Teaching Assistants' Use of Students' Work When Planning How to Address Misconceptions Inferred from It

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Given the crucial role that the graduate teaching assistants (TAs) play in undergraduate mathematics education, this study investigated how TAs interpret students' work and plan to address students' work in teaching. We analyzed TAs' interpretations and plans according to approaches towards addressing students' misconceptions discussed in the literature: viewing them as simply incorrect regardless of students' thinking behind them, viewing them as a reflection of students' flawed ideas to be confronted and replaced, and viewing them as resources for future learning. Our analysis shows that while TAs' interpretations were often sufficiently rich to support using student misconceptions as resources for future learning, none of their plans used them in this way. Rather, most TAs' plans were aligned with the confront-and-replace approach. Our results suggest that professional development could be designed to help TAs convert their rich interpretations into plans for using student misconceptions as resources for future learning.

Keywords: Graduate Teaching Assistants, Use of Student Work in Teaching, Interpretation of Student Work

Introduction

There has been growing interest in the role that graduate teaching assistants (TAs) play in undergraduate mathematics education (Kung & Speer, 2009; Speer et al., 2005). TAs have direct impact on their current undergraduate students' learning of mathematics by teaching classes, holding office hours, and working in tutoring centers (Kim, 2014; Ellis, 2014) and also will have long-term impact on mathematics education at the university level as the next generation of instructors (Ellis, 2014; Kim, 2014; Musgrave & Carlson, 2017). Recently there have been calls for more research especially about TAs' teaching (American Association for the Advancement of Science (AAAS), 2019). The current study addresses an important aspect of TAs' teaching: how TAs interpret and address their students' work. How teachers interpret students' work and plan to address errors has been studied intensively in K-12 teacher education literature (see the review by Stahnke et al., 2016). However, in comparison, we as a field still have limited understanding of whether and how TAs' interpretations of students' work relate to their instructional decisions. To understand how TAs make instructional decisions based on students' work we conducted a study where TAs analyzed students' written work on non-routine problems and planned to address the issues that they identified. The following research question guided our study:

> How do TAs use students' written work in planning how to address with the students any misconceptions reflected in that work?

Our question focuses on misconceptions because we will consider TAs inference of student understanding from written work and whether/how such inferences are related to their plans to address errors. To address this question, we adopted three ways to view students' misconceptions that we identified in the existing literature on how to use student errors or misconceptions in teaching, which we will detail in the following section.

Theoretical Background

Views on Misconceptions

Three different views of students' misconceptions that we identified in the literature (Smith et al., 1993; Li & Schoenfeld, 2019; Rake & Ronau, 2018) guided our analysis of TAs' interpretation of students' work: viewing misconceptions as simply incorrect regardless of students' thinking behind them, viewing them as a reflection of students' flawed ideas or thinking that need to be replaced, and viewing them as resources for future learning. The differences between these views of misconceptions are significant because they suggest different instructional approaches for dealing with misconceptions, such as simply reteaching the content that students showed the error for, confronting the error and replacing it with the expert counterpart, and providing context to reorganize and refine students' existing knowledge. We analyzed TAs' plans to address students' misconceptions in terms of these three instructional approaches. As summarized in Smith et al. (1993), studies that viewed students' misconceptions as "flawed ideas that are strongly held, that interfere with learning, and that instruction must confront and replace" (p. 115) represented a "fundamental advance from previous approaches that essentially divided student responses into two categories, correct and incorrect" by focusing "attention on what students actually say and do in a wide variety of mathematical and scientific domains" (p. 117). However, they also pointed out that "confront and replace" is problematic in practice because "misconceptions continue to appear even after the correct approach has been taught" (pp. 120-121). Smith et al. (1993) provided a constructivist view of "learning that interprets students' prior conceptions as resources for cognitive growth within a complex system of knowledge" (pp. 115-116) and suggested that refinement of students' existing knowledge for learning was more effective than replacement. Specifically, they argued that "instruction should help students reflect on their present commitments, find new productive contexts for existing knowledge, and refine parts of their knowledge for specific scientific and mathematical purposes" (p. 150). This reconceived view of misconceptions has been advocated for in recent approaches emphasizing sense-making in mathematics and understanding (Li & Schoenfeld, 2019) and in empirical studies investigating student reasoning behind incorrect responses (e.g., Makonye, 2012; Rake & Ronau, 2019). It should be noted that, from this view, errors or error patterns are different from misconceptions. Smith et al. (1993) used misconception as "a student conception that produces a systematic pattern of errors" and differentiated it from "simply designat[ing] a pattern of errors" (p. 119).

