Identifying Tensions in a Small Group Work on an Open-Ended Modelling Task

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We analyze students' cooperative small group work on an open-ended mathematical modelling task. Introduction of new ideas and techniques challenged traditional students' views on mathematics and conventional solution routines. This caused tensions in the student activity system pointing towards primary and secondary contradictions. To identify contradictions, their discursive manifestations were used.

Keywords: mathematical modelling, activity theory, contradictions, discursive manifestations of contradictions

The role of mathematics in life sciences has been continuously growing over past decades. Biology eventually replaced physics as the principal research partner for mathematics, and "after a century's struggle, mathematics has become the language of biology" (Steen, 2005, p. 22). An emergent need for professionals combining the knowledge of biology and mathematics stimulated changes in biology education, "the need for basic mathematical … literacy among biologists has never been greater" (Gross, Brent & Hoy, 2004, p. 85). New demands on the education of future biologists created major challenges for both disciplines calling for the reform of undergraduate biology education which "is burdened by habits from a past where biology was seen as a safe harbour for math-averse science students" (Steen, 2005, p. 14). New educational trends promoted interdisciplinary integration suggesting that "concepts from biology should be integrated within the quantitative courses that life science students take, and quantitative concepts should be emphasized throughout the life science curriculum" (Gross, Brent & Hoy, 2004, p. 86). Higher education institutions were encouraged to create strong interdisciplinary curricula integrating life sciences, mathematics, physics, and information science (Steen, 2005). In applications, mathematical modelling (MM) is often used to describe real-world problems in mathematical terms and analyze the real system gaining a deeper insight allowing to understand the future and take necessary actions. Motivated by the idea that MM can serve as a "didactical vehicle both for developing modelling competency and for enhancing students' conceptual learning of mathematics" (Blomhøj and Kjeldsen 2013, p. 151), in this paper we analyze the work of biology undergraduates on an open-ended MM task.

Design of the Study

An extra-curricular MM project for the first-year undergraduate biology students at a large Scandinavian university was designed to engage students into solution of biologically meaningful tasks. A group of twelve volunteer students (nine female and three male) met for five four-hour sessions where they were introduced to fundamentals of mathematical modelling (both theory and worked out examples) and worked on similar tasks in small groups of three to five students. Only two students had previously taken mathematics courses and ten were enrolled at the same time in a regular first-semester compulsory mathematics course. Our main goal was to demonstrate applications of different mathematical ideas and tools in life sciences and increase students' motivation for learning mathematics. Students' work on MM tasks was as a cooperative learning understood as "students working together in a group small enough that everyone can participate on a collective task that has been clearly assigned … without direct and immediate supervision of the teacher" (Cohen, 1994, p. 3). In the very first session students were introduced to basic ideas of modelling and a model of a modelling cycle illustrated with relevant examples. To get an idea of students' initial preparedness for mathematical modelling tasks, they were asked to work on an open-ended task 'Rabbits on the Road' (Harte, 1988, pp. 211–213).

> Driving across Nevada, you count 97 dead but still easily recognizable jackrabbits on a 200-km stretch of Highway 50. Along the same stretch of highway, 28 vehicles passed you going the opposite way. What is the approximate density of the rabbit population to which the killed ones belonged?

The work of two groups of students was video recorded and transcribed. The data set also includes answers to two self-administered questionnaires on a 5-point Likert scale on students' perception of mathematics and its relevance for biology. For more details about the activity, selection of tasks, and organization of students' work we refer to Rogovchenko (2021).

The research question we address in this paper is: *What contradictions arose during a small group work of biology undergraduates on an open-ended modelling task and how they were manifested*?

