Teaching Experience as a Frame for Contributions of Academic Mathematics Coursework to Teacher Learning

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We analyzed end of course reflections of graduate students in mathematics courses for practicing teachers. Participants were asked questions about mathematics content, habits of mind, and whether being a concurrent mathematics teacher while taking the graduate course impacted the value that they saw in either area. Participants were also asked to reflect on whether they believed the course would have been valuable when they were an undergraduate student. We first coded these reflections with Hoffman and Even's (2018, 2021) framework including essence, doing, and worth of mathematics. As a result of our analysis, we suggest extending their framework to mathematics and mathematics teaching, and to include content. Discussion includes suggestions for future research to understand how teaching mathematics reframes learning mathematics, and the importance of framing to understanding the uptake of mathematics courses for both prospective and practicing teachers.

Keywords: Content Courses for Teachers, Mathematical Knowledge for Teaching, Masters Degree Programs

Introduction & Background

We open by recalling a conversation that one of the authors often encounters when teaching a content course for secondary teachers, although this proposal will address content courses for practicing elementary, middle, and secondary teachers. The conversation begins when the topic of abstract algebra is brought up, a course where prospective teachers first learn modular arithmetic. A prospective teacher says, "I hated abstract algebra. I learned in that class that I don't like math." Yet, when practicing teachers in a graduate summer course, from the same district, learn modular arithmetic once more, they will say, "I really liked this class! I want to find a way to teach mods to my kids."

The two courses, the undergraduate abstract algebra course, and the graduate summer course, are taught by faculty in the same department. They address similar topics. Across different years, different faculty members, and different cohorts of teachers, these contrasting conversations repeat, as if in a time loop. Why does the conversation change, as teachers in this district cross the threshold from future teachers to current teachers?

While there is evidence that some teachers benefit greatly from taking graduate level coursework (University of Waterloo, n.d.), and there is evidence that content-focused professional development does impact teacher productivity across elementary and secondary (Harris & Sass, 2007), there is also evidence that it is pedagogical coursework and development that may ultimately make a larger impact on student outcomes (Baumert et al., 2010; Ottmar et al., 2015). Moreover, in some cases, the additional coursework may address content that overlaps with undergraduate coursework. There is a problem: the efficacy of undergraduate and graduate course taking, in terms of impact on teachers' teaching or in perceived usefulness of the course, is debatable and idiosyncratic.

The puzzle for us is this: mathematics teachers need more mathematics than they have learned in elementary, middle, and high school, in order to teach at that level. Yet taking mathematics as an undergrad--a natural place for candidate teachers to prepare mathematically to teach mathematics--seems to be met with adversity. There is a seeming paradox in how experience with teaching may mediate the uptake and relevance of mathematics courses.

Hence, we are interested in why teachers may find graduate and undergraduate courses useful or not. Our overarching research question is: How does teaching experience mediate teachers' uptake of mathematics? We address: *(RQ1) How do practicing teachers describe the contribution of graduate mathematics coursework on their learning? (RQ2) How would the same practicing teachers assess the relative value of this same mathematics coursework, had they taken the course as an undergraduate, prior to any teaching experience?*

In this study, we use *teachers* to refer to PK-12 classroom teachers, *students* to refer to PK-12 students, and we only use *instructor* to refer to tertiary level instructors.

Related Literature

In the existing empirical literature on the contribution of academic mathematics experiences to teacher learning, many practicing and prospective secondary teachers have been documented to find their mathematics experiences disconnected from teaching (e.g., Goulding, Hatch, & Rodd, 2003; Zazkis & Leikin, 2010) to be difficult in unexpected and sometimes unpleasant ways compared to secondary mathematics experiences (see Corriveau & Bednarz, 2017 for a review). At the elementary level, some prospective elementary teachers have found courses to be emotionally stressful and detrimental to their mathematical confidence, and to be disconnected from their teaching (e.g., Hart, Oesterle, & Swars, 2013). The literature also suggests that instructors of mathematics courses for elementary and secondary teachers design courses to combat disconnects and to support students' learning (e.g., Lai, 2019; Li & Superfine, 2018).

