Preservice Teachers’ Knowledge and Understanding of Fraction Division

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It is important for preservice teachers (PSTs) to have a deep and connected conceptual understanding of mathematical concepts they will teach in their future classrooms (Ball, 1990; Da Ponte & Chapman, 2015). For this reason, teacher education programs work to provide opportunities for preservice teachers' (PSTs’) to build understanding of the various ways students conceive of significant mathematical ideas and practices (e.g., Ball et al., 2008; Hill, 2010; Li, 2008; Li & Kulm, 2008; Lo & Luo, 2012; Tirosh, 2000). Therefore, it is important to investigate PSTs’ conceptions, connections, notions, and understandings of specific mathematics concepts so that results can be used to inform effective instruction in content courses for future teachers.

The goal of this comparative case study was to investigate PSTs’ *content knowledge* (Ball et al., 2008; Coskun et al., 2023) and *content understanding* (Bair & Rich, 2011; Hill et al., 2004, 2008) as the basis for measuring *cognitive types of teacher content knowledge* (Tchoshanov, 2011). The *cognitive types* include knowledge of facts and procedures (Type 1), knowledge of concepts and connections (Type 2), and knowledge of models and generalizations (Type 3). Our poster presentation will share results from the study for only *Type 2* with a content focus on fraction division. The research question for this portion of the study was: What are PSTs’ mathematical conceptions, connections, and notions of the fraction division concept?

We utilized a linear progression of constructs to guide the analysis of our data to answer the research question. That is, content knowledge □ cognitive types of teacher content knowledge □ specialized content knowledge (SCK) progression □ content understanding □ (conceptions, connections, and notions), with each construct rooted in the literature.

Data was collected from 18 preservice elementary teachers enrolled in mathematics content course at a public university in the Southeastern United States. The study analyzed both quantitative and qualitative data. For the quantitative data, a modified form of the Teacher Content Knowledge Surveys (TCKS) designed by Tchoshanov (2011) was used. For the qualitative data, pre- and post-term exit tickets were used. The SCK progression framework (Bair & Rich, 2011) informed the qualitative data analyzes.

Results also showed that Blair and Rich’s (2011) SCK progression framework was associated with levels of knowledge as measured by the TCKS. We found that 30% of PSTs with less than a numerical average value (< 0.5) of *Type 2* TCKS pre-post scores were those at Level 0 and Level 1 of SCK progression. Also, 40% of participants who were above average in TCKS scores were at Level 3 and Level 4 and the rest were at average. This study informed mathematics teacher educators and instructors of content mathematics courses for PSTs on the characterization of the kind and qualities of aspects of content knowledge and content understanding that PSTs possess and how to support them in specific mathematical concepts.

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Introduction

It is important for preservice teachers (PSTs) to have a deep and connected conceptual understanding of mathematical concepts they will teach in their future classrooms (Ball, 1990; Da Ponte & Chapman, 2015). For this reason, teacher education programs work to provide opportunities for preservice teachers' (PSTs’) to build understanding of the various ways students conceive of significant mathematical ideas and practices (e.g., Ball et al., 2008; Hill, 2010; Li & Kulm, 2008; Lo & Luo, 2012). The goal of this study was to investigate PSTs’ *content knowledge* (Ball et al., 2008; Coskun et al., 2023) and *content understanding* (Bair & Rich, 2011; Hill et al., 2008) as the basis for measuring *cognitive types of teacher content knowledge* (Tchoshanov, 2011).

Theoretical Framework

The study utilized a linear progression of constructs to guide the analysis of our data to answer the research question. That is, content knowledge → cognitive types of teacher content knowledge → specialized content knowledge (SCK) progression → content understanding → (conceptions, connections, and notions), with each construct rooted in the literature.

Methodology

Participants

- ✓ A total of 18 elementary preservice teachers in a mathematics content course were involved.
- ✓ Ten of the participants were selected for further qualitative analysis. Those were the top 5 and lower 5 PSTs with Teacher Content Knowledge Surveys (TCKS) scores.

Data Source and Design

Written Exit Tickets. PSTs were provided with both pre-and post exit tickets modified from the TCKS (Tchoshanov, 2011) as the qualitative data.

Instruments, Tools, Constructs, and Data Coding

Instruments and Tools

Analyzed with the SCK Progression Framework (Blair & Rich, 2011)

Five-level SCK Progression Framework

Indicators: entry (Level 0), emerging (Level 1), developing (Level 2), maturing (Level 3), and deep and connected mathematical knowledge for teaching (Level 4).

Data Coding: Bair and Rich’s framework to provide a fine-grained process for coding PSTs’ *content understanding* using the five-level progression of indicators.

Research Question

What are PSTs’ mathematical conceptions, connections, and notions of the fraction division concept?

Big Takeaways

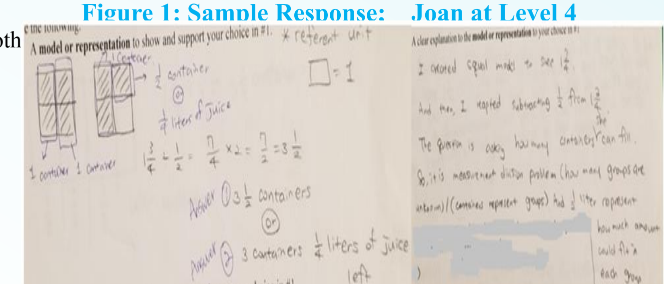
Content Knowledge

Blair and Rich’s (2011) SCK Progression Framework **Levels** appear to be associated with the Cognitive Type of Teacher Content Knowledge as measured by the TCKS (Tchoshanov, 2011).

Content Understanding

PSTs with strong and explicit fraction division conceptions, connections, and notions were at Level 4 or Level 3 but getting nearer to Level 4.

Figure 1: Sample Response: Joan at Level 4



Joan: “*the question is asking how many ½ liters containers can be filled? So, it is a measurement division problem*”. ***Fraction Division Conception***

Joan: “I created equal model to see $1\frac{3}{4}$ and then I *repeated subtracting* $\frac{1}{2}$ from $1\frac{3}{4}$”.

Joan’s model brings out (i) the problem structure, (ii) the action in the problem, (iii) appropriate representational connections to the problem situation and the context of solution, and (iv) the possible ways the solution can be expressed

Future Research

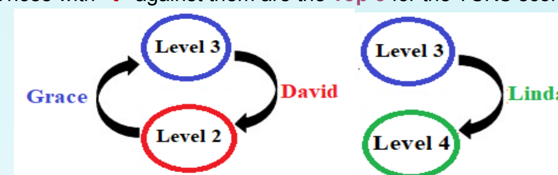
Investigating PSTs cognitive processes that interact with their conceptions, connections, and notions about specific mathematical concepts

Results

Level 0	Level 1	Level 2	Level 3	Level 4
Emily(**b**)	Sarah(**b**)	Kelly(**t**)	Linda(**t**)	Joan(**t**)
	Kate(**b**)	Grace(**b**)	David(**t**)	Lizz(**t**)
		Nina(**b**)		

PSTs with **“b”** against names are the **Bottom 5** for the TCKS scores

Those with **“t”** against them are the **Top 5** for the TCKS scores



Conclusion and Implication

The results of this study will inform instructors content mathematics courses for PSTs the need to integrate pedagogical strategies that will support the development of a deeper conceptual knowledge and understanding of any mathematical concept.

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