Balancing the Unbalanceable: A Theoretical Perspective for Teaching and Learning in Undergraduate Mathematics Classrooms

Candice M. Quinn
University of North Alabama

I propose a theoretical model for teaching, learning, and assessing mathematics learning in the undergraduate mathematics classroom combining participatory and acquisition views of learning. The model, Balanced Learning Needs Framework, was created from an organization of the 10 learning needs proposed by Sfard in 2003. I align the model with different forms of classroom instructional techniques used in an undergraduate Calculus I classroom. Included with the organization of the learning needs, this conceptualization has the potential to impact both teaching and assessment methods in undergraduate mathematics classrooms.

Keywords: participatory learning, acquisition learning, Calculus I, assessment

Many theories and frameworks exist to help researchers try to understand the phenomena of teaching, learning, and assessment within mathematics education (Radmehr & Drake, 2019). Dubinsky and McDonald (2001) stated how: “models and theories in mathematics education can support prediction, have explanatory power, be applicable to a broad range of phenomena, help organize one’s thinking about complex, interrelated phenomena, serve as a tool for analyzing data…” (p.275). In this paper I provide a framework designed for a larger study that was used to investigate learning in an undergraduate Calculus I course. This proposed framework helped to organize the researcher’s thinking about learning (Dubinsky & McDonald, 2001) in both social and individual classroom. Two main learning theories guided the construction of the proposed framework: situated and constructivist. Together, the theories and framework guided each aspect of construction of the Calculus course: setup, course design, classroom instruction, and assessment of learning.

Theoretical Background

Sfard (1998) metaphorically described students' development of knowledge under situated learning theory as participatory and under constructivist learning theory as acquisition. I consider a combination of the two learning theories (using their metaphors) along a continuum (Figure 1). This combination of situated theory as participatory learning and constructivist theory as acquisition learning is the theoretical foundation for proposed framework. Vygotsky’s sociocultural view of learning is placed in the middle of the continuum. Although this view of learning does place importance on the social process of learning, Vygotsky still considered the internalization of knowledge in the learner’s mind (Sfard, 2003).

While creating the framework, I reflected on both metaphors for learning, and considered knowledge as something that is both acquired in the individual learner's mind and co-created from participation within a community of practice. The largest difference between the two perspectives is regarding the emphasis and need of social interaction and where knowledge is constructed (i.e., in the mind of the individual learner or in the community). The framework is created with the notion of holistic learning where an individual both acquires knowledge (cognitive-acquisition perspective; Cobb & Bowers, 1999; Sfard, 1998) and constructs knowledge through participation within a community of practice (situated-participatory perspective; Cobb & Bowers, 1999; Sfard, 1998; Lave & Wenger, 1991).

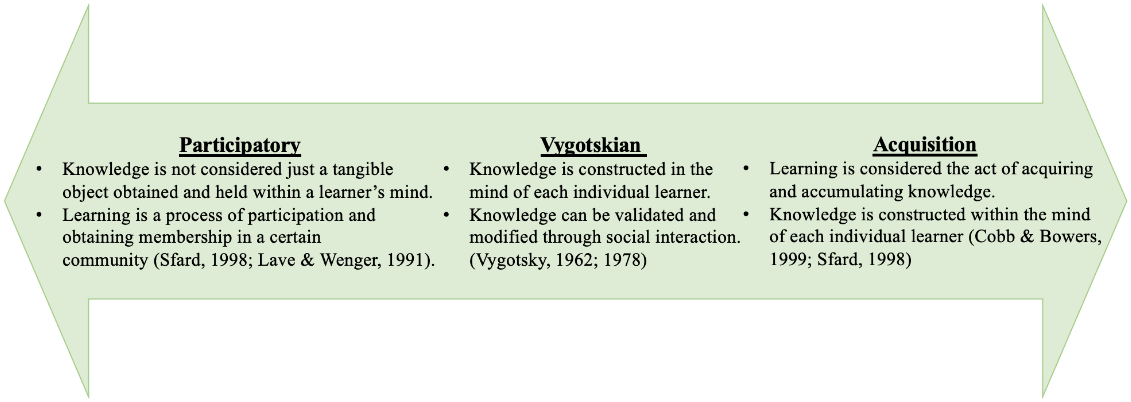

Figure 1. Participatory-acquisition Learning Continuum.