This literature has gaps. First, the literature does not distinguish between viewpoints of teachers before and after teaching. As our experience suggests, there may be a change of viewpoint that occurs after teachers enter the classroom. Second, the literature tends to report results from either multiple institutions, so it is hard to disentangle results from context, or from small case studies, meaning that it is hard to generalize. Our study addresses these gaps.

In designing our study, we observed that multiple guiding documents, in the US and beyond, emphasize the value of mathematical *content* and mathematical *habits of mind* (e.g., Ministry of Education, 2007; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). We also used the results of Hoffman, Even, and Mytlis on the contribution of academic mathematics to teacher learning (Hoffman & Even, 2018, 2021; Mytlis & Even, 2021). Their findings suggest that teachers and mathematicians view contributions in three categories: (1) the *essence* of mathematics, that is the nature of mathematics, or "what is mathematics"; (2) *doing* mathematics, or "how is mathematics done"; and (3) the *worth* of mathematics, for its own sake or for its applications. We interpret the *doing* of mathematics to be synonymous with *habits of mind*. We used content and habits of mind as a proxy for what may be explicitly taught in a mathematics course, and we use the results of Hoffman, Even, and Myrtis as a hypothesis for the learning that teachers perceive from these courses.

Conceptual Framework

Our past experiences influence our current perception. From our expectation of success at a task, to how much we value doing a task, to reaction time, our past experience can mediate how we respond to a current moment (e.g., Eccles & Wigfield, 2020; Pearson & Brascamp, 2008). In the case of teachers, prospective and practicing mathematics teachers differ in one significant respect: only one of them has past experience of classroom mathematics teaching. We hypothesize that this difference in past experience mediates how they experience mathematics learning, so that prospective teachers and practicing teachers may view the same mathematics content differently. Our study is an initial step to test this hypothesis by posing to practicing teachers who were enrolled in mathematics courses a counterfactual: would they have found this course just as valuable had they taken the course as an undergraduate prior to teaching?

We investigate potential differences in terms of framing (Goffman, 1974): how a person defines a situation. For instance, if one teacher defines a situation of learning as a "hoop to jump through", and a second teacher defines the same situation as "a place to learn how to do mathematics better", they may end up having different experiences of the same situation.

Participant Context

At Indigo University, the STEM Education Center, in collaboration with the Department of Mathematics, offers graduate mathematics courses taken by K-12 mathematics and science teachers (all institutional names are pseudonyms). Although many teachers enrolling in these courses are local, there are also teachers who enroll from out of state, as well as from countries outside the US. These graduate courses have been offered for almost two decades, and typically feature high end-of-course evaluations from the enrolled teachers.

We anticipated this site would be a fruitful context for our research questions for several reasons. The variety of backgrounds of teachers made it more likely that these teachers would have come from different undergraduate institutions. In this way, their collective viewpoints about experiences with undergraduate mathematics would reflect more than one particular institution or pedagogy. Moreover, we were interested in the case of a success story with mathematics. Frankly, we thought it would be more interesting and productive to document ways that experiences with graduate mathematics coursework are useful, rather than the opposite.

Data & Method

Participants included 46 practicing teachers who were enrolled in 9 mathematics courses in the summer masters program at Indigo University. Participants consented to sharing an end-of-course reflection assignment, which formed the data source for analysis. In total, the 46 teachers submitted 61 separate responses, as some teachers were enrolled in more than one course. Among these were responses from courses intended for elementary teachers and for middle and secondary level teachers. Instructors who taught the participants agreed to include three common prompts for reflection. One prompt posed to teachers the counterfactual situation that they had taken the course prior to classroom teaching:

> Suppose you had taken this course as an undergraduate, before having any significant teaching experience. Would you have found the experiences with habits of mind less or more valuable than now? Would you have found the content less or more valuable than now? For what reasons?

This was Prompt 3 of the reflection assignment. We report only on analysis of responses to this prompt. Prompts 1 and 2 asked about how their experiences with habits of mind and course content might change the way they taught an activity or topic in the future, and defined habits of mind as "approaches to creating mathematical ideas or working on math problems". Example habits of mind and course content specific to the particular course were included in Prompts 1 and 2 (e.g., "looking at a problem from multiple perspectives" and "using precise definitions" were example habits of mind; and "Pascal's triangle" and "linear regressions" were example mathematical content).

