What is so difficult about teaching precalculus for the first time? Graduate students' perspectives on learning to teach.

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This study seeks to explore and understand the challenges of teaching precalculus for the first time from the perspective of a novice Mathematics Graduate Student Instructor (MGSI). This qualitative multiple-case study followed three MGSIs through a semester of teaching to understand their needs and inform efforts to support instructors. Findings indicated novice MGSIs encountered many challenges including covering course content, estimating class time, facilitating activities, maintaining student engagement, and adjusting to variation in students' preparation. However, balancing dual responsibilities of student and instructor, working with students' negative responses to engaging teaching methods, and interpreting students' performance on assessments emerged as the most difficult challenges to overcome.

Keywords: Graduate Student Instructors, Challenges, Professional Development, Precalculus

Many graduate students aspire to careers in academia (Golbeck et al., 2016) and desire to be effective instructors. Research suggests graduate teaching assistants are concerned about their teaching effectiveness, communicating with students (Feezel & Meyers, 1997), dealing with student conflict (Park, 2004), and desire support for working with undergraduate students (Boyle & Boice, 1998). This study focuses on mathematics graduate student instructors (MGSIs), defined as a graduate student who serves as full instructor of record for an undergraduate mathematics course as part of their assistantship. Thus, they present course material, assess student learning, and assign final course grades. MGSIs are commonly assigned to teach introductory mathematics courses, despite being students themselves, having limited access to professional development (PD), and limited teaching experiences (Deshler et al., 2015; Ellis, 2014; Speer et al., 2005). They may face challenges including balancing teaching with their own coursework or research, awareness of low status of teaching at a research university, difficulties getting and interpreting feedback, working with undergraduates who bring negative mathematical experiences to their classrooms, and anxiety in dealing with these challenges (Hauk et al., 2009). Further, their previous experiences and competency with mathematics likely vary greatly from the undergraduate students they teach (Deshler et al., 2015). Additionally, MGSIs may need to develop pedagogical content knowledge such as understanding common student difficulties, different ways students approach tasks, and illuminating examples to share with students (Speer, 2008).

Although many universities have developed PD opportunities and supports for MGSIs, they vary greatly in depth and breadth (Belnap & Allred, 2009). Formal PD courses can improve STEM graduate students' college teaching self-efficacy, particularly for teaching methods, course planning, and assessment of student learning (Connolly et al., 2018). Efforts to develop and improve such programs benefit from a rich understanding of MGSIs needs and teaching experiences and focusing on novice or less experienced instructors is important as universities continually use new MGSIs to provide instruction to undergraduates.

To deepen the understanding of MGSIs' needs, a larger semester-long study explored how MGSIs understand their planning and teaching for student learning and their efforts to incorporate active learning methods into an undergraduate course by examining reasons they

gave for making pedagogical decisions and identifying their perceived challenges[1]. The larger study sought to address the research question: During their first semester teaching a new undergraduate mathematics course, how do MGSIs plan (design and reflect) and implement their plans, with a focus on their goals for student learning? To address this broad question, four sub-questions were created. Due to space limitations, this paper focuses on findings to only the final sub-question: *What challenges do MGSIs describe as they reflect on their planning and classroom instruction?*

Theoretical Framework

Jackson (1968) described three phases of teaching: preactive, interactive, and postactive. The preactive stage could be thought of as planning or what teachers do to prepare for class. This could include writing lesson plans, making decisions about content, selecting mathematical tasks, identifying instructional goals, or preparing materials. The interactive phase consists of the time spent in the classroom with students. Thus, it describes the time when the lesson plan is executed. The postactive stage of teaching follows the interactive stage and includes the teacher's reflective or evaluative thinking and their consideration of future lesson modifications.