Alignment of Learning Needs

Expanding upon Sfard's (2003) presentation of the 10 needs of mathematics learners, I constructed an organized model to represent these learning needs (LN) within the situated (participatory) and constructivist (acquisition) learning theories. This organization is a representation of how the LN can be systematized to enhance teaching and learning for undergraduate mathematics, as well as focusing on equitable assessments of learning. Each of the 10 LN corresponds to knowledge creation either within the mind of the learner or within the community of learners.

We can align the respective LN with the ideas of learning and knowledge to either an individual unit (i.e., acquisition) or a group unit (i.e., participatory). That is, each need may be internally driven (i.e., for internal knowledge construction) or socially driven (i.e., for participation within a community), or a combination of both. The majority of the needs as defined by Sfard (2003) are primarily driven either only by one or the other. I classify the LN into three main categories: internal needs, social needs, combination needs.

Internal Needs

There are four internal LN that each student has in order to construct mathematical knowledge: a need for *meaning*, *structure*, *repetitive action*, and *difficulty*. Constructivist views of learning attend to the internal needs of the learner and consider knowledge creation within the mind of the individual learner. Either Piagetian or Vygotskian, or both, perspectives of constructivism influence each of these needs.

All learners have this need to make *meaning* of the world around them and communicate their *meaning* of the world to others (Bruner, 1997; Sfard, 2003; Vygotsky, 1962). The creation of *meaning* is an internal process used by all learners to understand and make sense of the world around them (Piaget, 1953; Sfard, 2003). This view of *meaning* as internally constructing knowledge about the world around them and assessing one's internally constructed knowledge aligns with the acquisition view of learning.

The need for *structure* incorporates both the Piagetian notion of reorganizing mental schemes when acquiring new information and Vygotsky's hierarchical organization of new concepts (Bruner, 1997; Sfard, 2003). The need for *structure* incorporates what students already know (i.e., previous or prerequisite mathematical knowledge) and connects this knowledge with new ideas and concepts (Sfard, 2003). This view of *structure* is an extension of the need for meaning and builds on one's internally constructed knowledge, and therefore also aligns with the acquisition view of learning.

Through the process of *repetitive action* with mathematical ideas learners can reorganize their internalized knowledge (i.e., structure) to add the new mathematical object into their current knowledge schema (Piaget, 1953). This view of *repetitive action* is an extension of the need for structure, as a way of organizing new meaning, and rectifying one's understanding (Sfard, 2003) which considers knowledge to be something that is internally constructed. Therefore, *repetitive action* aligns with the acquisition view of learning.

The final internal need is the need for *difficulty*. Sfard (2003) stated that "true learning implies difficulties" (p. 366), what these difficulties are, however, is not the same for all learners. Therefore, learning requires a level of *difficulty* unique to each learner depending on previous constructed mental schema (Piaget, 1953). This corresponds to Vygotsky's Zone of Proximal Development (ZPD) in which each student has already built a certain level of ability and learning occurs inside of their ZPD with assistance (Vygotsky, 1978). This view of *difficulty* incorporates one's internally constructed knowledge and cognitive abilities, which aligns with the acquisition view of learning.

Social Needs

There are four social LN that each student has in order to construct mathematical knowledge: a need for *social interaction*, *verbal-symbolic interaction*, *well-defined discourse*, and *sense of belonging*. As with the internal LN, the social needs may be met with different means for each learner, and each are further described below.

Social interaction can be broadly defined as an exchange of ideas between an individual and another person via written or verbal communication (Sfard, 2003). Vygotsky (1962) suggested that *social interaction* is an essential part of learning especially for one to obtain a conceptual understanding of objects or ideas. *Social interactions* provide the space for students to challenge their preexisting notions about mathematical ideas. Lave and Wenger (1991) suggested that *social interaction* is essential for one to show knowing by doing within a community (i.e., situated learning theory). Therefore, the need for social interaction aligns with the participatory view of learning.

