field of mathematics.

Using the three-part model from Archer et al. (2015), we introduce the concept of mathematics capital. We build on the following three categories for our variables, analogous to Archer et al. (2015), that we posit are related to mathematics capital. The three variables, which we define below, are mathematics teaching and learning experiences, mathematics self-efficacy, and cultural capital. Additionally, we include a fourth variable: mathematics mindset. We include mathematics mindset to incorporate research from Boaler (2015) and Dweck (2006) that highlights the importance of how students' view themselves as mathematical learners and developing a growth mindset towards learning mathematics.

Mathematics teaching and learning experiences are defined as exposures to mathematics as process of doing (Cuoco et al., 1996), such that a student develops flexibility and fluidity within multiple mathematical representations (NCTM, 2014), and a student's responses to these experiences. Mathematics self-efficacy is rooted by Bandura's (1997) seminal work on self-efficacy where self-efficacy is defined as a person's perception of their ability to complete tasks. Specific to mathematics self-efficacy, developing agency and identity in mathematics is integral to the development of mathematics self-efficacy and outcomes in mathematics (Boaler & Greeno, 2000). Finally, cultural capital described by Bourdieu refers to a person's family background, educational attainment, and perhaps social refinement (1977, 1986). Although this is not inclusive, these short phrases can act as descriptors to understand Bourdieu's vision of cultural capital.

Mathematics capital is theorized using Bordieu's (1986) definition of social capital where social capital is a not only a person's social relations, but also the power that grows as a result of those social relationships. In other words, social capital is not just having relationships with people, but having these relationships and using them to harness forward movement in a social network. A nonexample of social capital is having positive social relationships, but not accessing these relationships for any form of person gain. In this study, we used structural equation modeling to investigate the relationship between mathematics capital, cultural capital, and the collection of one's mathematics mindset, self-efficacy and mathematics teaching and learning experiences. Most importantly, our model exhibits the independent variable that we are interested in characterizing: mathematics capital, where mathematics capital is defined as mathematical assets that actively influence a student's capability or power in the field of mathematics education and how the student utilizes these assets to exert power in the field of mathematics education. Identification of mathematics capital, and how it has historically been acquired and used by successful mathematics students, extends current mathematics education research as it relates to equity. Defining mathematics capital will help researchers and educators better understand how to take proactive steps to support students in more equitable ways. Thus, the purpose of our study is to characterize mathematics capital and understand its relationship with other variables related to student success in mathematics. To this end, we investigate the following research questions:

1. *Is mathematics capital measurable?*
2. *What relationships exist among mathematics capital, cultural capital, mathematics mindset, mathematics self-efficacy, and mathematics teaching and learning experiences?*
3. *Is mathematics capital distinctly different than the other variables in the study?*

Methods

Participants

The participants in this study were students enrolled in undergraduate mathematics courses at an urban midwestern university. Survey data were collected through Qualtrics from 219 participants where 104 participants were enrolled in foundational mathematics courses (e.g., Pre-calculus, Trigonometry, Quantitative Literacy), 79 participants were enrolled in either Calculus I, II or III, 8 participants were enrolled in Mathematics for Elementary Teachers I or II, and 28 participants did not respond to this question. No other descriptive data were collected in this study including gender, socio-economic status, age, or ethnicity. Participants were recruited through their mathematics instructor's request and were not offered any compensation. Participation was voluntary and collected anonymously.

Variables

Data were collected for the following variables: mathematics mindset (MIND), mathematics self-efficacy (SE), mathematics teaching and learning experiences (TEAC), cultural capital (CC), and mathematics capital (MC). Participants were asked to respond to survey questions using a 5-point Likert scale where options varied from strongly agree to strongly disagree. We used existing scales for both MIND and SE (Cribbs et al., 2021) where reliability had already been established. Our Chronbach's alpha values for the existing scales used for MIND and SE were .914 and .960, respectively. Thus, we proceeded with the validated scales as anticipated. For both TEAC and CC we used existing scales as templates to build items that mirrored those that had been previously validated but better fit our research. The TEAC items were based on research from Ottmar et al. (2014), while the items for CC were developed from existing research from Dumais and Ward (2010), Noble and Davies (2009), and Zabihi and Prodel (2011). Finally, to address our research question, we developed 15 new survey items that were used for MC (see Table 1).

