Bridging Mathematics: To Care for an Abstraction

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This theoretical poster engages with and presents dominant undergraduate mathematics topics as lively forces with rich potentialities for connection beyond disciplinary mathematics. More specifically, this poster offers both (a) a theoretical contribution to engage with undergraduate mathematics as situated, open, and lively (de Freitas & Sinclair, 2013; Mikulan & Sinclair, 2023); and (b) a potential bridge to resurface and generate connections between undergraduate mathematics and philosophical ideas that are often left out of mathematics classrooms. We invite our field to, as Mikulan and Sinclair (2023) write, *care* for an abstraction: "To care for an abstraction is to make sure you haven't extended it too far, forgotten the contingencies on which it depends, applied it flippantly or use it to foreclose thinking" (p. 70).

Doxiadis (2003), when inviting mathematicians to open up to other ways of sharing their work, suggested considering historical, philosophical, aesthetic, and other dimensions of mathematics to understand math differently. They propose to "[e]mbed mathematics in the soul…by embedding it in story" (p. 24). In resonance, this poster understands mathematics as embedded in worlds (rather than fixed, separated/separable from the 'real' world); always emergent, relational, and in transformation, carrying with it histories, desires, and possibilities that are neither predetermined/fixed nor confined within mathematics as a discipline. For example, the concept of *isomorphism* is often fixed to its algebraic definition and approached as the 'endpoint' of some learning trajectory (e.g., Larsen, 2013), narrowing down pedagogical conversations to metaphors of sameness or connections with homomorphisms (Rupnow, 2021). But what if instead we engaged with it as a *starting* point, as a spark? What stories can it awaken and how are they interconnected (or not) with(in) the world? Isomorphism then becomes a kaleidoscope of stories, awakening multiple wonderings: Why does isomorphism seek to preserve *these* specific structures (e.g., group operation), and what possibilities are foreclosed? Connecting isomorphism with notions of preserving a melody without, for example, consideration of an instrument's timbre could invite discussions of what might be left out when prioritizing certain structures at the expense of other dynamics. These stories, moreover, need not be reduced to auxiliary metaphors *towards* the definition of isomorphism and can invite other wonderings: Where does the persistent preoccupation for 'sameness of forms/structures' in dominant mathematics come from? This opens worlds of historical connections to follow, such as ties with coloniality and language (Barton & Frank, 2001; Gutiérrez, 2017) and often-ignored contingencies as well as connections between temporalities and abstractions (Mikulan & Sinclair, 2023), inviting us to trouble the belief that mathematics is 'universal' and 'objective.'

Understanding mathematics as open and entangled with lived/living histories has not only pedagogical implications but also political. The pervasive understanding of mathematics as abstract and thus objective is harmful in multiple ways, concerns that have been raised in K-12 literature (e.g., Fasheh, 2012; Martin, 2019) but are rare in undergraduate mathematics. Thus, we invite our field to open up to engage with mathematics as lively and situated, and question dominant narratives of undergraduate mathematics. As Wertheim (2019) writes, understanding mathematics as living "enables us to recalibrate our relationship with the subject, reframing it as a mode of engagement in which we can highlight its joyous potential for all people" (p. 70).