Analysis for RQ1: Characterizing the contribution of masters coursework in mathematics to teacher learning

We used a combination of inductive and deductive coding (Miles, Huberman, & Saldana, 2018). We began with deductive coding of responses according to Hoffman and Even's (2018) categories of the contribution of academic mathematics to teacher learning: Essence, Doing, and Worth of Mathematics. However, we noticed statements describing contributions of the masters'

course to teachers' learning that did not fit neatly into Hoffman and Even's descriptions, and wanted to differentiate between contributions for the purpose of mathematics from those for teaching. We inductively expanded the categories as shown in Figure 1 (next page). In tabulating the coding, we elected to use the unit of analysis of teachers rather than responses. We noticed that some teachers who had enrolled in different courses used the same phrasing across responses to different courses, and that this phrasing was essential to our coding.

Analysis for RQ2: Comparing relative value of coursework before and after teaching

We initially categorized responses as stating that the graduate course was *more valuable before teaching*, *more valuable after teaching*, *valuable either way*, or *none* (meaning, the response did not explicitly make a comparison of value before or after teaching). Then, as we noticed teachers stating that some aspects were more valuable before teaching and others more valuable after teaching, we inductively coded for the aspect of relative value (e.g., experience with habits of mind, content, technology).

Finally, we excerpted *frame* statements from responses, that is statements that explained something to the effect of, "Teaching experience made a difference in how I learned in the course because …". We surveyed these frame statements for themes in how teaching experiences framed the contributions of the mathematics courses to teacher learning. In tabulating this coding, we elected to use the unit of analysis of responses rather than teachers, because the comparison of contributions was course-dependent, and some teachers stated different comparisons for different courses.

Figure 1. Categories of contribution to teacher learning for RQ1

Contribution		Description: Statement that the course experiences …
Essence	Essence of Math	Changed a teacher's conception of "what is mathematics", or will help them convey to students a conception of "what is mathematics"
	Essence of Math Teaching	Changed a teacher's conception of the nature of teaching, including the possibilities of what teaching can look like
Doing	Doing of Math	Contributed to a teacher's mathematical habits of mind, or that the course will help teachers cultivate students' mathematical habits of mind, including struggling or persevering through mathematics
	Doing of Math Teaching	Statement that the course exposed teachers to teaching practices or activities they see as valuable or beneficial
Worth	Worth of Math	Statement that the course improved a teacher's sense of the value of mathematics, or helped a teacher convey to students the value of mathematics, either for its own sake, or because of its applications to science, technology, etc.. or that the course
	Worth for Math Teaching	Statement that the course improved a teacher's sense of the value of mathematical knowledge for the purpose of teaching
Content	Content Knowledge	Statement that a teacher developed specific content knowledge, or that the course will help teachers support students' acquisition of specific content knowledge, including connections among topics and better explanations (Shulman, 1986)
	Pedagogical Content Knowledge	Statement that a teacher developed specific pedagogical content knowledge, including detecting the potential of activities to facilitate learning; knowledge of student conceptions; or knowledge of or about multiple representations and explanations (Baumert et al., 2010)

Results

Characterizing the contribution of masters coursework in mathematics to teacher learning

Across our participants, the most frequently cited positive contribution of coursework was to the categories of doing mathematics (16 participants; 34.8%) and content knowledge (12 participants; 26.1%). Our prompt specifically mentioned content and habits of mind, and hence it is unsurprising that these categories are mentioned frequently. Table 1 summarizes the results.

Our main differences from Hoffman and Even are (1) introducing Content as a category and (2) extending contributions to both mathematics and mathematics teaching. This delineation allowed us to record that one possible benefit of these courses is the teaching itself: teachers mentioned the benefits of exposure to "great habits of teaching" (Doing of Math Teaching), or "realiz[ing] that how I am teaching now is not fulfilling what my expectations are for my students" (Essence of Math Teaching). Among our participants, 12 cited content knowledge as a contribution, for example, discussing the value of learning about concepts of calculus, or properties of number lines. Teachers explained the value in terms of being able to see more connections or providing better explanations in the future. All but 12 teachers cited a contribution of the graduate course in the responses analyzed.