Social interaction and being able to communicate one's ideas and knowledge require learners to use both *verbal* (i.e., talking and listening) and *symbolic* (i.e., reading and writing) representations of knowledge and thoughts (Sfard, 2003). This idea that knowing comes from having a word or symbol is grounded in discursive psychology (e.g., Harre & Gillet, 1995). Sfard (2003) described the importance of either a *verbal* or *symbolic* use to give meaning as "one just cannot construct the meaning of a concept before introducing a word or a symbol with which one can think about that concept. The sense of *understanding* [emphasis added] then develops through the use of the word or symbol" (p. 374). Therefore, *verbal* and *symbolic representations* are a need of learning mathematics because these representations provide the tools for social interactions to occur, and aligns with the participatory view of learning.

The need for *well-defined discourse* follows from the needs for social, verbal, and symbolic interactionism. *Discourse* in mathematics becomes well defined when sociomathematical norms centered around what is considered mathematical argumentation are established in the classroom (Yackel & Cobb, 1996). Engagement in mathematical argumentation requires students to have intellectual autonomy (Yackel & Cobb, 1996). Specifically, students would need to understand their own "mathematical capabilities" (Yackel &

Cobb, 1996, p. 473) as they judge and decide what constitutes a mathematical argument of another student. This notion of *discourse* in the classroom among students includes both social interaction and the use of verbal or symbolic representations of knowledge, and aligns with the participatory view of learning.

A *sense of belonging* is a human need to feel valued as a member of a community. A student's *sense of belonging* can be defined as their "sense of being accepted, valued, included, and encouraged by others (teachers and peers) in the academic classroom setting and of feeling oneself to be an important part of the life and activity of the class" (Goodenow, 1993, p. 25). Ultimately, if students do not feel they belong in a field of study, classroom, or group they are less likely to continue or engage with that group (Cheryan et al., 2009; Strayhorn, 2019), and thus makes this is an important need that must be addressed for all mathematics learners (Sfard, 2003). This need to belong aligns with the participatory view of learning because it includes a community or members outside of oneself.

Combination (Internal and/or Social) Needs

The last two LN either fall in the middle of the proposed continuum between (i.e., significance and relevance) or span (i.e., balance) both the participatory and acquisition metaphors for learning. First, *significance and relevance* as one LN is placed in the middle of the continuum. This need is what can drive the motivation for the learner (Sfard, 2003). If we consider the ideas from Piaget where new knowledge can only grow out of existing knowledge, then learners must internalize the *significance* of what they are learning as they reorganize their mental schemes (Sfard, 2003). If we consider instead the importance of social interaction in learning, the *significance and relevance* of new knowledge can come through social interactions and participation within a community (Lave & Wenger, 1991). Because the *significance and relevance* can be formed/made either internally by one's own drives and thoughts, or influenced by one's peers in their learning community, this LN is placed in the between the internal and social needs.

The last LN may seem obvious now and yet may be the hardest one to meet: the need for *balance*. This essential LN both corresponds to the requirement for the learning environment to maintain a balance between the participatory and acquisition metaphors for learning; as well as attempting to balance the other nine LN for each student. A further description of how the balance need is deciphered and represented is provided in the following section.

Balanced Learning Needs Framework

The framework presented (Figure 2) arranges the 10 LN along a continuum of two main metaphors for learning (i.e., participation and acquisition). This continuum is represented as a balance scale, where the *balance need* is represented as the lever, holding the remaining nine needs, placed upon a fulcrum that can pivot between the two learning metaphors. I chose this representation to demonstrate the importance of both social and individual LN. The idea is that if only one side of the balance is met in the learning environment, then learning is hindered for the students. The arrangement of the needs toward the participatory or acquisition side was done meaningfully; the vertical placement, however, is not meant to imply any order among those needs placed on each side. The placement on the left (or right) only implies these constructs *lean toward* the participatory metaphor (or acquisition metaphor) for learning.