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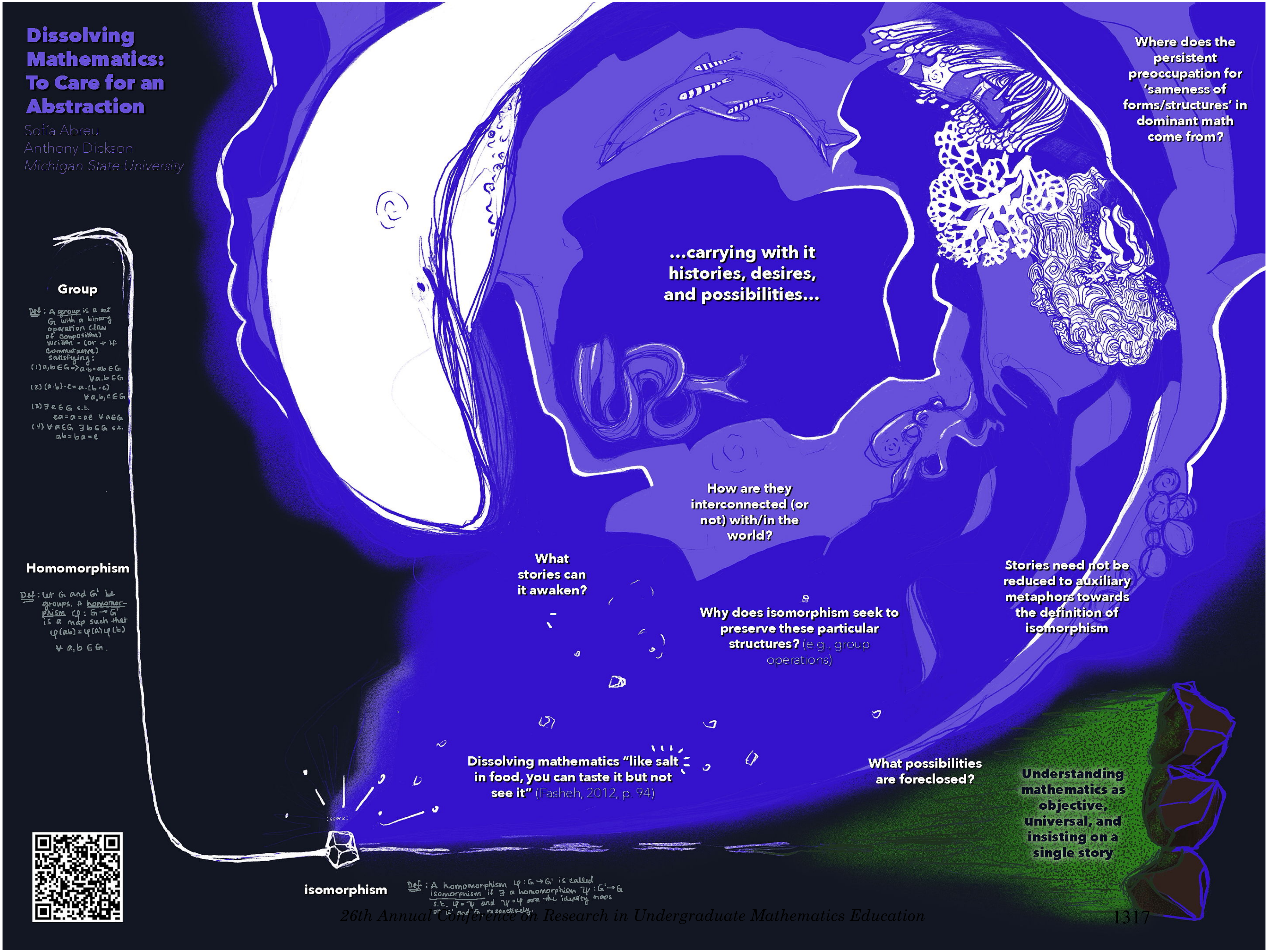

Dissolving Mathematics: To Care for an Abstraction
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Group
Def: A group is a set G with a binary operation (law of composition) written · (or + if commutative) satisfying:
(1) $a, b \in G \Rightarrow a \cdot b = ab \in G$ $\forall a, b \in G$
(2) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ $\forall a, b, c \in G$
(3) $\exists e \in G$ s.t. $ea = a = ae$ $\forall a \in G$
(4) $\forall a \in G$ $\exists b \in G$ s.t. $ab = ba = e$
Homomorphism
Def: Let G and G' be groups. A homomorphism $\varphi : G \to G'$ is a map such that $\varphi(ab) = \varphi(a)\varphi(b)$ $\forall a, b \in G$.
isomorphism
Def: A homomorphism $\varphi : G \to G'$ is called isomorphism if $\exists$ a homomorphism $\psi : G' \to G$ s.t. $\varphi \circ \psi$ and $\psi \circ \varphi$ are the identity maps on G' and G respectively.
...carrying with it histories, desires, and possibilities...
Where does the persistent preoccupation for 'sameness of forms/structures' in dominant math come from?
How are they interconnected (or not) with/in the world?
What stories can it awaken?
Why does isomorphism seek to preserve these particular structures? (e.g., group operations)
Stories need not be reduced to auxiliary metaphors towards the definition of isomorphism
Dissolving mathematics "like salt in food, you can taste it but not see it" (Fasheh, 2012, p. 94)
What possibilities are foreclosed?
Understanding mathematics as objective, universal, and insisting on a single story