Table 1. Survey items for TEAC, CC, and MC.

TEAC

While growing up:

1. I discussed solutions to math problems with my peers.
2. I worked and discussed math problems that reflect real life situations.
3. I solved math problems in small groups or with a partner.
4. I wrote a few sentences about how to solve a math problem.
5. I used visual representations (e.g. diagrams, tables, models).
6. I learned how to communicate ideas in mathematics effectively.
7. I worked with manipulatives (e.g. geometric shapes).

CC

While growing up:

1. I read.
2. I was read to.
3. I attended museums.
4. I was taught that education was important.
5. I was encouraged to go to college.

6. I participated in my education.
7. I participated in extracurricular activities.
8. I was taught about historic events.
9. I felt supported.
10. I did not have a lot of screen time.
11. I read music.

MC

While growing up:

1) I earned good grades in math.
2) Math was generally easy for me.
3) I used my math skills outside school.
4) I noticed patterns in math.
5) Experiences in my life helped me make sense of math problems.
6) I advanced in STEM because of my math skills.
7) I received opportunities, awards, or recognition because I was good at math.
8) Math made sense to me.
9) I worked through math problems even if they took longer than expected.
10) I worked through math tests quickly.
11) I usually knew when I had solved a math problem correctly.
12) I knew where math formulas came from.
13) I knew how to use math formulas.
14) I knew why math formulas worked.
15) I enjoyed math.

To attempt to measure TEAC and CC, we utilized 7 and 11 items, respectively, derived from existing scales. Our initial recorded alpha values were .805 for TEAC and 0.738 for CC. Through the process of investigation, we attempted to eliminate items but could not come up with a higher Chronbach's alpha value for either variable. Thus, we retained all 7 items for TEAC and all 11 items for CC as they developed the most parsimonious scale with acceptable reliability.

To attempt to measure MC, we utilized the 15 items listed above in Table 1. Our initial recorded alpha value was 0.943. Then, through the process of investigation, we reduced down to 6 items (retaining items (1), (2), (6), (7), (8), and (15) from above) which developed the most parsimonious scale with acceptable reliability. The recorded alpha value for this variable reduced to 6 items was 0.946.

Analysis

Data were collected using Qualtrics then exported from Qualtrics to SPSS (Ver. 29). In SPSS, initial analyses were performed to preliminarily assess the reliability of the survey measures. To assess the shared associations between the variables of interest, we used path analyses in structural equation modeling (M-Plus Ver. 7, Muthén & Muthén, 2012). The scale means for each construct were modeled simultaneously and the interrelations were explored.

Results

The constructed scores of mathematics capital spanned the entire range of the scale. However, most of the responses (57.20%) were between "Somewhat agree" and "Somewhat disagree" with the remaining 14.4% below towards "Strongly disagree" and 28.40% above towards "Strongly agree". Overall, the distribution of scores was less normal and more platykurtic than ideal. However, there was nothing else to suggest that the measure was not a valid reflection of the overall distribution of mathematics capital.

Path analyses in structural equation modeling (M-Plus Ver. 7, Muthén & Muthén, 2012) were used to examine the associations in the variables of interest. Figure 1 shows the correlations between each pair of variables that were interrelated. Self-efficacy was positively related to mathematics mindset ($r = .36$, $p < .05$) and teaching and learning experiences ($r = .27$, $p < .05$). Higher reported self-efficacy was linked with a more positive mathematics mindset in addition to more positive teaching and learning experiences, with 12.96% and 7.29% shared variance, respectively.

