Multivariable Calculus Instructors' Reports of Resource Use: Resources Used and Reasons for Their Use

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Given the challenge of visualizing the main constructs of two-variable functions and their differential and integral calculus, it is important to consider the use and perceived potential of resources to contribute to students' understanding of multivariable calculus. This case study considers how four instructors attempt to utilize resources in their multivariable calculus teaching and their motivations to do so. We study how these instructors think about the digital and non-digital resources that they use to foment students' understanding. We also look at their reporting of the ways that instructors, students, and resources interact in multivariable calculus to determine if resource use is meant to facilitate visualization, reasoning, or communication. With this, the study proposes to contribute to the discussion of resource use in multivariable calculus.

Keywords: multivariable calculus, digital resources, visualization

Introduction

Multivariable calculus is an important tool for modeling phenomena in science, technology, engineering, mathematics, and other fields of knowledge. There is a growing research base on the teaching and learning of two-variable functions that includes the study of related definitions, basic ideas, and geometric understanding (e.g., Martínez-Planell & Trigueros, 2021; Dorko & Weber, 2014; Martínez-Planell & Trigueros, 2012, 2019; Trigueros & Martínez-Planell, 2010; Weber & Thompson, 2014; Yerushalmy, 1997). There are also studies about the differential calculus of two-variable functions (e.g., Harel, 2021; McGee & Moore-Russo, 2015; Martínez-Planell et al., 2015, 2017; Tall, 1992; Trigueros et al., 2018; Weber, 2015) and their integral calculus (e.g., Jones & Dorko, 2015; McGee & Martínez-Planell, 2014; Martínez-Planell & Trigueros, 2020). All of this is discussed in a survey article on multivariable calculus (Martínez-Planell & Trigueros, 2021). There are some studies on the use of non-digital resources (e.g., McGee et al., 2012, 2015; Sherer et al., 2013; Wangberg et al., 2013, 2020). However, the literature on the use of digital technologies is somewhat sparse (e.g., Alves, 2014; Habre, 2001; Ingar & Silva, 2019; VanDieren et al., 2020).

To further study the use of digital resources, we seek to study how multivariable instructors report that they use resources in their teaching of multivariable calculus. More specifically, we consider how multivariable instructors report that digital, digitally generated, and other resources are used to nurture students' understanding and their reports of how students, instructors, and resources interact in the construction of multivariable calculus understanding. This study attempts to contribute to answering these questions by focusing on four instructors as they discuss the resources they use and the reasons for using them to determine if they are focusing on visualization, reasoning, or communication.

Theoretical Framing

Didactic Triangle and Didactic Tetrahedron

The didactic triangle is a model that can be used to organize and discuss instructional phenomena by considering three key components, the teacher, the students, and the content, as well as interpreting the links between the three components in the context being studied. Observing that "artifacts are not passive resources that teachers and students draw on but 'actively' shape activities," Rezat and Strasser (2012, p. 644) argue to extend the didactic triangle to the model of the didactic tetrahedron (displayed in Figure 1). The idea of the didactic tetrahedron was previously presented by Tall (1986). This seems particularly appropriate in our study since we focus on artifacts used, including both digital and non-digital resources that instructors leverage in their instruction.

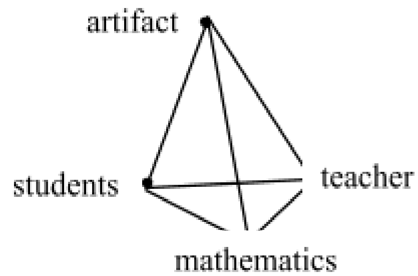

Figure 1: Didactic tetrahedron model (Rezat & Strasser, 2012).