However, Clark and Peterson (1986) note that many researchers do not differentiate preactive and postactive thinking when studying teacher planning. They argue that teaching is a cyclical process and that a teachers' reflections may influence their plans for subsequent classes. Thus, they conceptualized teacher planning to include both preactive and postactive thinking. Similarly, the Mathematical Association of America defined design practices as "the plans and choices instructors make before they teach and what they do after they teach to modify and revise for the future" (Abell et al., 2018, p. 89) reiterating the notion that these phases of teaching may be combined. Yet Clark and Peterson do contend that the "kind of thinking that teachers do during interactive teaching does appear to be qualitatively different from the kind of thinking they do when they are not interacting with students" (1986, p. 258) and maintain the distinction between planning and the interactive phase of teaching.

Methods

Methodological Approach

In order to deeply explore MGSIs perspectives on planning and teaching, a qualitative multiple-case study methodology was selected. Case study is defined as "an empirical method that investigates a contemporary phenomenon (the "case") in depth and within its real-world context" (Yin, 2018, p.15). The research design provided ample reflection time and space for MGSIs to describe their goals, decisions, and challenges. The site, timeframe, course, and participants defined the bounds of this case (Yin, 2018). Each case is comprised of an MGSI serving for the first-time as a full instructor of record for precalculus, an undergraduate service course in the mathematics department, as part of their teaching assistantship during fall 2019 at a large research university in southeastern United States.

Participants and Setting

All MGSIs teaching precalculus in fall 2019 were invited to participate in this study and three male MGSIs in their early twenties volunteered. All three were in their second year of graduate school, cared about their teaching, and were considering academia as a potential career track. None had experience teaching precalculus and all had served as a calculus teaching

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assistant during their first year of graduate school. At the end of the study, each MGSI chose a pseudonym. Chen was an international graduate student and the other two, Willie and Patrick, were domestic graduate students. These MGSIs were actively receiving university support for learning to teach as they were provided with a peer mentor, were enrolled in a one-credit, year-long pedagogy course, and had access to lesson plans that included problem sets and teaching suggestions.

Precalculus is the prerequisite course to the calculus sequence which most STEM majors are required to complete. It is a four-credit hour course covering topics in algebra and trigonometry and is considered a service course, meaning the majority of students enrolled in the course are not mathematics majors. The course typically meets Monday through Thursday and enrolls about 38 students in each section. At this university, precalculus is a common teaching assignment for second-year graduate students.

Data Collection

To answer the research question and provide a rich understanding of MGSIs challenges and teaching experiences, data was collected throughout the fall 2019 semester and included (1) interviews, (2) lesson observations, (3) weekly journals, (4) course documents, and (5) a focus group. MGSIs were interviewed at the beginning of the semester to understand their prior teaching experiences and beliefs about teaching and student learning. They were asked to select a total of three lessons, roughly one each month, where they planned to seek student engagement. These lessons were observed and video-recorded, lesson plans were collected, and followed by a semi-structured interview (50–100 minutes) that explored the MGSIs' goals, planning and lesson design, and perceived challenges. Other forms of data included weekly journal entries (responding to 3–6 prompts), participant observations from the pedagogy course for MGSIs, their course assignments, and interviews with their peer mentors. At the end of the semester, MGSIs participated in a focus group seeking to gain insight into their overall experience as first-time instructors of record. The focus of this study was on understanding the MGSIs experience of learning to teach undergraduate precalculus.

Data Analysis

The purpose of this study was to understand MGSIs' needs by examining their thinking and teaching experiences in order to inform efforts to support their development. Thus, dramaturgical coding was chosen as the coding scheme. According to Saldaña (2016), it is useful for exploring underlying psychological constructs and "attunes the researcher to the qualities, perspectives, and drives of the participant" (p. 146). It includes six types of codes: objectives, conflicts, tactics, attitudes, emotions, and subtexts. Objectives typically referred to MGSIs goals for student learning. Conflicts, or obstacles, referred to aspects of teaching MGSIs identified as challenging or difficult and are the focus of this paper. Tactics provided insight into MGSIs' lesson design and classroom practices. The attitude codes included the MGSIs orientations about teaching and learning. Analysis of a pilot study suggested the emotion and subtext codes were not useful for answering the research questions, thus they were omitted from later analysis. A fifth category called influences was added to capture references to external influences on MGSIs planning.