Table 1. Contributions of coursework in mathematics, with unit of analysis being teachers (n = 46)

Contribution	Math/Teaching	# Participants mentioning contribution (%)
Essence	of Math | Math Teaching	0 (0%) | 2 (4.3%)
Doing	of Math | Math Teaching	16 (34.8%) | 7 (14.2%)
Worth	of Math | Math Teaching	0 (0%) | 2 (4.3%)
Content	CK | PCK	12 (26.1%) | 5 (10.9%)

Analysis for RQ2: Comparing relative value of coursework before and after teaching

We found that teachers parsed the relative value in terms of habits of mind, content, the course as a whole, and technology. We coded *valuable either way* when teachers wrote about benefits in both directions. We coded more *valuable AFTER teaching* only when teachers wrote about benefits in one direction (resp. *BEFORE)*. See Table 2. We only coded *course as a whole* when the teacher did cite a specific aspect, but unambiguously stated relative value.

Table 2. Aspects of contribution and their relative value, with unit of analysis being responses (n = 61)

	Course as a whole	Habits of Mind	Content	Technology
More valuable AFTER teaching	4 (6.6%)	24 (39.3%)	18 (29.5%)	1 (1.6%)
More valuable BEFORE teaching	3 (4.9%)	9 (14.8%)	11 (18%)	0 (0%)
Valuable either way	1 (1.6%)	6 (9.8%)	6 (9.8%)	0 (0%)

We found in total 27 framing statements. Analysis of framing statements suggests that overwhelmingly, teachers found an aspect more valuable after teaching because they could envision how the aspect could be applied to their teaching – and that they could not do so before. As one participant stated, ""Without teaching experience, I would not be thinking about those

things as we go along." For brevity, we discuss only habits of mind here. Participants mentioned that habits of mind were useful now, and not previously, primarily because they now saw the value of habits of mind to students ("... now that I have had experiences working with students, I understand the importance of being able to solve using multiple strategies") or because they could better empathize with students ("When I actually had the experience of teaching in a classroom, I was surprised how many students struggled with the content. Having to struggle through habits of mind problems helped me to see what my students go through daily. I think if I would have taken this class earlier, it would not have resonated with me as much.").

Discussion & Conclusion

We undertook this study because of a local phenomenon that teachers seemed to have differing evaluations of the value of similar mathematics coursework, depending on whether they took the material after or before teaching experience. We posited that teaching experience mediated the uptake of course experiences in some way. Consequently, we examined how practicing teachers describe the contribution of graduate mathematics coursework to their learning, and how they would assess the relative value of this same coursework, had they taken the course as an undergraduate, prior to any teaching experience. We posed to teachers in an end-of-course reflection assignment the counterfactual situation of taking the same course as an undergraduate.

We found that most teachers cited some contribution of the graduate course to their learning, including the emphasis of the course on habits of mind, or the course's content. We also found that the plurality of responses indicated that teachers felt this course was more valuable after teaching than prior to teaching. To investigate why this may be the case, we examined framing statements. In these statements, a common theme was that without teaching experience, teachers did not realize the worth of habits of mind, or of content, for cultivating these aspects in students. Another common theme was that struggling in graduate coursework gave teachers empathy for their students, which would ultimately help the teachers be more understanding of students in the future.

We view our main contributions to be twofold. First, we extend Hoffman and Even's (2018, 2021) theoretical framework, and elaborate the possibilities for how academic mathematics coursework can contribute to teacher learning. Not only may they learn about mathematics, but they may also rethink their own teaching, or find teaching practices and activities modeled by the instructor they wish to bring into their classroom (cf. Wasserman & McGuffey, 2021). Second, we introduce the notion of framing as an explanation for why teachers may find courses useful or not, and how utility may shift over time. Prior work on teachers' conceptions of academic mathematics experiences have not differentiated between viewpoints before and after teaching experience. But as our results suggest, undergraduates and practicing teachers may frame their mathematics learning very differently. We suggest that further work investigating these frames can contribute to improving courses for both populations, as well as potential differences to work into course design with intention.

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