Activity Theory and Contradictions

Activity theory (AT) provides a versatile tool for inquiry into various aspects of interactions in a teaching and learning process. The main unit of analysis in AT is the activity system composed of six core elements. A mediational triangle is formed by a subject (an individual or a group of individuals) dealing with an object (a desired outcome) via mediating tools or artefacts (material and conceptual). The object embodies the meaning and purpose of the system, and the analysis of the system is based on point of view of the subject. The activity of the subject is directed towards the object and is transformed into outcomes through the tools. The base of the triangle represents the contextual characteristics of the activity system. It is composed of the community (individuals and groups that share with the subject an interest in the same object), the rules (explicit or implicit) that regulate the actions of the subject towards an object, and the division of labor (horizontal and vertical) stipulating the division of tasks between the community members. The model of the activity system we use in this paper is represented in Figure 1. With the main focus on the student group, the six core elements of the activity system are contextualized as follows. The subject is a student group acting towards the object which is related to desired outcomes (engaging in a MM activity; learning new practices; gaining new skills, solving assigned problems). To achieve the goals, tools (instruments, mediating artefacts) are used including MM tasks; problem-solving strategies; mathematical symbols, concepts, procedures, and results. The object and tools can eventually change impacting in turn the activity. A wider community includes the MM project group, fellow students, family, friends, educational officials, academics influencing the activity system. The system is regulated by the rules -- social-mathematical norms, university regulations, class culture, educational pedagogy, expectations of peers. The division of labor defines the roles and responsibilities of community members along with their expectations of each other's roles.

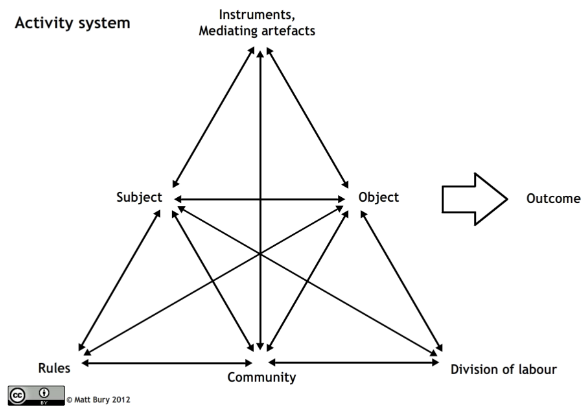

Figure 1. A model of the second-generation activity system. Adapted under CC-by-SA-3.0, Bury (2012).

Contradictions and Their Discursive Manifestations

Each activity system continuously evolves through collective actions; it is not isolated and interacts with other activity systems. In activity systems, "equilibrium is an exception, and tensions, disturbances, and local innovations are the rule and the engine of change" (Cole & Engeström, 1993, p. 8). External influences are appropriated by the system and modified into internal factors which impact the system's development. The third generation of AT identifies contradictions as "historically accumulating structural tensions within and between activity systems" emphasizing "the central role of contradictions within an activity system as sources of change and development" (Engeström, 2001, p. 137). Contradictions should not be perceived as system's deficiencies but rather as opportunities for transformations that change the activities. Implementation of new teaching approaches, ideas and tools disturbs activity systems and creates tensions which manifest contradictions in the system that should be resolved to improve the quality of education. The process of using contradictions to promote learning and change is referred to as expansive learning and should be understood as "construction and resolution of successively evolving contradictions" (Engeström & Sannino, 2010, p.7). Our recent research discusses tensions arising in the modeling project analyzing two activity systems for students and for the project team (Rogovchenko, 2023).

Contradictions are the systemic phenomena which develop in time; they cannot be identified directly. Contradictions should be "approached through their manifestations" (Engeström & Sannino, 2018, p. 49), and we use the term 'tensions' for the latter. Tensions signal internal contradictions in the activity system both within its core elements (primary contradictions) and between these elements (secondary contradictions). In a network of interacting systems tensions may point towards tertiary and quaternary contradictions (Engeström, 1987) which are out of the scope of this paper. Engeström and Sannino (2011) introduced a methodological framework for the identification and analysis of discursive manifestations of contradictions distinguishing four main types described in Table 1. We use this tool to analyze the transcripts of students' small group work on a MM task.

Table 1. Types of discursive manifestations of contradictions (adapted from Engeström & Sannino, 2011, p. 375).