Seven sources of data were coded for each MGSI: their semester-long journal, three lesson interviews, the background interview, and two written assignments from a pedagogy course. All codes were created in a hierarchical manner; each code was placed under the categories of objectives, tactics, conflicts, attitudes, or influences (level 1). To maintain MGSIs perspectives in the analysis, coding included in vivo codes (Saldaña, 2016). This paper focuses on the analysis

of the conflict codes which included codes such as "run out of time" and balancing workload. Code mapping (Saldaña, 2016) included reviewing quotes from categories at levels 2 and 3 to confirm that quotes described the same concept, discussing unclear passages with a colleague, and listing, comparing, and organizing codes into categories by rearranging or combining similar codes, was utilized at four points in the analysis: after coding the first piece of data (Chen's journal) and again after completing coding the data from each MGSI. The codes were then condensed into themes, by grouping level 2 codes into larger categories. The analysis sought to identify areas MGSIs described as problematic from their perspective. As MGSIs expressed many common difficulties, findings are reported only through cross-case analysis.

Throughout data collection and analysis, a research log was created containing memos to document the research process, reflect on lesson observations and interviews, generate future questions, discuss concerns, and contemplate next steps in the research process (Saldaña, 2016). All interviews were transcribed using software and were cleaned by the author. All coding was done in Dedoose and coding proceeded one MGSI at a time so that each instructor could be studied individually before making comparisons across cases. Tentative findings were shared with and reviewed by MGSIs for trustworthiness.

Findings

MGSIs limited teaching experience likely contributed to some of their challenges, particularly as they were trying out new teaching methods and thus struggling to facilitate activities or estimate time students would need to complete problems or tasks. Further, they were teaching the course for the first time and still grappling with determining the depth and length of time to spend on mathematical concepts. The difficulties with managing class time and course pacing could easily be attributed to their stage of beginning to learn to teach and are not elaborated upon in this paper due to space limitations. Further, MGSIs noted times in the semester, such as before breaks or near the end of the semester, when students seemed generally less engaged. MGSIs were able to manage most of these challenges with support from mentors and peers. Thus, this paper focuses on other more enduring challenges highlighted in the table.

Table 1: Summary of Challenges and Aspects of the Challenges

Phases of Teaching	Challenges
Preactive: Planning	(1) **Limited time for planning**; (2) Course content coverage
Interactive: Implementing Lesson Plans	(1) Estimating time needed for tasks or activities; (2) **Students' responses to teaching methods** (3) Facilitating lesson logistics; (4) Maintaining semester-long student engagement
Postactive: Reflecting on Teaching Experiences	(1) **Interpreting students' performance on quizzes or tests**; (2) Interpreting students' course preparation

Preactive Challenges: Preparing to Teach

MGSIs are simultaneously both a student and an instructor. As a result, Willie expressed in his journal that he was "essentially working two full time jobs." Learning to balance these responsibilities and mange workloads was a challenge for all MGSIs in this study. Throughout the semester, MGSIs found weeks where the time needed for their own coursework conflicted with their responsibilities as an instructor. Both Patrick and Chen identified this as their greatest teaching challenge in the final journal entry. Chen reflected,

> Time management is probably the most difficult in the past semester. This semester is the first time that I taught MATH 115 [precalculus]. We meet four times a week, so, I have to do a lot of lecture preparation (which I have been trying to align with homework, quiz, and exam). Even though we have the lesson plans in online material and worksheets, I still usually go over the book, look for extra material online, and compare my notes to other peoples' notes. These planning worked fine when I am not busy, however, toward the end of the semester, I find it difficult to balance planning the lesson with focusing on my own study.

Patrick identified the same challenge and stated, "there were weeks this semester where it felt as though I had to choose between doing well in my role as a MATH 115 instructor and doing well in my courses. This is a no-win situation." In the focus group, MGSIs collectively agreed they spent most of the day teaching, holding office hours, or preparing for teaching in some way and then would go home and complete their own graduate coursework in the evening or over the weekend. These MGSIs chose to spend a large amount their time on preparing to teach and wanted to be effective instructors. Willie explained,

> I feel like if you want to have a good class, it takes a lot of preparation. It's pretty much writing all those assignments and grading. You know, if you really want to be a good teacher, you're going to spend more time.