The Teaching Triad

Jaworski's (1994) teaching triad (displayed in Figure 2) may be thought of as an expansion of the "teacher" node in the didactic triangle. It consists of three interrelated elements: sensitivity to students, mathematical challenge, and management of learning (Jaworski et al., 2017). *Sensitivity to students* relates to the teacher-student link. It describes the teacher's knowledge of students and the teacher's attention to affective, cognitive and social student needs. It includes the ways in which the teacher interacts with individuals and how the teacher guides group interactions. *Mathematical challenge* relates to the teacher-mathematics link. It describes the challenges that a teacher offers to students to promote mathematical thinking and activity. It includes the assigned tasks, the posed questions, and any teacher emphasis on metacognitive thinking. The *management of learning* construct extends the didactic triangle to the wider context of the classroom. It describes the teacher's role in the constitution of the classroom learning environment by the teacher and students. It includes, among other things, classroom groupings, instructional planning of tasks and activities, selection and use of instructional resources, and classroom norms that are set. The three components are interrelated.

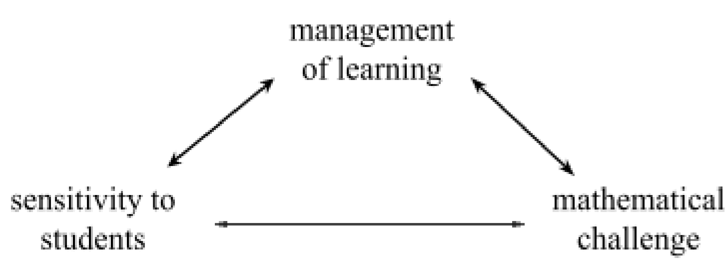

Figure 2: The teaching triad (Jaworski, 1994).

Spatial Literacy

The broader sense of being literate refers to being competent and knowledgeable in a certain area (Moore-Russo & Shanahan, 2014). Spatial literacy includes the abilities to visualize objects in three-dimensional space, to reason about their properties and relations, and to communicate with others about these objects and relations. Moore-Russo and colleagues (2013) introduced a framework that describes spatial literacy in terms of three domains (shown in Figure 3): visualization, reasoning, and communication. *Visualization* is the process of generating cognitive representations of spatial objects through visual images that may or may not be facilitated by external representations or physical actions. For example, a student can internally visualize a paraboloid when looking at a representation of a paraboloid on a computer screen, or when listening to the word "paraboloid", or when out of habit or by memorization the student imagines the geometrical object given the symbolic representation $z = x^2 + y^2$ without reasoning about or justifying the relationship or communicating about it with others. *Reasoning* is defined as the cognitive processes that individuals use to form conclusions or make judgments from a given set of premises; it is the process of organizing, comparing, or analyzing spatial concepts. For example, a student is reasoning when mentally using transversal sections to justify that the graph of $z = x^2 + y^2$ is a paraboloid. Reasoning may or may not involve geometric imagery, as when using an entirely symbolic mental argument to conclude that $z - x^2 = y^2$ is also a paraboloid. *Communication* is the exchange of information with others using resources such as language, written symbols, gestures, etc. to convey ideas that relate to spatial objects or relationships. For example, a student explaining to another why the graph of $z = x^2 + y^2$ is a paraboloid is engaging in communication. At the same time, the student may or may not be engaged in reasoning and/or visualization depending on the argument in the student's mind. The different domains overlap as suggested in Figure 3. Although we will not give examples of each possibility, they may be imagined using variations of the above paraboloid examples.

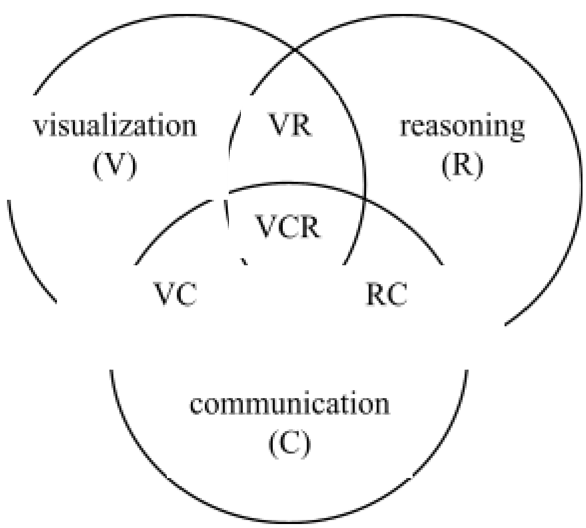

Figure 3: Spatial literacy domains (Moore-Russo et al., 2013)

Research Questions

This case study considers how university instructors reflect on teaching multivariable calculus specifically related to the resources they use. The specific research questions follow.

