Using Refutation to Address a Graphical Misconception in Calculus

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Background

Refutation

- Refutation texts have been shown to repair misconceptions held by students in science contexts.[6]
- Students have shown greater gains in multimedia learning environments when misconceptions are addressed.[5]
- Students prefer expository refutation over expository text and view refutation as more efficient.[3]
- There is less research reflecting the use of refutation in mathematics contexts and there is little, if any, research exploring refutation with the use of graphical representations.

Connecting *f* and *f'*

- Students have difficulty coordinating multiple graph features at once (e.g., how can a graph be decreasing and concave up?)[1]
- Students also struggle to interpret the derivative of a function when it is presented graphically.[4]
- Sketching the graph of *f* based on a graph of *f'* involves coordinating the properties of each and representing that graphically.

Research Questions

1. What is the effect of refutation videos and graphs on student performance?
2. Is there potential value in using refutation in a mathematics education setting?

Design & Procedure

Pilot Study

Pre-test → Expository Video n=96 → Post-test

Experiment

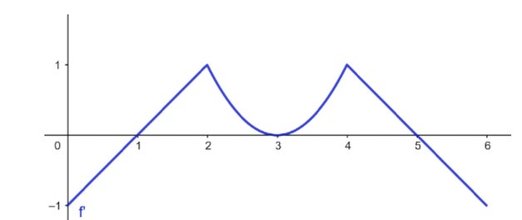
Pre-test → Refutation (Target n=50) / Expository (Target n=50) → Post-test → Spatial Reasoning Test

Materials

Pretest: 5 items
- 2 true/false
- 3 multiple choice

Example item:

The graph of f', the derivative of f, is shown to the right. Decide if the following statement is true or false:
The graph of f is increasing for 4 < x <5
a) True
b) False

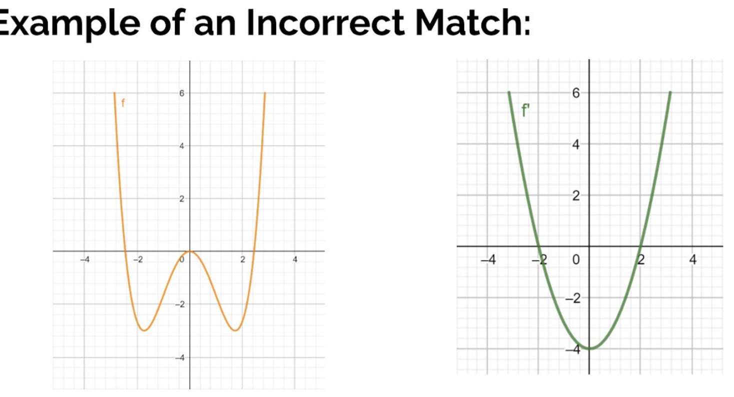


Refutation

Some students think that the behavior of ' matches the behavior of f. That is
- When f is positive, f' is positive
- When f is negative, f' is negative
- When f is 0, f' is 0

For example, students might match the following graphs of a function f and its derivative f'…

Example of an Incorrect Match:



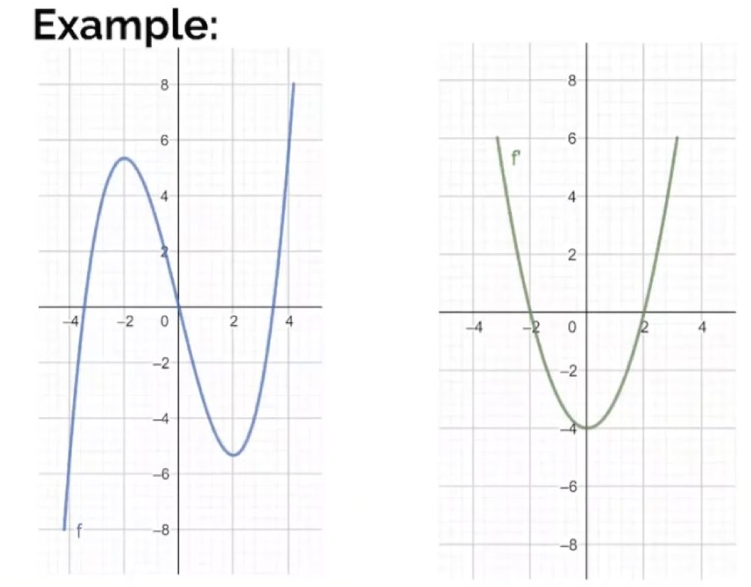
Notice that the graph on the right matches the behavior of the graph on the left. However, the idea that the behavior f matches the behavior of f' is **not true!**

Exposition

Since the derivative value represents the slop of the tangent line the relationship is characterized as follows:
- When f is **increasing** the slope is positive, so f' is **positive**
- When f is **decreasing** the slope is negative, so f' is **negative**
- When f attains a maxima or minima, the slope is flat so f' is **0**