MGSIs reported that planning active learning activities tended to take more time to prepare than traditional lecture-type lessons. Experienced faculty often cite lack of time to redesign courses as a reason for not incorporating active learning (Johnson et al., 2018; Plush & Kehrwald, 2014). Thus, it may be that examples of these types of activities are more valuable to share with new instructors. To a lesser extent, MGSIs occasionally mentioned concerns about feeling behind schedule, pacing material, or selecting problems of the right difficulty level.

Interactive Challenges: Implementing Lesson Plans

A conflict MGSIs experienced with implementing lesson plans came from some students' responses to MGSIs efforts to create student engagement. MGSIs experienced conflict when their students either provided feedback indicating they disliked an activity or appeared frustrated by or not engaged in an activity. These challenges often related to lessons when MGSIs planned activities intended to engage students. MGSIs tended to reference a few students or a group of students rather than an entire class when reflecting on this difficulty. For example, Chen reflected in an interview on feedback he received from a few of his students on both a mid-course evaluation and directly following a discovery type lesson he designed to help students make meaning of asymptotes. He stated, "they don't like when I do this kind of stuff, have them struggle and conclude an idea from the lesson. A few people don't like that." Similarly, Patrick reflected on student feedback related to a jigsaw activity in his journal. He wrote, "many comments suggested that they prefer a standard lecture because the way that I explain topics is clearer and more organized" than their peers' explanations.

Beyond preferring lecture-style instruction, MGSIs occasionally encountered students who opposed non-lecture teaching methods. Willie discussed at length difficulties he encountered with one group who struggled to complete a jigsaw activity about quadratic functions. Willie shared in the interview,

> I just remember it was three red faces right in front of me. I was like, Oh no, that's not good! I tried to help. [It] just wasn't sticking. They weren't putting in effort. They just want to come into class, take notes and leave is what it felt like. They were unhappy.

Willie confirmed the students were unhappy with the *teaching method*, explaining, "I had one of them come up to me and said, can we not do this again? So yeah, she was not happy with the method." Willie tried to provide hints to move the group along, but their progress lagged well behind other groups and their attitudes did not improve.

Although these MGSIs participated in some PD related to facilitating student-to-student interactions, MGSIs perceived success with group work varied greatly in this study. Patrick and Chen repeatedly expressed difficulties getting students to engage in group work, indicating that students tended to work alone despite their efforts to encourage collaboration. Patrick stated in his journal, "I am having trouble getting my students to interact with each other." And elaborated in the first lesson interview,

> Maybe I can find some way to make my lessons so that they are helping each other a little more or at the very least so that it's not just silent in my classroom. Sometimes it is. A lot of times it is… I'll say you can work on these problems. You can work with people around you or people next to you. They just work by themselves mostly. I don't know if it's a matter of they just want to try it [alone] or what.

Despite the limited success, both continued to try various approaches to group work. Chen shared, "I'm not gonna give up doing group work so, but I have to like find a better way to do it." Both suggested time of day could have impacted their students' engagement. Many students appreciate aspects of group work yet report dissatisfaction if they prefer to work at a different pace than their group, are uncomfortable, shy or introverted, or need more clarity on a topic (Uhing et al., 2021). MGSIs perceived these types of teaching challenges as difficult to overcome, and during the focus group they expressed a consensus understanding that it is not possible to please all students.

Postactive Challenges: Reflecting on Teaching

MGSIs discussed challenges related to understanding their precalculus students, particularly their performance on quizzes or tests, but also around students' prior preparation, and how students appeared to prepare for their current course. In this study, quizzes and tests refer to written assessments of students' knowledge of mathematical content which MGSIs collected and graded. MGSIs were troubled that some students at times struggled to demonstrate mathematical knowledge, understand concepts, recall facts or procedures on quizzes or tests, or repeated errors that MGSIs attempted to address in their teaching despite MGSIs best efforts to include student-centered teaching methods.