1. Which **resources** do university mathematics instructors report that they use or have used in the past for teaching multivariable calculus? What reasons do they give for their selections? How do they report that they use the resources?

2. Which, if any, **spatial literacy domains** (i.e., visualization, reasoning, communication) do instructors mention and focus on? Is this the same for all instructors or does it vary by instructor?

Methods

The data reported here are part of a case study that involved four university instructors from four different U.S. universities. We studied written responses to a series of 24 survey questions and present the results of the eight questions that directly address the resources used and the reasons for their use below. The instructors were part of a convenience sample of multivariable calculus professors from four different universities in four different states working on a project whose main purpose was to help students develop a deeper understanding of multivariable calculus concepts through hands-on, physical 3D explorations using guided learning activities and innovative, pedagogically designed 3D-printed surfaces and solids. All four instructors were currently using the digital CalcPlot3D platform (Seeburger, 2018). Since this paper focuses on the instructional decisions that guide the work of teaching, specifically of teaching multivariable calculus, we analyze data from the instructors in regard to their teaching rather than analyzing student thinking, student engagement, or how students make meaning of the content at hand.

An instructor's response to a survey question was considered the unit of analysis. The research team independently reviewed the survey responses multiple times. During their independent review, they marked ideas that appeared frequently. The "bottom-up" analysis yielded possible directions that the analysis could take. The team met to discuss this first pass over the data, comparing their notes. They decided to revisit the data individually while considering possible frameworks that could be used to frame the research. The team met twice to determine the framing of the study and then decided on specific categories to use for the coding. Each team member coded the data set independently before coming together to discuss the coding until a consensus was reached. During this process, the team pulled out representative quotations from the data set. The results were analyzed both in terms of each instructor as well as in terms of general tendencies across the group of four instructors.

Results

We begin by reporting our results for the first series of research questions: Which resources do university mathematics instructors report that they use or have used in the past for teaching multivariable calculus? What reasons do they give for their selections? How do they report that they use the resources? In our discussion, we will refer to the four surveyed instructors as M, P, S, and T, and we consider their use of digital resources then their use of other types of resources.

Instructor M has used CalcPlot3D since initially teaching multivariable calculus. For out-of-class assignment, she tried Survey Monkey and a publisher's online homework system before settling on WeBWorK, since she could embed video lectures and CalcPlot3D labs in that platform. She has also used Jamboard and Flipgrid for more student interaction. She has used Matcha.io as an online, mathematics editor. Instructor P initially used Cabri Geometry and Excel before writing his own Visual Basic, Java, and Java Script applets. CalcPlot3D followed from this. This instructor also used other digital resources like Smartboards, which he used to record lessons and an online custom OER textbook on the LibreTexts platform together with the WeBWorK online homework system and student-produced videos. While some digital textbooks might be what Viglietti and Moore-Russo (2018) call "static resources" (i.e., text-heavy resource that relies on typed text and digital pictures), the custom OER textbook used by P leveraged interactive figures. Instructor S initially used Mathematica and GeoGebra before she adopted

CalcPlot3D. Instructor T initially relied on Mathematica with some incorporation of Desmos and Tikz for two-dimensional plots before he moved to using CalcPlot3D. The reasons given by the four instructors for using digital resources in multivariable calculus and for changing their initial graphing software (which only applied to P, S, and T) are detailed in the next paragraphs.

Even though their initial choices of software varied, their reasons for using digital resources were fairly consistent. The responses from all four instructors showed the importance of *mathematical challenge* to them and the value they placed on students visualizing ideas of multivariable calculus, convinced that doing so would improve student understanding. M mentioned that digital resources "give students a deeper understanding of the material beyond the computational and symbolic which they are used to" stating that digital resources "challenge students to think and reason visually in 3D." P reported that "students need to see the relationships of calculus in motion to develop the kind of intuition for the definitions and constructs involved." S claimed to use digital resources every class "to create illuminating visual demonstrations" and to show students the insights that could be gained through *visualization*. S felt that digital resources help foster "a spatial awareness and understanding that connects to the material they [students] are learning." T felt digital resources helped students "attach geometric meaning to various course concepts."

