Opportunities to Learn Provided by a Commonly Used Business Calculus Textbook

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Contributing to the growing body of research on opportunities to learn provided by undergraduate mathematics textbooks, this study reports on the analysis of optimization examples given in a commonly used business calculus textbook in the United States. Findings of this analysis indicate that the textbook consistently encourages students to interpret critical numbers and extrema, in addition to encouraging students to verify whether or not critical numbers produced minimum/maximum values of the objective functions from which they were determined. The analysis further revealed that the textbook rarely encourages students to include units of extrema, provides limited opportunities for students to reason about objective functions that have more than one critical number, and provides minimal opportunities for students to reason about absolute extrema problems, respectively. Recommendations for different stakeholders, including business calculus textbook authors and business calculus instructors, are included.

Key words: Opportunity to learn, optimization problems, business calculus, textbook analysis

Textbooks play an important role in students' learning of mathematics at all levels. In fact, according to Reys et al. (2004), "… the choice of textbooks often determines what teachers will teach, how they will teach it, and how their students will learn" (p. 61), a sentiment that is shared by several other researchers (cf. Alajmi, 2012; Begle, 1973; Kolovou et al., 2009). Although much research as reported on opportunities to learn about various mathematical topics at the K-12 level (cf. Alajmi, 2012; Charalambous et al., 2010; Dole & Shield, 2008; Jones & Tarr, 2007; Pickle, 2012; Stacey & Vincent, 2009; Stylianides, 2009; Thompson et al., 2012), there is a paucity of similar research at the undergraduate level, which is the motivation for the present study.

The study reported in this paper is part of a larger study (Mkhatshwa, 2016b) that examined: (1) students' reasoning about optimization problems, (2) opportunities to learn about optimization problems via two course lectures each of which was attended by approximately 300 students, and (3) opportunities to learn about optimization problems provided by a course textbook. The current study reports on opportunities to learn about optimization problems via examples provided by the course textbook (Haeussler et al., 2011), a widely used textbook in the teaching of business calculus in the United States (Mkhatshwa & Doerr, 2016a).

Consistent with findings reported by other researchers (cf. Reys et al., 2004), one major finding of the larger study is that the presentation of optimization problems during course lectures closely followed the presentation of optimization problems in the textbook. Findings of the present study suggests that the textbook encourages students to interpret critical numbers and extrema, and to verify extrema using various methods such as the first or second derivative test, respectively. In addition, the textbook rarely encourages students to include units of extrema, provides limited opportunities for students to reason about objective functions that have more than one critical number, and provides minimal opportunities for students to reason about absolute extrema problems. In light of the findings of the present study, we recommend, among other things, that business calculus textbook authors provide more opportunities for students to reason about units of extrema, objective functions that have at least two positive critical numbers, and absolute extrema problems (Mkhatshwa, 2019) in economic contexts.

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