

A Genre Analysis of Mathematical Remarks

Valentin A. B. Kühle
Auburn University

This report describes the preliminary results of a genre analysis of mathematical remarks, which was conducted using Bhatia's (1993) seven-step genre analysis model. As part of the analysis, discussion threads on Mathematics Stack Exchange about mathematical remarks were analyzed as well as a corpus of remarks from first-year mathematics lecture notes of a British university. An overarching communicative purpose and seven non-mutually exclusive "mini communicative purposes" (Hyon, 2018, p. 30) of the genre of mathematical remarks are identified.

Keywords: Discourse, Genre Analysis, Mathematical Remarks

The American Mathematical Society's (AMS) Publications Technical Group (2017) observed:

In mathematical research articles and books, theorems and proofs are among the most common elements, but authors also use many others that fall in the same general class: lemmas, propositions, axioms, corollaries, conjectures, definitions, remarks, cases, steps, and so forth [...] (p. 2)

To account for differences between elements of this list, the *amsthm* package supports three theorem styles: plain (e.g., for theorems and lemmas), definition (e.g., for definitions and axioms), and remark (e.g., for remarks and notes). By being named alongside theorems and proofs as well as receiving their own theorem style by the AMS, "remarks" hold a privileged place in the writing of research mathematicians. The purpose of the study outlined in this report is to describe the genre of mathematical remarks using genre analysis, in particular by using Bhatia's (1993) seven-step genre analysis model.

As my work is of preliminary nature, I will only elaborate on a subset of the seven steps (i.e., steps 1, 2, 4, and 6). Further, in the interest of space, I will only report on findings regarding the communicative purposes of the genre of mathematical remarks, that is, I omit my findings regarding the form of the genre. This choice is motivated by the definition of genre I draw on (i.e., Swales's [1990] definition), which places communicative purpose at the center of a genre. Thus, this report's overarching question, which also guides much of the genre analysis, is: What are the communicative purposes of mathematical remarks?

Background Literature

As discussed by Hyon (1996) as well as Bhatia and Salmani Nodoushan (2015), researchers from three different (Anglophone) research traditions have sought to study genre: (a) New Rhetoric studies, (b) systemic functional linguistics (SFL), and (c) English for Specific Purposes (ESP). But, as Bhatia (2012) remarked, "[g]enre theory, in spite of these seemingly different orientations, covers a lot of common ground" (p. 241). For an in-depth comparison of the three traditions, see Hyon (1996) and Hyon (2018). In short, differences between the traditions that were more pronounced in the 1980s and 1990s have begun to fade in recent times with particularly the New Rhetoric and ESP traditions having ever-greater overlap (Hyon, 2018). I draw on the ESP tradition and provide a few brief remarks on this tradition below.

English for Specific Purposes

The ESP tradition of genre theory draws on Swales's (1990) definition of genre, which—in abbreviated form—reads:

A genre comprises a class of communicative events, the members of which share some set of communicative purposes. [...] In addition to purpose, exemplars of a genre exhibit various patterns of similarity in terms of structure, style, content and intended audience. (p. 58)

Building on Swales's (1990) definition of genre, Bhatia (1993) developed a seven-step genre analysis model attending to purpose and form: (a) placing the given genre-text in a situational context, (b) surveying existing literature, (c) refining the situational/contextual analysis, (d) selecting corpus, (e) studying the institutional context, (f) levels of linguistic analysis, and (g) specialist information in genre analysis. As Bhatia (1993) explained about these seven steps: These steps have been artificially separated for the sake of convenient formalization and systematic discussion. Moreover, it is not the intention to suggest that in all such investigations, the analyst must go through all the stages listed above and certainly not in that order. (p. 40)

For an application of Bhatia's (1993) model, see Singh et al. (2012). For four reviews of Bhatia (1993)—as well as his response to these reviews—see Nielsen et al. (2017).