The reasons that Instructors P, S and T gave for changing from their initial choices of graphing resources to a different digital resource (in this case, CalcPlot3D) also show *sensitivity to students* in their concern for students' affective, cognitive, and social needs. The most cited reasons were that not only is CalcPlot3D free (i.e., students do not incur any cost), but more importantly it requires no specific syntax so that it is easier to use. Instructor P said that he "had to change platforms a number of times to keep my visualizations usable and easier to get students to play with themselves." When explaining why she switched digital resources, S reported that "CalcPlot3D requires no coding syntax, unlike Mathematica, which I have used before for this course. It is more intuitive for users than GeoGebra, which makes it much more accessible to students." She stated, "my students could immediately jump into graphing with the program without any training." Instructor T adopted CalcPlot3D because "the syntax is simple enough that I feel good about having students work with it, and the fact that it's web-based is great."

Issues related to the *management of learning*, that included providing formative feedback, also figure among the reasons given by the instructors for using digital resources. Instructor comments often dealt with software features that allow for the creation and use of in-class instructor demonstrations or independent student work. Instructor M said, "I have embedded video lectures into webwork with 'exit ticket' questions that students answer immediately following watching a video. I also embed CalcPlot3D labs in webwork to give students immediate feedback." Instructor P stated that he would use a scenario presented with CalcPlot3D as a common focus for student discussion board assignments, and he "encouraged students to visually verify their results using CalcPlot3D" during student video presentations. Instructor S reported that she liked "the self-explanatory nature of the [CalcPlot3D] program and the powerful options that are built in to help instructors easily craft in-class demonstrations." Instructor T said, "there are a lot of visualizations I spent hours on creating in Mathematica, only to later realize that CalcPlot3D had already built in…the software is robust enough that I can usually render the objects that I want to render, and I'm calibrating my expectations of what I can ask students to do with the software independently and with how much guidance." T reported using CalcPlot3D as "visuals I include in lectures, assignments, and worksheets."

As for non-digital resources, there was also a wide variety of resources that have been used by three of the instructors including: 1x1x1 blocks (M), Play-Doh (M), paper cylinders to demonstrate parametric curves lying on surfaces and finding tangent planes (M), a hill on campus to explain partial derivatives (S), extendable pointers with arrows (S), a machine-shop-produced tabletop 3D coordinate system (S), coffee stirrers and fishing line (T), giant arrows made from wooden dowels for vectors (T), the K'nex toy pieces for xyz axes (T), various foods that were sliced to represent integration (T), and cardboard sheets for planes (T). The incorporation of such a wide variety of resources shows the instructors' *sensitivity to students*' cognitive needs. These resources shed light on how instructors have changed their *management of learning* for certain topics by bringing what they believed to be the most relevant resources.

The use of 3D-printed surfaces, a non-digital yet digitally generated resource, were reported by all four instructors. This is of particular interest since it is the focus of a project in which the four instructors participate. All four instructors were aware and mentioned that CalcPlot3D has the capability to produce STL files that can be used to print a 3D surface (or curve) model using a 3D printer. The instructors report using 3D-printed surfaces either just a few times a semester or some of the time. The most common reasons given for using 3D-printed surfaces related to *mathematical challenge*; they included their beliefs that: 1) embodied learning is privileged by the potential interactivity and manipulability of the physical object and 2) the 3D model had the potential to bridge the gap between the 2D renderings of a screen and the 3D reality. Instructor M mentioned student engagement when using 3D-printed surfaces, she claimed the engagement came from students being "able to hold and point to them" and believed in the importance of showing "an additional visual representation of the surface beside the 2D version on the screen." Instructor P shared that since a 3D-printed surfaces are "more concrete than the 3D renderings on the 2D computer screen using CalcPlot3D, it can be quite helpful for students, making the surfaces tactile" and "used it to help us confirm our various traces of the surface drawn on the Smartboard." Instructor S mentioned, "I wanted to provide students with a physical object to touch, to hold at various angles, and to point to features as they discuss concepts with their classmates. Having a tactile object removes any ambiguity that could arise from viewing a 3D object on a 2D screen." While Instructor T claimed that both 3D-rendered and 3D-printed objects afford avenues for students to be able to attach geometric meaning, he believed, "3D models have some advantages, like being able to draw on the models; they are easy to handle and view from multiple angles, in a more natural way than computer images." The reasons for using 3D printed surfaces are indicative of the instructors' *sensitivity to students*' cognitive needs. However, there were comments, especially from Instructor T about how they also affect the *management of learning* since there are non-negligible constraints to their use in class (e.g., number of models needed, class time issues in passing out and collecting the models).

