Introductory Calculus Instructional Practices Around Student Prior Knowledge

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Calculus is a pivotal course for postsecondary students and serves as a gatekeeper to advanced mathematics courses and STEM majors. This study considers the frequency with which instructors report engaging in practices that may help to strengthen introductory calculus students' prior knowledge. Using survey responses (N=136), this study reports on results from a set of questions that measures these practices. Results illustrate that instructors may use various practices to address students' prior knowledge in their introductory calculus classroom. Additionally, instructors provided examples of practices they use in response to student prior knowledge that are not explicitly considered in the literature. This implies that a closer look at what is happening in the classrooms may be key in understanding the current state of how instructors are dealing with student prior knowledge in introductory calculus.

Keywords: Introductory Calculus, Instruction, Student Prior Knowledge

Introductory calculus is an essential course for many students; it can bridge the secondary-tertiary transition. As such, it is often a culmination of students' K-12 knowledge and a gatekeeper to more advanced mathematics (Bressoud, 2015; Rasmussen et al., 2019). Despite its importance, students continue to struggle to make it through the course in postsecondary settings (Bressoud et al., 2013). One issue facing instructors at the postsecondary level, many of whom have been trained as mathematicians and not as teachers, is the varying levels of prior knowledge that students have when they enter the classroom. One reason for the difference in students' prior knowledge might be the various paths to get to postsecondary introductory calculus (i.e., having seen calculus in high school or not) (Sadler & Sonnert, 2018; Schraeder et al., 2019). Research has shown that student understanding of prerequisite knowledge connects to student success in calculus (Rasmussen et al., 2019); therefore, it is important to understand how instructors engage in practices that support the development of students' prior knowledge. Studying instructional practices that may strengthen student prior knowledge can help researchers understand more deeply practices that might be leveraged in the classroom to help more students succeed.

Theoretical Background

Introductory Calculus Instruction

Despite research on the impacts of and barriers to success in postsecondary calculus, mathematics instruction at the postsecondary level remains comparatively unexamined (Speer et al., 2010). Some researchers have looked at the instructional format in calculus, finding that calculus classes typically feature lecture and demonstration (Biza et al., 2016; Larsen et al., 2015). Other researchers have shown that instruction in introductory calculus classrooms may be more varied. For example, while lecture continues to be the most common mode of instruction for postsecondary mathematics, recent arguments suggest that this mode can help to model mathematical discourse (Viirman, 2021). In addition, Bressoud and Rasmussen (2015) found that while university instructors considered themselves fairly traditional (i.e., they thought that calculus students learned best from clear, organized lectures), they were open to trying other instructional approaches. Finally, several studies have pointed towards instructional practices in calculus courses that move away from lecture; recent work in math and related STEM fields has shown that university faculty do attempt to combine content- and student-centered practices and that these practices increase student achievement (Bressoud & Rasmussen, 2015; Hovey et al., 2019; Rasmussen et al., 2019; Tang & Titus, 2002).

Prior Knowledge & Introductory Calculus

In my dissertation study, I frame prior mathematics knowledge as the mathematical reasoning and skills students bring into the classroom from their previous mathematics classes or experiences. I draw here from Tobias' (1994) definition of domain knowledge: "domain knowledge deals with familiarity with general information in an area, even though it may not be specifically referred to in a particular passage" (p.39). A key difference between my definition of prior mathematical knowledge and Tobias' definition of domain knowledge is that I choose to include the idea of 'mathematical experiences' that may fall outside mathematics content. I borrow the concept of mathematical experiences as part of prior knowledge from Campbell et al. (2020), who put forth "how students understand core ideas based on their own lived and cultural experiences" (p. 68) as their definition of prior knowledge. By combining these definitions of prior knowledge, I can consider students' specific mathematics content exposure and how this exposure happened (e.g., procedural instruction in algebra courses). I also conceptualize prior mathematics knowledge as distinct from prerequisite knowledge because not all prior mathematics knowledge is important for calculus understanding.

One reason that instructional support of student prior knowledge is important is that it allows students to connect old and new ideas, creating opportunities to make sense of them. Neumann (2014) argues that student prior knowledge is brought to the surface in encountering new ideas. Furthermore, learning occurs when this prior knowledge and new ideas are worked through together. One way to understand where opportunities for prior knowledge and new ideas to be worked together is to examine instructional practices.