Method

Data Collection

I collected two sets of data, each for a different step of Bhatia's (1993) seven-step genre analysis. For step 2 (i.e., surveying existing literature), I compiled all discussion threads related to mathematical remarks I could find on Mathematics Stack Exchange (MSE). The search was conducted on August 29, 2022 and consisted of searching for “remark” using the MSE search function. The top 500 results (sorted by “relevance”) were then examined to determine whether the question asked pertained to conventions about writing mathematical remarks. I then inspected each discussion thread's “related” section to find any additional remark-related questions but found none. All ten relevant discussion threads were downloaded.

For step 4 (i.e., selecting a corpus), I compiled all remarks from the PDF lecture notes of all mandatory courses for first-year mathematics majors at my British undergraduate institution. These eight courses are: Foundations, Differential Equations, Analysis 1 & 2, Introduction to Abstract Algebra, Linear Algebra, Mathematics by Computer, and Geometry & Motion. These PDF files were written by the lecturers of these classes and uploaded or sold (in printed form) by the mathematics department (before or after the semester) for a small fee (£1–3). During the compilation process, three situations occurred: (a) remarks were the notes' only labeled exposition (in which case the remarks were included), (b) remarks were not the notes' only labeled exposition (in which case only the remarks were included), or (c) the notes contained no exposition labeled as remarks, only exposition labeled as “note” (in which case the “note”s were included). “Note”s were excluded (case b) or included (case c) to account for some authors possibly using the term synonymously with remarks, whereas others may have chosen to make a distinction. I ended up with a corpus of 31 remarks.

Given the preliminary nature of this work, rather than trying to create a large representative corpus of remarks (e.g., by compiling remarks from talks, lecture notes, textbooks, and research papers), I chose to focus on remarks from first-year lecture notes because this corpus was of a manageable size. Further, I suspected that this corpus might emphasize different communicative

purposes than the discussion threads on MSE—a difference of pedagogical interest to me. Thus, for step 6, I am studying the communicative purpose(s) of a sub-genre of mathematical remarks: the remarks made to first-year British mathematics majors in lecture notes.

Data Analysis

To analyze the genre of mathematical remarks, I am using Bhatia's (1993) seven-step genre analysis model. The four steps I detail in this report are steps 1, 2, 4, and 6. For step 1 (i.e., placing the given genre-text in a situational context), I drew on my knowledge and experience. Step 4 (i.e., selecting corpus) has already been described in the Data Collection.

For step 2 (i.e., surveying existing literature), I analyzed the MSE discussion threads by using structural coding (Saldaña, 2009) to identify segments of the data that: (a) asked a question (or made an assertion) about the form of mathematical remarks (e.g., "Can I have a 'remark' after a 'lemma' before its 'proof'?"), or (b) asked a question (or made an assertion) about the purpose of mathematical remarks (e.g., "Remarks may also be used to provide motivation"). As described in the introduction, I will only report on the latter analysis in this report.

For step 6 (i.e., levels of linguistic analysis), Bhatia (1993) outlined three levels of linguistic analysis for analyzing one's corpus. I report on my level 3 analysis (i.e., the largest grain size), which Bhatia (1993) referred to as structural interpretation of the text-genre. Such an analysis may break down a genre into different moves, each of which has its own "mini communicative purposes" (Hyon, 2018, p. 30). Given that remarks are typically short texts of only a few sentences, I considered it unlikely to find a series of moves the genre follows. Nevertheless, I used the idea of "mini communicative purposes" to identify different sub-purposes that remarks can have. To identify these purposes, I process coded (Saldaña, 2009) the remarks in my corpus.

Preliminary Results

As Bhatia (1993) specified, researchers need neither follow all his steps nor follow them linearly. With this in mind, I share my preliminary results for steps 1, 2, and 6 below.

Step 1: Placing the Given Genre-Text in a Situational Context

To place the given genre-text in a situational context, Bhatia (1993) encouraged analysts who come from the relevant speech community to draw on their own experience and background knowledge. As a mathematics education researcher with a background in mathematics, I am familiar with the communication of proof-based mathematics and consider myself a member of the relevant speech community. Thus, in this section, I share experience-based observations I made before beginning my analysis. For the purposes of this report, I include only my observations about the communicative purpose(s) of mathematical remarks.