Behavior of f		Behavior of f'
$f(x)$	Is increasing	$f'(x) > 0$
$f(x)$	Is decreasing	$f'(x) < 0$
$f(x)$	Attains a local Maxima/minima	$f'(x) = 0$

Example:

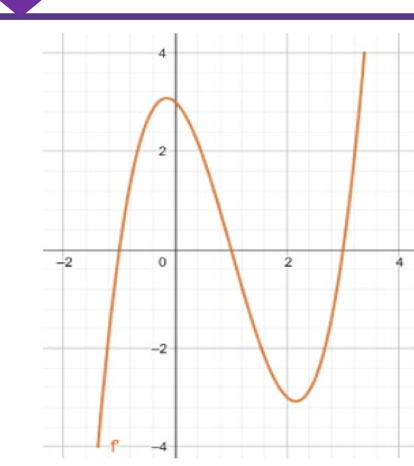


Exposition
(see right)

Post-test: 11 items
- 4 true/false
- 6 multiple choice
- 1 free response

Example item:

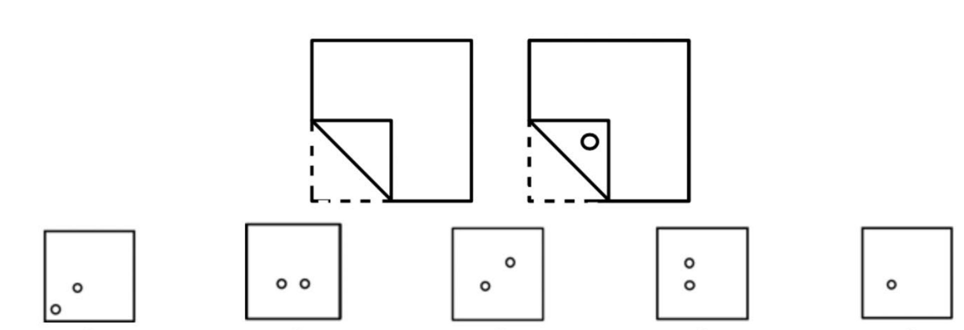
The graph of f', the derivative of f, is shown to the right. For -2<x<4 what are all the intervals in which the graph of f is increasing?
The graph of f is increasing for 4 < x <5
a) -1 < x < 1 and 3 < x ≤ 4
b) -2 < x < 0 only
c) -1 < x < 0 and 3 < x < 4
d) -2 < x < 0 and 2 < x ≤ 4
e) 3 < x ≤ 4 only



Spatial Reasoning Test:

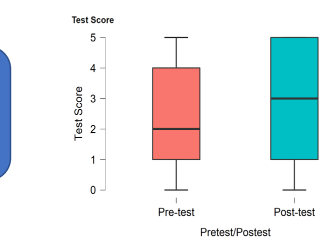
Spatial Reasoning has been shown to influence Calculus learning (Cromley et al., 2017).

Example item:



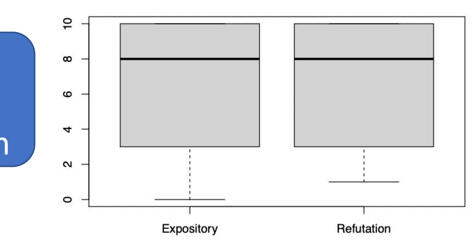
Results

Pre/Post Test Overall



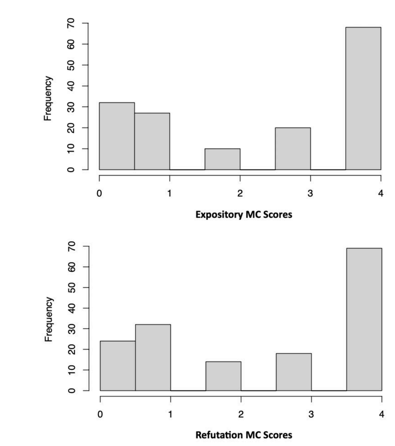
- Saw increases in learning overall

Post Test Score Comparison



- No significant differences in conditions were observed

Question Type Score Comparison



- Grouping by question type (T/F, Multiple Choice (MC)) yielded similar results

Analysis of FR Question

Expository	Refutation
39.3%	40.3%

- Same prevalence of misconception
- Differed in nature

Condition	Example
Expository	"I decided based on where the graph was decreasing"
Refutation	"From the video it stated that f and f' mirror each other"

Discussion

- Results suggest that our lessons improved learning overall
- Refutation failed to improve learning when compared to expository, and some cases reinforced the misconception
- The misconception was prevalent (40% of incorrect responses)

Implications

We interpret the results in several ways:
1) Acknowledging limitations of refutation (see Zengilowski et al., 2021)
2) Additional instructional supports are needed for building robust conceptions related to this topic
3) Using refutation in this way may reinforce existing prior conceptions about f and f'

References

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