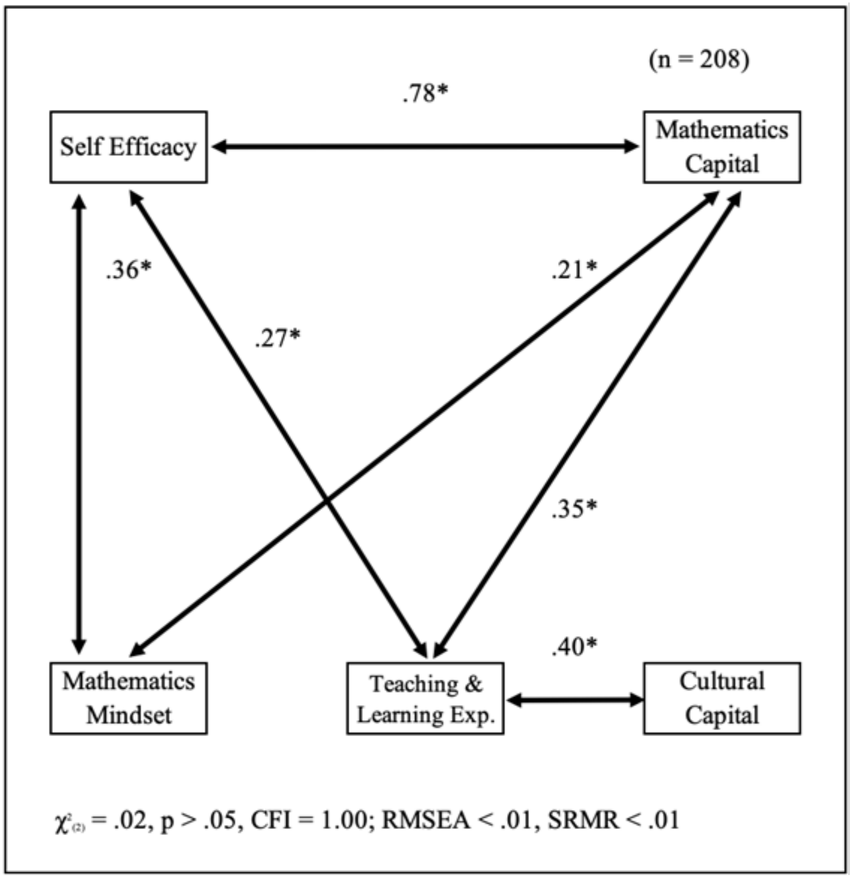

Figure 1. The shared association between the variables of interest.

Meanwhile, cultural capital was only significantly related to teaching and learning experiences ($r = .40$, $p < .05$). In other words, more cultural capital was related to more positive teaching and learning experiences with 16.00% of the variance shared between the two. Notably, however, cultural capital was not meaningfully associated with any of the other variables.

Finally, the measure of mathematics capital was positively related not only to mathematics mindset ($r = .21$, $p < .05$) but also to teaching and learning experiences as well ($r = .35$, $p < .05$). Here too, higher reported mathematics capital was linked with a more positive mathematics mindset in addition to more positive teaching and learning experiences, with 4.41% and 12.25%

shared variance, respectively. Perhaps most interestingly, mathematics capital's strongest association was with self-efficacy ($r = .78$, $p < .05$). In other words, the participants who reported more mathematics capital were also more likely to have high self-efficacy with 60.84% shared overlap in the variance. Lastly, mathematics capital and cultural capital were not related to each other in the current study. The resulting model ($n = 208$) was a good fit to the data ($\chi^2_{(2)} = .02$, $p > .05$, CFI = 1.00; RMSEA < .01, SRMR < .01).

