Metacognition in Proof Construction: Connecting and Extending a Problem-Solving Framework

Sarah Sparks
University of Northern Colorado

Gulden Karakok
University of Northern Colorado

Keywords: Metacognition, Problem Solving, Proof

Mathematical proofs are an integral component of undergraduate mathematics education. Providing students opportunities to develop the necessary writing technique, and deductive reasoning skills to communicate mathematical ideas via proof is a central goal of undergraduate mathematics programs (Committee on the Undergraduate Programs in Mathematics [CUPM], 2015). For undergraduate students, developing these skills and utilizing them appropriately can be an arduous feat and has implications for their success in a mathematics course or program (Alcock & Simpson, 2002). Due to the difficulty and importance of proof, the mathematics education community has contributed to a varied body of research related to the teaching, learning, and cognitive skills relevant to mathematical proof.

Research in the cognitive processes related to proof construction have provided insight into how undergraduate students, graduate students, and mathematicians interact with the proof construction process. Inglis and Alcock (2012) demonstrated that the processes with which each of these three groups construct proofs varies and that proof construction is not uniform even within a group. Their participants also showed inconsistency between how they said they approach proofs and the actual processes they implemented when given a proof construction task. To address these different approaches and their implications for the teaching and learning of proofs, we need to know more about what beliefs and judgements individuals have about their own cognitive processes during proof construction, and how these impact the choices they make during this process. Metacognition, defined to be "any knowledge or cognitive activity that takes as its object, or monitors, or regulates any aspect of cognitive activity; that is, knowledge about, and thinking about one's own thinking" (Lerman, 2020, pp. 608-609) captures exactly this aspect of an individual's thinking.

Existing metacognition research in mathematics education has focused on the relevant metacognitive processes that occur during mathematical problem solving more broadly. Proof related tasks can be a subset of mathematical problem solving, and therefore many of the findings from problem-solving based metacognition research may be applicable to proof. However, research shows that there are aspects of the proof construction process that may not be addressed in other problem-solving research (e.g., Savic, 2015). The language, structure, and reasoning required in proof constructions can be viewed to be unique to mathematical proofs and form a special writing genre (Selden & Selden, 2013). Savic (2015) also found that when the multi-dimensional, problem-solving framework developed by Carlson and Bloom (2005) is utilized in context of a proof task, adjustments of the framework are needed for the proof processes; in particular, there is a need to study metacognitive processes in proof more directly.

In this poster, we discuss the research that aims to explore the phenomenon of individuals' metacognitive processes during the construction of mathematical proofs. We identify the areas of the Carlson and Bloom framework in which metacognitive processes could potentially be important for proof construction, and we will demonstrate these with active engagement during poster presentation with the audience. During this interactive poster presentation, participants will be asked to share their thinking processes on proof tasks while we highlight the aspects of their metacognitive processes.

References

Alcock, L., & Simpson, A. (2002). Definitions: Dealing with categories mathematically. *For the learning of mathematics*, 22(2), 28-34.

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational studies in Mathematics*, *58*(1), 45-75.

Committee on the Undergraduate Programs in Mathematics. (2015). *Curriculum guide to majors in the mathematical sciences.* Washington DC: Mathematical Association of America.

Inglis, M., & Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*, *43*(4), 358-390.

Lerman, S. (2020). *Encyclopedia of mathematics education*. Springer International Publishing AG.

Savic, M. (2015). On similarities and differences between proving and problem solving. *Journal of Humanistic Mathematics*, *5*(2), 60-89.

Selden, A., & Selden, J. (2013). Proof and problem solving at university level. *The Mathematics Enthusiast*, *10*(1), 303-334.