MGSIs' feelings, and perhaps efficacy for teaching, seemed to be negatively impacted by student performance on graded assessments and at times they pondered to what extent they were accountable for student learning. In some ways, this presented an emotional challenge for MGSIs as they expressed feeling responsible for student performance on quizzes and tests. Patrick wrote in his journal: "When quiz scores are lower than normal, it's hard not to feel like it's my fault somehow." Chen stated in the third lesson interview, "I feel like as an instructor I should be somewhat responsible for how they learn." Willie also expressed feelings of responsibility for student learning during the third lesson interview when he asked, "I mean 70 is like the minimum that I want it. So, when I get below it, I'm like, did I do something wrong?" MGSIs contemplated their responsibility for student learning. Chen expressed this in his journal when reflecting on things he could do to help students remember concepts and concluded "I don't know if it's the students or my fault that they don't learn, I don't know." In the final journal entry Willie expressed,

> The most difficult thing for me was watching kids not do well and knowing I've tried everything I could and offered everything I could to help them. It made me feel bad as a teacher and bad for the student. However, I have to learn that I cannot help everyone!

Processing students' performance on tests or quizzes and communicating with students about low scores can be difficult for instructors. MGSIs responded to students low scores in a variety of ways including curving scores, offering extra credit, dropping quizzes, and allowing students to correct exams for points. The experience of seeing students struggle with problems on a quiz or test that had been, from the instructor's perspective, clearly covered during class was a source of conflict for these MGSIs whose goals included helping students develop mathematical thinking and procedural skills so that they could succeed in both current and future courses (Monastra & Yee, 2021). This tension may cause distress, disappointment, or discouragement for MGSIs and may have been especially salient in this study as MGSIs spent a lot of time reflecting on their goals for student learning. Further, MGSIs put a great amount of effort into their teaching and then did not always receive the benefit of large amounts of student success.

Discussion

This study's findings suggest that MGSIs need opportunities to reflect on and discuss their teaching, support and training for working with undergraduate students, and planning time. Cognitive coaching techniques (Costa & Garmston, 2018) may be helpful for promoting reflection. Graduate student PD can address a wide range of topics, yet often focuses on generic teaching skills (Park, 2004) or "administrative responsibilities and mechanical aspects of teaching" (Speer, 2008, p. 313). These topics are important and necessary, yet they may not be sufficient for supporting MGSIs. Further, their teaching does not always lead to the desired results. MGSIs may experience tension between their goals for student learning and the actual learning they see demonstrated on tests or quizzes. Acknowledging and reflecting on these tensions by pondering questions about instructors' responsibility for motivating their students may be an important area for PD. Allowing MGSIs to reflect on who should be accountable for student learning may lead to fruitful discussions.

Further, MGSIs may encounter some negative responses or pushback from their students. Student attitudes have been identified by faculty as a reason to not utilize reform materials or teaching methods (Deslauriers et al., 2019; Henderson & Dancy, 2007) and student stress can impact instructors' problem selection on assessments (Yerushalmi et al., 2010). Further, undergraduate STEM students perceive they learn more and prefer well-delivered lectures when compared to the increased cognitive effort required in active learning settings (Deslauriers et al., 2019). Discussing productive ways to respond to students' attitudes towards teaching methods, tips for creating learning environments that promote student engagement with tasks, and discussions or suggestions related to setting class norms, features of high-quality tasks for group discussion, various models of group work, and question posing techniques would appear to be beneficial for graduate student instructors. Some of these topics were addressed in these graduate students' PD, which emphasizes the point that support cannot be limited to a one-time training. Rather novice instructors need *sustained* discussion and models of teaching methods as well as opportunities to share success and struggles with implementing various teaching methods in their classrooms. Finally, PD providers and mathematics departments must be aware of the time strain some graduate students may be experiencing as time is identified as a barrier to instructors adopting reform teaching practices (Henderson & Dancy, 2007; Johnson et al., 2018). Future research into the impact of graded assessments on MGSIs self-efficacy or teacher identity and how MGSIs understand student motivation would be helpful.

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