Building on Neumann's (2014) claims, Campbell et al. (2017) developed a framework to study how postsecondary courses might cluster by pedagogies and included instructional attention to supporting prior knowledge in their pedagogical characterizations. They found that both lecture and active learning strategies can be used to support student prior knowledge. Campbell et al. (2020) explain that while strides have been made in postsecondary institutions to have student-centered classrooms (which may promote discussions and use of prior knowledge), instructor attention to prior knowledge in college classrooms is still uncommon. This line of inquiry provides evidence that support of prior knowledge (though perhaps rare) does occur in postsecondary classrooms and that there may be observable instructional practices among those who consider prior knowledge in their instruction. I further this line of inquiry by asking the following research question: *How and to what extent do introductory calculus instructors report engaging in practices that promote students to make mathematical connections to their prior knowledge?* Results that answer this question help us better understand if and how these practices are being put into use by introductory calculus instructors around the U.S.

Methods

Participants and Procedures

The data for this study were collected using a survey distributed to postsecondary instructors at institutions across the United States in Spring 2022. The population of interest for this survey included full-time instructors at 4-year universities with a Carnegie classification of R1, R2, Doctoral, or Arts & Sciences Focused Baccalaureate who have taught introductory calculus at least twice. The choice to sample full-time instructors at these types of institutions was informed by research on where most students take introductory calculus (Blair et al., 2015). After excluding responses that did not participate past questions about demographic information, there was an analytic sample of 142 respondents. I received 42 responses from R1 instructors, 32 responses from R2 instructors, 39 responses from Doctoral institution instructors, and 29 responses from instructors at Baccalaureate Arts & Sciences focused institutions. In addition, 61 participants identified as female, 79 participants identified as male, 1 participant identified as non-binary, and 1 participant chose not to disclose gender identity. 13 participants identified as Asian, 3 participants identified as Black or African American, 117 participants identified as White, and 8 participants chose to not identify their race. In addition, 70 participants identified as Tenured Faculty, 15 as Tenure-Track Faculty, 57 participants as other full-time faculty. Finally, 23 participants indicated that they had taught introductory calculus 2-5 times before the survey, 32 had taught the course 6-10 times, and 87 had taught it more than 10 times.

Survey Construction

The survey data used here was intended to capture how often instructors engaged in practices that have been theorized to strengthen student prior knowledge. I developed a set of questions to measure this construct, drawing from the C-SAIL Mathematics Teacher Survey (California Version, 2017; Edgerton & Desimone, 2019). As written, this survey did not specifically measure practices that engage student prior knowledge. However, it was appropriate for this purpose because the questions were structured and validated to ask about instructional practices. I selected specific items based on theoretical grounding in prior literature. For example, I chose items asking about discourse in the classroom, teaching foundational skills, and presentation of multiple strategies (Chappell & Killpatrick, 2003). As the C-SAIL Mathematics Teacher Survey was written primarily for use in K-12 classrooms, I adjusted wording for the calculus and postsecondary contexts when necessary. It included 12 items measuring frequency on a 5-point Likert scale that indicated instructors engaged in these practices: Never, Once or Twice a Semester, Once or Twice a Month, Once a Week, or Almost Every Class Session. A higher score indicates that an instructor engages in the practice more often.

Additionally, data presented here was gathered from a single open-ended question at the close of the survey, "How, if at all, is your instruction affected by the mathematical understanding students bring when they *come into* your class at the start of the semester?" This question was intended to gather information on any instructional practices that my survey questions may not have caught.

Analysis

Analysis for this set of survey data occurred along three lines. First, de-identified data from the participants was used to validate the survey. Second, the survey data were analyzed to understand which practices instructors used more frequently. For these analyses, only data from instructors who had completed the full set of questions was considered (N=136). Finally, qualitative data was open-coded (Corbin & Strauss, 2014), to understand additional practices espoused by instructors; 127 instructors responded to this question.