Given that our TAs sometimes just noticed errors or error patterns from student work samples (and/or connected it to error patterns that they noticed from their previous teaching experience) and proceeded with their plan to address the errors without providing interpretations, in our current study, we analyzed TAs' interpretation of students' errors or misconceptions in terms of the three views described above, focusing on whether it was viewed as a flawed idea to be replaced or a reflection of their knowledge to be used as a resource of learning, and whether and how it was reflected in TAs' planning of how to address the students' errors or misconceptions, i.e., reteaching, confronting-and-replacing, or reorganizing and refining.

Perceiving, interpreting, and responding to students' thinking

Addressing students' misconceptions starts with perceiving and understanding those misconceptions. Given that TAs spend a lot of time trying to make sense of students' written work while they are grading or during small group work in class, our study design adopted a setting where TAs would have opportunities to read written students' work samples, interpret what is written, and devise plans for their next instructional moves. Several existing studies have similarly adopted this setting; they involved student work samples, most of which included errors, that were developed, based on, or adopted from, existing research (e.g., Son, 2013; Cooper, 2010). As noted in the recent review by Stahnke et al. (2016), most such studies focus on content at the elementary level, such as proportional reasoning (Hines & McMahon, 2005; Son, 2013), geometry (Son & Sinclair, 2010), fractions (Jakobsen et al., 2013), algebraic or numeric computations (Cooper, 2010; Magiera et al., 2013) and have involved preservice teachers (PSTs). Stahnke et al. (2016) made common observations based on the results of these studies: (a) "pre-service teachers had difficulties in perceiving and interpreting" among other things "common misconceptions and student errors", and (b) teachers' own "difficulties with mathematics tasks influenced their perception and interpretation" (p. 14). They also showed that (c) teachers' proposed approaches to addressing students' misconceptions or errors were either '"reteaching' (Cooper, 2009) or showing students' how to do it correctly (Son, 2013)" (p. 14). Moreover, a large number of PSTs did not use student work at all in their response to the students, other than to say that the student's method is wrong or they briefly mentioned it as "a stepping-stone" to correct the student error (33 and 13 out of 75, respectively, Son, 2013, p. 62). Although these studies did not explicitly adopt the views and approaches towards misconceptions we discussed above, the approaches to student misconceptions that PSTs adopted that are in the literature are aligned with "reteaching" or "confront-and-replace" approaches to misconceptions, rather than with using student misconceptions as a resource for reorganizing and refining students' existing knowledge, and are connected to PSTs' difficulties with interpreting students' errors in terms of the students' thinking reflected in their written work.