Manifestation	Features	Linguistic cues

Dilemma	Expression or exchange incompatible evaluations	"On the one hand […] on the other hand", "yes, but"
Conflict	Arguing, criticizing	"no", "I disagree", "this is not true"
Critical conflict	Facing contradictory motives in social interaction, feeling violated or guilty	Personal, emotional, moral accounts, narrative structure, vivid metaphors
Double bind	Facing pressing and equally unacceptable alternatives in an activity system	"we", "us", "we must", "we have to", pressing rhetorical questions, expressions of helplessness

Methodology

Our research focus is mainly on the student group whose traditional views on mathematics and conventional solution routines were challenged in the project when new ideas and techniques of MM were introduced. It was expected that tensions in the activity system would arise, in particular, in relation to student engagement, understanding of a modelling task, comprehension of the mathematical content, and approaches to solution of a modelling task. Since tensions in the system were anticipated, deductive thematic analysis of the data was conducted to identify the presence of contradictions in our activity system. The list of tensions-related themes was generated, and the search for discursive manifestations of contradictions was made by the authors based on the descriptions and linguistic cues in Table 1. Full session transcripts were analyzed for two groups working on the task, the transcripts of a 42-minutes long recordings with 5,076 words and 7,874 words respectively. The word count includes speaker labels (Student 2 or Teacher), time stamps (01:00 or 01:26-01:38), and short comments of transcript writers ("inaudible" or "students work silently"). The initial screening of full transcripts was made, and eight episodes were selected for the analysis in this paper. Separate coding was used for identifying four types of discursive manifestations of contradictions (as in Table 1) and four types of tensions related to (i) student engagement, (ii) understanding of a modelling task, (iii) comprehension of the mathematical content, and (iv) approaches to solution of a modelling task. Tensions that were identified were in turn coded in relation to core elements of the student activity system and associated primary or secondary contradictions. Both authors independently coded each of the eight episodes; the count of discursive manifestations and tensions is shown in Tables 2 and 3.

Table 2. Discursive manifestations of contradictions identified by the authors.

Manifestation	First author	Second author
Dilemma (D)	3	4
Conflict (C)	4	5
Critical conflict (CC)	6	6
Double bind (DB)	7	6

Table 3. Tensions identified by the authors.

Tension	First author	Second author
Engagement (E)	2	2

Understanding of the task (U)	8	9
Comprehension of the mathematical content (MC)	2	2
Approaches to solution (A)	6	7

The disputed issues were reviewed again, the authors discussed the codes applied in the episodes and negotiated common meanings coming to the agreement.

Analysis

Eight episodes were selected from the transcripts to illustrate tensions identified through discursive manifestations of contradictions. For each episode we provide the codes used in the analysis of students' work.

Episode 1

Student 1: Eh, and then we have to assume how many cars are running over a rabbit then.
Student 3: Oh, God…
Student 2: Not everyone runs over a rabbit!
Student 1: No, it is not so! And that is not even a half either, it is maybe five percent. (C, A)
Student 2: Right.
Student 1: Five percent is one out of 20.
Student 3: But one out of 20 sounds like quite a lot of rabbits then. (D, A)
Student 1: Yes.
Student 3: So, one out of 20 is not so wrong. Take five percent then.
Student 1: Five percent.
Student 3: We just have to put… we just have to put something like this. (DB, A)
Student 1: But no! We have 97 dead [rabbits]. But if we assumed that they died in the last 24 hours, then we divide it by the number of cars. Then we find that percentage. (DB, A).

Episode 2

Student 5: Can't we just divide the number of individuals in the area and find out how many are there in each square meter?
Student 1: But we don't know what the area is, we only know the stretch. We don't know how wide it is. (DB, U)

Episode 3

Student 3: No, but let's just assume, as she says, that we have 20 miles that way and 20 miles that way, then we have 20 miles on the stretch.
Student 1: Yes.
Student 3: But then there is still a very large area, for only 20,000 rabbits. (D, U)
Student 1: Yes, I think, I don't know if this is what they mean by density.]So, density.
Student 3:]No.
Student 1: There is no point in calculating density per hypothetical square kilometer. (C, A)
Student 2: You never know how big it is as well…
Student 1: So, I do not know if that's what they mean by density, or if it is just a part of the population they're looking for. It is like 'What is the approximate density of the rabbit population?'