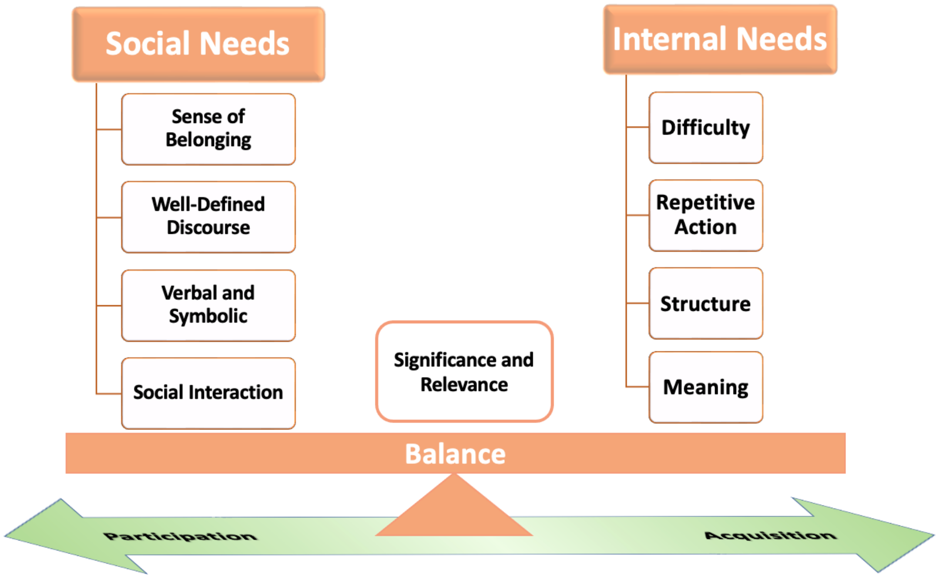

Figure 2. Balanced Learning Needs Framework

The author posits that the Balanced Learning Needs (BLN) framework presented here is a multidimensional construct with interrelated elements that impact student learning in either social or lecture settings. The framework can be utilized by academics to reflect upon the learning environment within their undergraduate mathematics courses and the implications for instructional and assessment practice. In the next section, I provide an example of how different classroom instructional methods from a Calculus I course that align with each learning need.

BLN Framework with Classroom Instruction

Sfard (2003) suggested "to meet learners' multifarious needs, the pedagogy itself must be variegated and rich in possibilities. The learning individual is a complex creature with many needs that must all be satisfied if the learning is to be successful" (p. 384). Educators must help create learning environments that allow students the opportunity to meet each need through equitable instruction (Sfard, 2003). Therefore, equitable instruction refers to the instructional techniques used to support the development of the 10 needs for *all* students and promote a balance between the needs, the metaphors of learning, and the students. For instance, collaborative group work is an equitable instructional practice that, if implemented with fidelity, students are given opportunities to explain their thinking and justify their mathematics using both verbal and written methods (Perry, 2018).

Often, it is difficult to connect theories of teaching and learning with the actual implementation of classroom activities and assessments. In Table 1, I provide an outline of how classroom activities and instructional practices meet each of the LN in the BLN framework. The course components described in Table 1 were implemented in a Calculus I classroom that also included individual and group testing methods. I provide a rationale for the assessment methods in the next section. Note, in the table below, *lecture* refers to instructor-centered classroom moves (i.e., instructor reviewing prerequisite material or concepts), without student-to-student engagement. Additionally, lecture may include whole class discussions and clarifications of mathematical language and symbols.

Table 1. Learning Needs Matched to Course Instructional Techniques.