Research Methods

Research Setting, Participants, and PD

We conducted our study in a single-variable calculus course, Calculus I for STEM majors, at a mid-sized public U.S. university. At the institution, Calculus I is offered as large coordinated lectures with 100-150 students taught by faculty instructors, and with discussion sections for 25-30 students taught by TAs. In the discussion sections, TAs mainly solved textbook problems based on questions from students. At the time of the study, 16 TAs at the institution taught discussion sections. We invited all 16 TAs to participate in our study. From these, five TAs volunteered. Those TAs were considered overall "strong" PhD students in the department, four were in their 3rd-4th year in the program with years of experience teaching calculus discussion sections and one was in the second semester of their first year in the program and also with experience teaching discussion sections. The TAs also participated in a bigger study that included classroom observation, collection of students' work, and interviews, as well as professional development (PD). However, we will only focus on the PD in this proposal. The authors (one mathematics educator, and one mathematician who has also conducted mathematics education research) conducted four 75-minute PD sessions that were one or two-weeks apart. Each session centered around one worksheet (with non-routine problems) and started with one of the TAs presenting their solutions to the problems as a prompt for discussion about how to make the solution better mathematically, or friendlier to students, or about their own difficulties when they tried to solve the problems. Then, the PD analyzed 2-4 student written solutions to the worksheet problems that we collected from the previous years. The student work samples we chose were examples of common misconceptions based on what is reported in the existing literature on students' thinking about calculus, and included evidence of student's reasoning (based on the researchers' determinations). Based on their analysis, TAs wrote responses to the following three prompts: "Please describe in detail what you think each student did in response to this problem," "Please explain what you learned about these students' understandings", and "Pretend that you are the TA of the student. What questions/problems might you pose next? Why?" There were also additional verbal prompts from the facilitators, such as "where may they be coming from?" and "why do you think the student answered that way?". Then, they discussed their noticing, interpreting, and planning with other TAs. These prompts were adopted from the noticing literature (Jacobs et al., 2010) to collect what TAs' were attending to, how they were interpreting student work, and their plan to address what they noticed and interpreted. In this proposal, we will focus on the observations we made about the TAs interpretations and plans.

Analysis

The data we used was the TAs' written responses to the three prompts and their discussions during the PD. We first transcribed the TAs discussions during the PD and typed out their written responses. Then, we classified chunks of written responses or verbal participation in the PD (divided by who spoke and the misconceptions or errors that were addressed) into noticing, interpreting, and planning. Then, the two authors categorized the chunks into several codes based on open coding (Strauss & Corbin, 1998). Once we came up with all the codes that were descriptive of the data, we again coded about 20% of the chunks, chosen randomly from each TA and from each PD session, to check inter-coder reliability. Our results matched except for a few chunks for which we discussed and revised the description of the codes until we reached agreement. Note that we analyzed all of the interpretations and plans proposed during PD, not only the ones that TAs eventually settled on by the end of the PD sessions.

Results

TAs' interpretation of student work

We analyzed TAs' interpretations in terms of the three views towards misconception discussed earlier: incorrect regardless of student reasoning behind it, reflection of student's flawed ideas or thinking that needs to be replaced with the right one, or reflection of student's existing knowledge that can be refined and reorganized to accommodate new concepts/context to learn. In our analysis, we first identified types of TAs' interpretations that indicated they considered students' work as a reflection of students' mathematical thinking. Our analysis indicated that in most chunks of interpreting student work (83 among 107), TAs interpretations indicate that they considered the students' work as a reflection of the students' mathematical thinking. Among these 83 chunks, 35 interpretations regarded a student's thinking about a concept, relation, theorem, or procedure as being different from what is mathematically accepted, 35 interpretations regarded a student's application of knowledge as in an inappropriate context, and 13 regarded a student's thinking as taking an approach deemed correct but with mistakes or as not carrying the approach out to its conclusion. The remaining 24 chunks, where TAs interpretations did not indicate that they considered the students' work as a reflection of their mathematical thinking, included interpretations such as that the student is just guessing, that they are computing without thinking, and that students' work is unclear, with no further interpretation.

The 35 interpretations regarding a student's application of knowledge in an inappropriate context are particularly important for promoting instruction that helps "students reflect on their present commitments, find new productive contexts for existing knowledge, and refine parts of their knowledge for specific scientific and mathematical purposes" (Smith et. al. 1993, p. 150) because of the straightforward way such interpretations can be translated into plans to help students understand the appropriate context for their approaches and how the present context is different. For example, three TAs interpreted the following student work (Figure 1) as an example of applying limit laws in an inappropriate context.