Discussion

Our most important finding was that mathematics capital was directly and significantly related to almost all other variables in the model: mathematics teaching and learning experiences, mathematics self-efficacy, and mathematics mindset. This result suggests that increased mathematics capital is correlated to higher levels of mathematics self-efficacy, more positive mathematics teaching and learning experiences, and a growth-oriented mathematics mindset. This result also indicates that mathematics capital might be a candidate for exploring the potential mediating role in explaining the other associations we observed (e.g. the relationship between self-efficacy and teaching and learning experiences). While we did not measure the directionality between relationships in this analysis, work is ongoing to continue to understand and define the relationship between mathematics capital and these variables.

The strongest association in our model was between mathematics capital and self-efficacy. While there is research highlighting mathematics self-efficacy as a predictor of students' mathematics outcomes (e.g., Ayotola & Adedeji, 2009; Bonne & Johnston, 2015; Fast et al., 2010), our results contribute to the literature by highlighting the importance of the relationship between mathematics self-efficacy and mathematics capital. However, the nature of the relationship between these two variables must be further teased apart. Specifically, future research should investigate the items used to measure mathematics capital and mathematics self-efficacy to ensure that these two variables are psychometrically distinct. Given the strong correlation between mathematics capital and self-efficacy, we found that there was a 60.84% shared overlap in the variance, which was not high enough to claim these variables were the same. As variable validation continues for mathematics capital, we expect future research to help delineate between mathematics capital and mathematics self-efficacy.

We were surprised to find that our model showed no significant relationship between mathematics capital and cultural capital. As we continue to confirm the existence of mathematics capital, it is important to differentiate mathematics capital from other forms of capital. This distinction reiterates that mathematics capital is its own entity and does not overlap with cultural capital. This result also suggests that mathematics capital cannot be defined within cultural capital, and instead, requires its own definition. Said differently, mathematics capital is not the intersection of cultural capital and other mathematics variables such as self-efficacy, mathematics mindset, or mathematics teaching and learning experiences: mathematics capital is its own measurable, quantifiable variable.

While our analysis showed that mathematics capital and cultural capital are distinct forms of capital, it brings up the question of whether capital is transmutable. We expected to find some relationship between the two forms of capital due to the inherent transmutability of capital (Bourdieu, 1986). Thus, the lack of relationship between mathematics capital and cultural capital, raises the question: why is there no statistically significant direct relationship between these two forms of capital? Notably, mathematics capital and cultural capital are both statistically related to mathematics teaching and learning experiences. Therefore, any association between mathematics capital and cultural capital only exists inasmuch as they are both related to mathematics teaching and learning experiences. In addition, there may be other variables that were not included in our model, such as financial capital, that are associated to both mathematics capital and cultural capital. Thus, more research is needed to determine the relationships between these variables and different forms of capital.

Conclusion

A primary limitation of our study was the sample size. With 219 responses, we were able to investigate interrelations between the variables we were measuring. However, collecting a minimum of 300 responses would allow us to do latent variable modeling of the associations between the variables as latent constructs (i.e., to measure the error free associations between them). Data collection is currently ongoing to reach this sample size so that our analysis can be extended. We also plan to extend our analysis to investigate potential differences in student subgroups. Of particular interest are the differences between students who are enrolled in a calculus sequence course pursuing STEM majors and students who are non-STEM intending.

In addition to our quantitative analyses, our current survey allows students to respond to two free-response questions to gather qualitative data. In this portion of survey, we are looking to compare and contrast the measured variables, as quantitatively reported by the participants to the descriptions in their free-response questions. We are curious how students report their self-efficacy, mathematics teaching and learning experiences, and their mathematics mindset on a scale and how these compare to their experiential descriptions of these variables.

Our ultimate goal is to understand mathematics capital so that we can learn how it is acquired and better support students in attaining this capital. We view mathematics education as a holistic and comprehensive undertaking, as Bourdieu saw many capital-related concepts (1986). Developing students' mathematics capital requires understanding and describing how mathematics capital intersects with students' teaching and learning experiences in mathematics, their mathematics self-efficacy, their mathematics mindset, and their cultural capital. Having this holistic approach to mathematics education will help us as we continue to uncover more relationships between these variables and discover new ones.

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