Student 3: To obtain density, we have to have the whole area; we can't use just a stretch. You must have something like a square. (DB, A)

Episode 4

Student 4: Yes, you have to say how many remain alive, that is something.
Student 2: Yeah. Not all [rabbits] who jump over there die.
Student 2: No, and there were only 28 cars, and you don't know for how long the bodies have been laying there. (C, U)
Student 4: No, that's exactly it. 'Easy recognizable,' then it's like that, then it's like they have just been ran over, so you can see what they are. (C, U)

Episode 5

Student 4: No, you find kilometers per rabbit. Ninety-seven, what did you get?
Student 3: Hm?
Student 4: Ninety-seven divided by two hundred
Student 3: That's half a rabbit per kilometer
Student 4: Yes, approximately one half
Student 2: So, half a rabbit dies per kilometer
Student 4: Yes, per kilometer
Student 3: Every two kilometers there is a dead rabbit.
Student 4: Yeah, but that's hell is not right, it's not logical at all! (CC, U)

Episode 6

Student 3: Since he] drives two hundred kilometers an hour
Student 2: What if we just round it [velocity] up to a hundred?
Student 4: Sure.
Student 3: It makes it much easier to round up to a hundred.
Student 4: Yes, but we have, we have this bizarre thing, so it will be the same anyway. (CC, U)
Student 2: Hmmm, this was a frustrating task. You think it's a math problem, but then you know nothing. So, you can't solve it. (CC, U)
Student 4: Yes, we have to, we have to [solve it] somehow. Find our own numbers. (DB, U).

Episode 7

Student 1: Won't it be too many, because it gets a bit more complicated when you start pulling so many different factors into
Student 4: Yeah, but it is only getting … The answer will just be better and better (D, U)
Student 1: That's about it.
Student 4: Then there will be fewer factors that will screw us up. (CC, U)
Student 2: Just assume something about all the factors in the end!
Student 1: I wonder if it is possible to solve this task here. (CC, U)
Student 4: Sure, it is.

Episode 8

Student 4: I'm really just looking forward to getting an answer, because this was really annoying! (CC, U)

Student 2: Hmm, but I think I will be just as annoyed then, because there is no correct answer. But it will be fun to hear what can be assumed. (C, U)

Conclusions

Activity theory acknowledges that contradictions can potentially transform the activity system, but this may not necessarily happen. Contradictions are present in each activity system; they may originate from multiple perspectives, cultural and historical traditions, different interests of members of an activity system. Experiencing problems, conflicts, disturbances that originate from the contradictions in the activity system, individuals or groups of individuals attempt to change the system to alleviate tensions. Nevertheless, resolution of contradictions by individual actions alone is not possible, significant changes in the activity system require cooperative actions leading to the development of a historically new form of activity. Ultimately, "an expansive transformation is accomplished when the object and motive of the activity are reconceptualized to embrace a radically wider horizon of possibilities than in the previous mode of the activity" (Engeström, 2001, p. 137).

In our MM project, primary contradictions were observed within the subject (engaged learners vs passive participants), tools (new methods of population dynamics vs student's traditional mathematics toolkit), rules (student-oriented learning in MM sessions vs teacher-centered pedagogy in regular classes), object (scientifically based understanding of phenomena vs intuitive interpretation of the reality), division of labor (lack of scaffolding vs increased volume of the independent student work). Secondary contradictions were manifested between the rules and object (students' engagement with extra-curriculum modelling tasks vs the need to perform well in the exam; instructor's wish to infuse teaching innovations in students' learning vs curriculum and time constraints), between the tools and division of labor (new thinking required by MM tasks vs learners' expectations how the learning of mathematics should be organized), and between the community and object (creating numerate mathematicians and students' conventional perceptions of mathematics and their previous bad experience with it).

Although the work on an open-ended modeling task was challenging for students, they mentioned the work with assumptions as the best thing in the project: "It made me think in a different way than usual and it was exciting. Making assumptions was new to me." "The assumptions. To understand what assumption is important and which is not." We observed that students interest in mathematics was gradually increasing, and they were opening up for new ways of learning the subject. Students' answers prompted that the use of MM in teaching biology undergraduates stimulates their interest in mathematics and its interdisciplinary applications: "It was interesting to try to solve problems without knowing all about them." "It was social and challenging; it was also very interesting to experience new educational methods." Students' positive feedback on the project indicates possibilities for the expansive learning in the activity system which can transform students' learning of mathematics by enriching the standard curriculum with biologically meaningful mathematical tasks.

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