1. Find $f'(0)$ if $f(x) = \{g(x)sin\left(\frac{1}{x}\right), \quad if\ x \neq 0\ 0, \qquad if\ x = 0$, and $g(0) = g'(0) = 0$.

$$\lim_{x \to 0} f(x) = 0 \times \lim_{x \to 0} \sin\left(\frac{1}{x}\right) = 0 = f(0)$$

Figure 1. Part of Student Work Sample that TAs Interpreted as Unwarranted Generalization

One TA said, “the student tried to distribute the limit symbol over products while the limit does not exist for one of the factors. This also reflects reliance on memory rather than understanding,” and another TA elaborated further by saying “they don't know limit when h goes to 0, that $sin(\frac{1}{h})$ does not exist. So, they assume that it exists and 0 times something that is 0.” Here, the TA’s interpretations indicates that the student’s application of the limit law in this context was incorrect, but also notes contexts where their method would have worked (i.e., when the limit of each factors exists). Translating this interpretation into an instructional plan that uses the student’s misconception as a resource for new learning to promote knowledge refinement could happen if the TA planned to discuss types of limit problems where the student method of distributing the limit worked, and specifying the difference between those problems and the current problem as a rationale for why the method is not applicable any more. This way, the student’s existing knowledge will be refined with boundaries of where that particular method is “correct” and new ways of approaching a similar-looking but different context can be learned in comparison to the old way. In next section, we will discuss whether and how TAs’ plans were in line with such a view of considering students’ misconception as resource of new learning.

TAs’ plans to address students’ work

To understand what TAs planned to do to address student misconceptions, we first coded each TA’s plan, and categorized them based on the conceptualization of how to use student work: reteaching without using it, confront the misconception that the error is showing and replace it with the correct counterpart, and using it as resource for learning. As we discussed, the most productive use of students’ error in this regard would be using students’ misconceptions as resource for student learning. Our analysis shows that most TAs’ plans involved confronting students with their errors while none of the TAs plans used students’ misconceptions as a resource for student learning. Three main instructional approaches were (a) giving students opportunities to realize their error (52 plans), (b) pointing out the error and have them fix it or teach them how to fix it without giving such opportunities (12 plans), and (c) solving the problem without addressing students’ errors and starting with a simpler problem, giving hints, or scaffolding them with small steps (7 plans). Plans included in (b) and (c) are clearly aligned with “confront-and-replace” and “reteach” approaches. Plans included in (a) could potentially align with using students’ misconceptions as resources for learning and knowledge refinement, but further investigation of these plans showed that they did not.

The plans included in (a) could be categorized into several further subcategories, the most prominent of which were (i) asking students to explain the part of their answer that includes an error (11 plans), (ii) providing students opportunities to check if their answer has the desired properties, hypotheses that their answer is assuming when students applied a theorem, or consistency within their answer (16 plans), and (iii) giving students counter examples to their methods/responses or providing examples or tasks for students to realize the error (13 plans). Seven plans involved a combination of these approaches, and the remaining plans combined these with some additional guidance on a next step assuming that the error was fixed by students.

A common feature of most of these plans is that they stop with the student realizing their error. Only a small number of plans included next steps after students realized their errors and most of these next steps were about restarting the problem by scaffolding students with small steps or having the student try a simpler problem than the originally given problems. Therefore, these plans confront a student with their error and either leave the student to try again or give steps to replace the student's incorrect method with a correct one, but do not use the student's misconception as a resource for refining their mathematical thinking. An example of this can be seen from the discussion in PD of the student work sample in Figure 2.

Suppose for some function $h(x)$, we know that $h(0) = 2$ and that $h'(x) = \left(1 + x^3\right)^{\frac{1}{3}}$. Draw the graph of $h(x)$.

$h'(x) = (1+x^3)^{1/3}$ $\quad f(x) = (x)^{1/3}$ $\quad g(x) = 1+x^3$

$\downarrow$ $\quad\quad \downarrow$

$\frac{3}{4}(x)^{4/3}$ $\quad x + \frac{x^4}{4}$

$$h(x) = \frac{3}{4}(1+x^3)^{4/3} \cdot x + \frac{x^4}{4}$$

$$\boxed{h(x) = \frac{3x(1+x^3)^{4/3} + x^4}{4}}$$

Figure 2. Student work sample TAs plan to address by asking the student to check if their answer has the desired property.

