A Coordinate System Framework

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Coordinate systems underlie many important topics in undergraduate mathematics, such as functions, equations, and graphs. By providing a framework that describes the aspects of coordinate system thinking, it offers a foundational framing to examine a wide range of characterizations of student reasoning in undergraduate mathematics. This proposal introduces a conceptual framework that emphasizes how one engages with an object viewed in a single coordinate system and multiple coordinate systems. The framework consists of four sub-parts: Naming, Locating, Re-Naming, and Re-Locating.

Keywords: linear algebra, calculus, coordinate system, change of coordinate system, framework

A coordinate system promotes student spatial thinking by imposing a visual aspect to undergraduate mathematics that may be abstract and computational. Many researchers have investigated how undergraduate students reason about coordinate systems when transitioning from one system to another in the context of precalculus, calculus, and linear algebra (Hillel & Sierpinska, 1993; Arcavi, 2003; Montiel, Vidakovic, & Tangul, 2008; Sayre & Wittmann 2008; Montiel et al., 2009, 2012; Lee et al., 2020; Moore et al., 2014; Hillel, 2000; Selby, 2016; Wawro et al., 2013; Zandieh et al., 2017). Despite these studies, there is no overarching framework for interpreting student reasoning across a variety of mathematical contexts. Therefore, this paper proposes a framework to provide a foundation for interpreting a wide range of undergraduate students' thinking with coordinate systems.

The coordinate system framework encompasses two fundamental processes for representing objects within a single coordinate system: Naming and Locating. (1) Naming involves assigning numbers and symbols to spatial and geometric objects, such as points, vectors, lines, and curves in accordance with conventions established by a coordinate system. For example, (2,3), <7,5>, and $y=-3x^2+5x-1$ are all examples of objects represented through the naming process. On the other hand, (2) Locating is the process of plotting symbolic expressions into visual representations, such as dots or graphs in space, using a coordinate system. An example of this process is graphing the function $y = x^3 - 10x$. These two processes can be extended to include reasoning about an object in the context of multiple coordinate systems: Re-Naming and Re-Locating. (3) Re-Naming involves describing the coordinates of a spatial object using a new coordinate system that is laid atop the object without moving it. In other words, the object does not receive a new name until a new coordinate system is imposed over its location. An example of Re-Naming would be taking a point (1,1) in the Cartesian coordinates renaming it as $(\sqrt{2}, \frac{\pi}{4})$ in the Polar coordinates. (4) Re-Locating involves plotting a symbolic expression that does not change into different spaces and transforming the shape of an object into a different shape by mapping points in the first coordinate system to points in the second coordinate system. Although the transformed location may appear different from the original, the two shapes are images of each other.

By using an overarching lens, researchers can better analyze student reasoning by focusing on how students think about each part of a problem, and how they leverage their understanding of coordinate systems to reason about multiple coordinate systems. This approach can also serve as a useful framework for teachers and researchers when designing tasks that involve coordinate systems.

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