Learning Need	Course Component	Description of how the component of the course was designed in an attempt to meet each learning need
Meaning (Internal)	Lecture and web-based homework	Allows students to work through problems and build their own internal meaning.
Structure (Internal)	Lecture and web-based homework	During lectures, instructor attempts to build structure in lecture by identifying connecting concepts from student's previous knowledge. Homework allows students to extend and organize new information to their previous knowledge.
Repetitive Action (Internal)	Lecture and web-based homework	Individual practice time in lecture and the homework time on the computer allow the students multiple problems to practice.
Difficulty (Internal)	Lecture and web-based homework	Instructor designs lecture and homework problems with a certain level of difficulty to allow the students to productively struggle when building their knowledge.
Social Interaction (Social)	Group Setting and web-based homework	Student-centered activities during group work require students to talk through problems and social interact to complete the learning activity. Students could choose to either work alone or with a peer on the homework allowing for more possible time for social interaction.
Verbal and Symbolic (Social)	Lecture and Group Setting	During lecture, after students initially explored a concept during group work, instructor provides the students with the formal terminology and symbolic representations of the concepts. During group work, students needed to communicate with one another using both verbal and symbolic representations for the mathematics.
Well-defined Discourse (Social)	Lecture and Group Setting	During lecture, the instructor sets the sociomathematical norms and models mathematical discourse that is needed for the students to engage in mathematical argumentation. During group work, the students establish the group's norms and engage in discourse.
Sense of Belonging (Social)	Lecture and Group Setting	During the lectures, instructor attempts to encourage all students to participate in whole class discussions and feel that they could share any idea, thought, or question. During group work, instructor monitors each group to ensure each student felt comfortable: (1) with their group members, (2) to voice their ideas, and (3) encouraged to share their ideas with their group.
Significance and Relevance (Combination)	Lecture, Group Setting, and web-based homework	Because this need is driven by the learner and can be met either while working alone or in a group setting, this need can be met for any learner within any one or all of the course components.
Balance (Combination)	The course as a whole	The instructor attempts to maintain balance by providing equal opportunity for each need to be met in their designed course component.

An instructors view of where knowledge is constructed largely impacts the environment of the classroom and how information is presented to students. Although student-centered instruction has been shown to increase student learning outcomes (e.g., Bressoud, 2011; Freeman et. al, 2014), traditional lecture is still a prominent form of instruction in undergraduate calculus courses (Bressoud et al., 2015). Note in the table above, both lecture and student-centered activities were used in the classroom as instructional techniques. As both aspects of classroom instruction can help students construct their mathematical knowledge. Additionally, web-based homework was included as an instructional technique, as there has been a large increase in the use of web-based homework systems in undergraduate mathematics education (Serhan, 2019).

Discussion

The creation of the BLN framework came from both anecdotal and empirical evidence from students' formative and summative assessments in Calculus I classrooms. Many tests in a mathematics classroom assess what is "easy" to measure and not what is "worth" measuring (National Research Council, 1993), especially at the undergraduate level. Consider the following four main purposes of assessments: an individual student's proficiency of the subject matter in knowledge and skills; an individual student's performance after working within a group; group effectiveness and productivity; and students' interpersonal skills (Webb, 1995). In mathematics classrooms, however, students are usually formally tested only on individual expertise of the subject matter knowledge. The testing of individual knowledge and skills helps perpetuate the competitive nature of learning mathematics. Although competition can also be a motivation for learning, many women in mathematically intensive areas of study can feel isolated in competitively focused environments. Therefore, if a learning environment is created with an attempt to balance the LN of each student using the BLN framework, then a shift in assessment also needs to be considered. For instance, increased use of student-centered teaching strategies creates a mismatch between the learning and assessing contexts: students are asked to work in groups to learn together cooperatively, but are tested individually and potentially competitively. If we consider assessing knowledge in the space it is constructed (i.e., assessing the constructed knowledge on the lever at both sides of the fulcrum) then group testing may be a necessary addition to undergraduate mathematics courses.

Limitations and Future Research

I recognize that the proposed BLN framework being enacted in a classroom might differ depending on the pedagogical beliefs of the instructor. The framework was created from the perspective of one researcher, and still requires further empirical testing and validation. I have begun to explore this framework on the social side using group testing environments. Preliminary results did indicate how a lowered sense of belonging for female students impacted group performance on a group exam (Quinn, 2021). That is, their sense of belonging decreased and they underperformed in the group setting. Additionally, measuring the needs in a collaborative learning environment verses traditional lecture environments may help to show the *balance* needed between the pedagogy and learning environments in the classroom to help all students learn. Future testing of the framework will include individual and group testing methods to measure the internally constructed knowledge (i.e., individual exams) and socially constructed knowledge (i.e., group performance).

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