The following excerpt shows one TA's response to one of the researchers' questions about next steps, after the TA presented their plan of having the student differentiate their answer to show that it would not result in the derivative originally included in the problem (Figure 2):

Researcher 1: Let's say you know they…took the derivative and they got the wrong answer and they got confused, what would you do?

TA E: Both integration and differentiation to them is just a bunch of following. They don't understand really why the nuts and bolts of why this exists. So I would just be like if you have learned this, you have learned it the wrong way, this formula does not exist and one way to clearly see that is the derivative is not the derivative that we want it to be. But if you want to know why it's actually wrong then take an analysis course or something.

In this excerpt, the TA re-emphasized why the contradiction should tell them that what they did was wrong, and this excerpt exemplifies that TAs often do not have explicit plans to use students' misconceptions as a resource for learning. In fact, all plans to the problem in Figure 2 involved starting over by asking what they know about the original function given the derivative without attempting to "integrate". An approach that uses the student's misconception as a resource for learning could use some of the TAs' interpretations of the work in Figure 2 as an "unwarranted" generalization of a differentiation rule (i.e., chain rule) to integration. The plan could focus on how to properly implement the student's idea of "going backwards" with a differentiation rule (in this case, integration by u-substitution) and discuss the differences between contexts where that works and the given problem. Then, one could start navigating the different methods to accommodate the new, similar looking, but different context. However, we did not see any plans that take this type of approach for any student work samples in our data.

Discussion and Conclusion

Our analysis of TAs' plans to address students' misconceptions showed that their plans mainly focus on addressing what student did wrong in the solution by providing students opportunities to realize their errors and potentially fix them, or by directly teaching how to fix the errors. These results are more positive than just "reteaching" the concepts or procedures without addressing students' errors, which was one of the main approaches PSTs adopted in some of the existing studies in K-12 literature (Cooper, 2010; Son, 2013). However, TAs' plans were mainly aligned with the "confront-and-replace" view of misconceptions, which has been criticized in the existing literature. Moreover, in most of their plans, TAs did not include any plans to provide alternatives after giving students opportunities to realize their error, other than just starting over the problem in a mathematically correct way. With these approaches, students will have opportunities to learn that what they did is wrong, but are not helped to understand what problems in their thinking led to such a wrong answer. Thus, this way of addressing student errors is not supporting learning as expanding students' thinking (Smith et al., 1993).

In contrast to their planning, a significant number of TAs' interpretations of students' misconceptions were aligned with a desirable view of misconceptions based on the literature, which is as a resource of new learning. In these interpretations, TAs identified the methods that students may have adopted, and the contexts where their method might have worked in the past, although it did not work for the problems that were given to them at the time. Although we did not see any evidence from our data that this way of interpreting misconceptions was reflected in TAs' planning of addressing students' errors, we found that there is potential to expand this view to their planning. Since our TAs have ability to identify where the students' methods would have worked in the past, they could share that with their students to acknowledge that their attempt to solve the problem had a rationale that was supported by their learning in the past, and then help them realize the differences between the contexts where their methods worked and the current problem context. This way, students' previous knowledge about the concept or procedure would be refined and reorganized in their learning of new way of addressing the problem. We anticipate that this way of planning that is aligned with what is recommended by mathematics education literature on how to address student misconceptions as resource of learning (Smith et al., 1993; Li & Schoenfeld, 2019; Rake & Ronau, 2018) could be promoted in PD for TAs. The TAs in our study were capable of and showed a tendency to attribute potential reasons for students' "wrong" choice of methods to the problem to their past "successful" experiences of using them. PD could promote this way of interpreting when applicable and provide opportunities to think about how to incorporate such interpretations in their planning to address students' misconceptions. This type of PD has the benefit of helping TAs learn about an approach to teaching that is broadly applicable and could be used in a variety of courses regardless of their content and focus.

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