We now report our results for the second series of research questions: Which, if any, spatial literacy domains (i.e., *visualization*, *reasoning*, *communication*) do instructors mention and focus on? Is this the same for all instructors or does it vary by instructor? Concerning the spatial literacy domains, the data from all four instructors were similar and provided evidence of a seemingly balanced instructional focus on VR (*visualization-reasoning*) and VC (*visualization-communication*), with some VRC (*visualization-reasoning-communication*) instances.

In the VR overlap, instructor comments focused on issues related to students being able to see the idea and relationships in multivariable calculus as they were trying to make meaning of the topics or trying to solve tasks that had been assigned to them. Instructor M mentioned how "challeng[ing] students to think and reason visually in 3D… gives students a deeper understanding of the material beyond the computational and symbolic which they are used to." Instructor P talked of benefits to student understanding when they are "rotating the surfaces in their hands to gain perspective." S claimed, "Visualization through CalcPlot3D gives students a spatial awareness and understanding that connects to the material they are learning. Instead of going through calculations blindly, students develop a spatial understanding of the concepts."

In the VC overlap, Instructor M related that "Each group got a different model so some students 'connected' with their model referring to it later in the class (e.g., 'Remember in our group's model there was that point that....')." While some of the comments focused on student-to-student *communication*, a number of comments related to instructor-to-student *communication* where instructors used resources in their lectures to help visual clarify the items being presented through demonstrations and examples. Instructor P stated, "I have brought in a couple 3D printed surfaces for students to see and to point out certain features."

The overlap of VRC was also noted, but not as often as comments that involved either VC or VR. As shared earlier, Instructor P encouraged students to use CalcPlot3D as a common anchor that allowed them to visually communicate their arguments in online discussion forum posts as well as in student-produced video presentations. Instructor S observed, "during office hours, students will reach for one of the 3D surfaces in order to ask their question. Likewise, they will reach for a surface to explain the answer to a question that I have posed to them." Instructor T commented that the "models are easy to handle and view from multiple angles (in a more natural way than computer images), and I find they work well for in-class group work" that would naturally involve *communication* among the students and for "office hours" that would involve *communication* between the instructor and a student.

Conclusions

We found that concern for students' learning and the conviction that geometrical understanding is crucial in multivariable calculus, moved the four instructors to consider the use of a variety of resources in their teaching. Three of the instructors started with a graphing resource other than what they were using at the time of the interview (CalcPlot3D). Ease of use for them and for students was the major factor influencing their change in digital technology. It seems that over time (and with more instructor experience) the attention to the use of technology for teaching shifted from being instructor-centered (demonstrations to present in class) to student-centered (activities to engage students and group work). In the spatial literature domains, there was about an equal number of *visualization-reasoning* and *visualization-communication* comments and some which also suggested *visualization-reasoning-communication* activities. The observations resulting from our study suggest that the interviewed instructors were from the start *sensitive to students'* needs and that future faculty adopting digital resources in their teaching could benefit from support in issues dealing with the *management of learning*: the creation of demonstrations and activities using their software of choice, being able to discuss constraints and difficulties using particular software for graphing in multivariable calculus, coordinating the use of graphing software and digital homework systems like WeBWorK, using digital technology in evaluation, sharing materials, and so on. Similarly, the instructors seem to find great potential of digitally generated resources, particularly 3D-printed models, in helping with student *visualization*, *reasoning*, and *communication*. However, the creation and subsequent use of these models in class seems to also require as much support as digital resources.

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