Designing Tasks to Teach Mathematical Induction

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We conducted a study on the teaching and learning of mathematical induction, based on the principles of didactical engineering, with undergraduate students enrolled in a Discrete Mathematics course at a four-year university. Students' performance on induction tasks improved. We focus here on designing tasks with the potential to foster inductive reasoning and give meaning to the Principle of Mathematical Induction.

Keywords: Mathematical Induction Tasks, Didactical Engineering, Undergraduate Students

The recent research on mathematical proofs is rich, but not many studies specifically address mathematical induction. A proof by mathematical induction may be used whenever we have a statement $P(n)$, where n is a natural number. According to Ernest's (1984) framework, the steps of a mathematical induction proof are:

- basis step, checking that $P(n_0)$ is true (n_0 is the smallest value for which $P(n)$ is true),
- inductive step (checking that $P(k) \rightarrow P(k+1)$, $k \geq n_0$), and
- invocation of the Principle of Mathematical Induction (PMI).

We report on a study on the teaching and learning of proofs by mathematical induction, with undergraduate students enrolled in a Discrete Mathematics course taught by the first author at a four-year university. We focus on the application of the principles of didactical engineering (Artigue, 1994) to design tasks with the potential to foster inductive reasoning and give meaning to the PMI. Figure 1 illustrates the phases of the didactical engineering process.

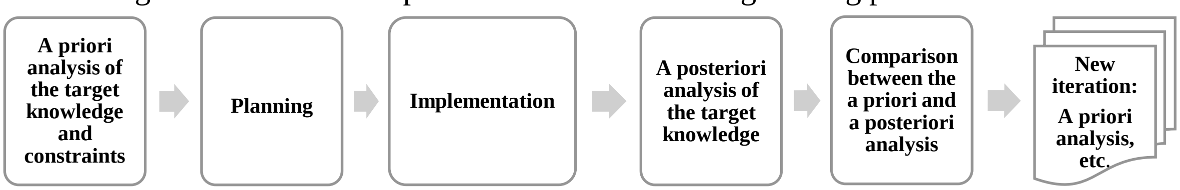

Figure 1. The didactical engineering process adapted after Artigue (1994)

At the beginning of the fourth iteration, the a priori analysis revealed that our students had difficulty giving meaning to mathematical induction as a method of proof. Although our students' performance improved with each iteration, the induction step remained procedural in nature. Ernest (1984) drew our attention to the usefulness of analogies to the PMI. Harel (2001) used tasks that defined sequences recursively and fostered inductive reasoning, thus giving meaning to the PMI. Other studies highlighted the importance of visual representations (Stylianou and Silver, 2004) and gestures used during the teaching and learning process (Kokushkin, 2020). Based on research and our own experience with the teaching and learning of mathematical induction, we designed, implemented, and refined tasks with the potential to foster inductive reasoning and give meaning to the PMI. One such task asked the students to create their own analogy to the PMI (Postelnicu and Gonzalez, 2020), while another task used visual representations. We discuss the use of visual representations, intended to help the students define a sequence recursively and find the general term of the sequence and the formula for the sum.

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