Results

Reliability

Internal consistency of the items was analyzed using a correlation matrix and Cronbach's alpha. Inspection of the correlation matrix showed that all items except for one ("How often do you have students solve practice problems using explanations or solution strategies you have modeled in class?") had correlation coefficients greater than 0.3. The one item with no correlation coefficients greater than 0.3 was dropped from the analysis, leaving 11 items for analysis. These 11-items had a high level of internal consistency, as determined by Cronbach's alpha of 0.81.

Survey Response Instructional Practices

Overall, participants in this sample reported that they engaged in some form of instructional practice that may strengthen student prior knowledge. I calculated a composite score for each participant by averaging their item scores. I then calculated the mean across the participant sample. The mean was 3.77, indicating instructors in my sample participate in some of these instructional practices between once or twice a month and once a week. Additionally, there were practices that instructors were engaging in less or more (see Table 1). For example, instructors reported assessing students in prerequisite skills the least, with a mean of 2.93 (nearly once or twice a month). They also reported most often building calculus knowledge by connecting it to prior knowledge, with a mean of 4.42 (between once a week and every class).

Interestingly, the lower reported practices seem to align with a more procedural approach to strengthening student prior knowledge (e.g., reteaching or simplification of problems). In contrast, the higher reported practices involved a less procedural approach (e.g., engaging in discourse, connecting calculus content to students' experiences). Interestingly a one-way ANOVA showed no significant difference between instructors at different institution types for each item or across items. This implies that regardless of institution type instructors are engaging in similar practices.

Table 1. Means for Instructional Practices for Student Prior Knowledge.

"On average, across the span of a semester, how often do you engage in the following practices in your introductory calculus course?" (N=136)

Practice	Mean	Std. Deviation
Build calculus content by connecting it to prior knowledge	4.42	0.80
Correct or refine students' use of language when they are presenting their ideas	4.14	0.91
Provide opportunities for students to discuss their mathematical reasoning in class	4.20	1.08
Assess students in procedural skills taught prior to calculus	2.93	1.26
Reteach content using simpler numbers if students are struggling	3.47	1.15
Teach foundational skills students have not yet mastered	3.29	1.01
Have students apply prerequisite mathematics skills in the context of calculus concepts	4.40	0.82
Simplify the context of word problems by re-writing them	3.07	1.36

Present multiple solution strategies for a given problem or problem type	3.84	0.83
When a problem can be solved multiple ways, use strategies that students most often use	4.07	0.93
Connect calculus to students' experiences	3.68	1.01

Short Answer Responses

During the open coding of the short-answer responses to the question, "How, if at all, is your instruction affected by the mathematical understanding students bring when they *come into* your class at the start of the semester?" several themes emerged about how introductory calculus instructors perceive changes to their instruction based on their understanding of students' prior knowledge. Initially, this coding prompted me to divide the responses into five categories: instructors who said that their instruction was affected and gave me specific examples, instructors who said that their instruction was affected but gave me no or vague examples of instruction, instructors who said that it did not affect their instruction, instructors who gave an unclear response, and instructors who gave a student-focused response. Many (71 out of the 172 responses), instructors provided examples of instruction they engaged in response to students' prior mathematical knowledge. Although many of these align with the surveyed practices, a few new ideas came up. For example, explicit review was a common response. One respondent explains, "I assume most students need a "reminder" of the prerequisite topics. If they ask [or] seem to need more I do more review as part of my in-class explanations of problems." In addition, many instructors seemed to anticipate and build review into their plans for the course. This indicates that instructors can articulate practices they engage in to activate and connect to student prior knowledge and that they may do so in different ways than the literature has previously noted.

Discussion

These results are emerging as part of a more extensive dissertation study but indicate that introductory calculus instructors are mindful of the need to activate students' prior knowledge in the course. Additionally, they report engaging in practices that might help students strengthen their prior knowledge during introductory calculus instruction. Further, they might be engaging in more and less procedural instruction to support students (e.g., reteaching prerequisite skills versus engaging students in discourse). This finding is in line with previous research (Hovey et al., 2019; Rasmussen et al., 2019). Qualitative data supports the idea that instructors can identify and describe practices they engage in to respond to students' prior knowledge. Future research is needed to understand exactly how these practices are implemented and which are most accessible or provide the best student outcomes. Work in this vein can help the field move towards an incremental improvement approach (e.g., Litke, 2020) to better support calculus students without asking current instructors to overhaul their entire instructional approaches to introductory calculus.

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