Reasoning Students Employed When Mathematizing During a Disease Transmission Modeling Task

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Few studies explicitly focus on students' understanding of differential equations for representing real-world phenomena (Lozada et al., 2021). Rowland and Jovanoski (2004) studied the meanings students have for the terms of an already constructed differential equation and found that across contexts, students tend to conflate amount with rates of change of amounts. Our goal is to document students' reasoning while constructing differential equations that represent real-world phenomena (called mathematization) in a way that helps instructors connect their students' emergent models with normatively correct models. We frame students' reasoning while mathematizing using theories of quantitative, co-variational, and multi-variational reasoning (Carlson et al., 2002; Jones, 2018; Thompson, 2011). Using this lens builds on previous findings that the operations students use on quantities reflect the quantitative relationships perceived by the student, (Larson, 2013), and that the models that students could potentially make depend on (and are constrained by), the quantities the student imposes onto the situation (Czocher et al., 2022).

The purpose of this study was to (1) catalog the quantities students impute while modeling a disease transmission scenario from first principles and (2) document students' rationales when combining those quantities. We wanted to know: What are participants' existing ways of reasoning when writing equations to model of the spread of a disease throughout a community? To answer this question, we present work from STEM undergraduates who participated in a set of individual cognitive task-based interviews via Zoom. Data were analyzed by first identifying the quantities participants imposed onto the task scenario and then documenting the participants' (inferred) reason for using specific arithmetic operations$(\mathrm{e.g.}, +, -, \div, \times)$ on quantities.

Our results reveal the rationales for using arithmetic operations to depict specific aspects of the dynamics of spread of disease throughout a community. All three participants constructed models for the instantaneous rate of change of the sick and healthy populations, though at times using unconventional notation. All three participants constructed a model for the change in the healthy population by covarying healthy people and sick people. They did this by noting that the decrease in the healthy population is equal in magnitude to the increase in sick population. Two participants depicted this relationship using the arithmetic operation $-$, while one participant, Rua, chose not to use any notation to depict this relationship. We infer that Rua was covarying the number of sick people and the number of healthy people, but not multi-varying sick, healthy and time. Our contribution extends what is known about students understanding of differential equations by reporting on how covariational and multivariational reasoning played a role in students' construction of a differential equation that models disease transmission. We infer that covariational and multivariational reasoning were observable because our participants were working on differential equations modeling task focused on disease transmission.

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Introduction

- Few studies exist on students' understanding of differential equations for representing real-world phenomena (Lozada et al., 2021).
- Students tend to conflate amounts of change and rates of change of amounts in many different contexts that require differential equations (Rasmussen & King, 2000; Rasmussen & Marrongelle, 2006; Rowland & Jovanoski, 2004; Mkhatshwa, 2018).
- **Goal:** document students' reasoning while constructing differential equations that represent real-world phenomena (called mathematization) in a way that can inform instructors about how to connect their students' emergent models with normatively correct models.

Theoretical Perspective

Mathematical model - a conceptual system consisting of **elements, rules governing interactions**, the **relationships between elements**, and **operations**. Expressed into the world through different representations (Lesh & Doerr, 2003)

Elements are:

- **Quantitates** - triple of an object, attribute, quantification (Thompson, 2011).
  - Ex attributes: length, volume, cardinality, speed, and density (Ellis, 2007).
  - A quantity is cognitively distinct from (mathematical) variables.

Relationships between elements and operations are described by

- **Covariational reasoning** - cognitive activities for coordinating two varying quantities (Carlson et al., 2002)
- **Multivariational reasoning** - the cognitive activities for coordinating more than two varying quantities (Jones, 2018)

RQ: What are participants' existing ways of reasoning when writing equations to model the spread of a disease throughout a community?

Results

- 3 participants constructed a model for the change in the healthy population by covarying healthy people and sick people.
  - All noted that the decrease in the healthy population is equal in magnitude, and inversely related to, the increase in sick population.
  - 2 depicted this relationship as $\frac{ds}{dt} = -\frac{dh}{dt}$
  - 1 depicted this relationship as $\frac{ds}{dt} = \frac{dh}{dt}$
- We infer that the decision to use/not use the symbol– is rooted in the participants multivariation of sick, healthy and time.

Methods

Data collection

- Cognitive task-based interviews with 20 STEM majors who completed or were enrolled in differential equations
- Report 3 computer Science majors on a Disease Transmission (SI compartment model) task.

Analysis

- Identify quantities participant imposes onto task
- Note arithmetic operations used on quantities
- Document participants' reason for using that arithmetic operation
  - quantitative, covariational, and multivariational reasoning.

Rua

$$\Delta S(t) = (H(t) \cdot S(t) \cdot r) \qquad \Delta H(t) = (H(t) \cdot S(t) \cdot r)$$

Rua: It's (referring to $\frac{dW}{dt}$) the same (as $\frac{dI}{dt}$). Um, I think there was one of the equations that we were working through, where I believe the aspect of negativity or positivity was something I was kind of trying to internalize or view, probably, where I think this is a similar outlook just depending on how you're looking at it. Uh, it would either-- you know, if that rate is something that's representative of decay or growth with regards to sick, then it should be the opposite for healthy, and vice versa. If it was growth or decay for healthy, then it should be the opposite-- I guess that's because they're inversely related-- for the sick.

Winnow & Roion

Winnow:

$$\frac{ds}{dt} = \frac{m^{5}}{s(t) \cdot h(t)} \cdot h(t) \cdot r \qquad \frac{dh}{dt} = -\left(\frac{m}{s(t) \cdot h(t)} \cdot h(t) \cdot r\right)$$

Winnow: That would show the rate of change of healthy people, so I guess it would be a negative number because the healthy people would be-- the number of healthy people would be decreasing. Later says: We got was minus 5 healthy people because we had 5 new sick people

Roion:

$$\frac{\partial I}{\partial t} = 1.\left(a \cdot \left(\underbrace{c \cdot w(t)}_{\text{well}} \cdot \underbrace{I(t)}_{\text{infected}}\right)\right) \qquad \frac{\partial W}{\partial t} = -\left(a \cdot c\ I(t)\right) \cdot w(t)$$

Roion: OK. So for this one (referring to $\frac{dW}{dt}$), it'd pretty much be the same (as $\frac{dI}{dt}$) but in the negative direction, as the amount of well people would be decreasing as more infected…

ER: OK. So are you saying that $\frac{dW}{dt}$ is the same as $\frac{dI}{dt}$ but negative?

Roion: Yes.

ER: OK. And what tells you that that's the case?

Roion: It's because as more people are infected, that means less people in the community are actually well. So it's decreasing as time goes on, as more of the infected people interact with the well people.

Discussion

- We extend what is known about students' understanding of differential equations by reporting on how covariational and multivariational reasoning played a role in students' construction of a differential equation that models disease transmission.
- We infer that covariational and multivariational reasoning were observable because our participants were working on modeling disease transmission using differential equations.

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