Context. Most of my relevant experience comes from studying mathematics at a British university. Lectures at the university consisted of a chain of labeled "blocks" (e.g., definition, theorem, proof, remark). These blocks—including remarks—were both spoken and written. Remarks were announced in spoken discourse (e.g., "Let me make a remark.") and written discourse (e.g., "Remark: The function ..."). The only spoken discourse not recorded on the boards was unlabeled exposition, whose purpose it typically was to motivate material.

Communicative purpose. A remark's purpose is to highlight a piece of information that does not fit into the mathematical quartet of definition, statement-to-be-proved, proof, and example but is too important to be left as unlabeled exposition. Remarks can, for instance, relate one theorem/proposition/lemma to another (e.g., one being a special case of another), warn the

reader about a potential confusion, indicate an alternative proof, highlight an idea or nuance that may have been lost amid mathematical notation, or share an alternative notation.

I suspect that the distribution of communicative purposes of remarks in a text depends on the intended audience. For instance, “highlighting an idea or nuance that may have been lost amid mathematical notation” might be a more prevalent purpose for remarks aimed at newcomers to proof-based mathematics, whereas remarks “relating one theorem/proposition/lemma to another” might be more prevalent when the audience is research mathematicians.

Step 2: Surveying Existing Literature

In his call to survey existing literature, Bhatia (1993) took an expansive view of possibly pertinent literature. Particularly relevant to this study is the inclusion of practitioner advice. Thus, as described under “Data Collection,” I searched MSE for “remark” and found ten relevant discussion threads. After analyzing the discussion threads, I identified the following purposes of mathematical remarks:

1. to be able to refer to some piece of exposition (this requires that the remarks are numbered);
2. to draw attention to a piece of exposition;
3. to isolate (important or noteworthy) exposition tangential to the main exposition;
4. to give the reader a break (e.g., with an historical anecdote);
5. to provide motivation;
6. to state why some hypotheses are necessary or natural;
7. to state why one cannot hope to get stronger results;
8. to justify a seemingly unnecessarily circuitous route;
9. to address a potential distraction in the reader’s mind;
10. to point out pitfalls; and
11. to point out (and possibly explain) the following of a certain convention.

Step 6: Levels of Linguistic Analysis

Analyzing my corpus of mathematical remarks, I identified the following non-mutually exclusive (mini) communicative purposes of remarks:

1. justify the author’s choices on a meta-level (e.g., why they made a particular choice);
2. respond to a possible reader assumption or reaction (e.g., pointing out a pitfall);
3. point out something the reader may have overlooked (e.g., pointing out a border case, pointing out an additional implication);
4. introduce new or different notation and terminology;
5. point out an alternative (e.g., to a definition, an equation);
6. give advice; and
7. share background or additional context (e.g., historical anecdote, practical application, related concept).

As aforementioned, these “mini-purposes” of remarks are not mutually exclusive and, while studying their intersections, one combination of moves stood out due its frequency and structure: purpose 2 followed by purpose 7. That is, around half the time, in addition to responding to a possible reader assumption or reaction, authors chose to share background or additional context.

Discussion

Synthesizing across steps 1, 2, and 6, I observe that there are different grain sizes of communicative purposes of mathematical remarks. The overarching communicative purpose

appears to be to separate some exposition from the main exposition—either for elevation or isolation. For example, whereas some remarks warn of a pitfall and are numbered to be able to refer to them more easily (i.e., elevated exposition), other remarks isolate exposition tangential to the main exposition to the extent that they can be skipped—one of the analyzed remarks even included the phrase “feel free to ignore this remark.” Discounting the possibility of reverse psychology, mathematical remarks are thus sometimes (almost) integral (elevated exposition) and at other times skippable (isolated exposition). I suspect this duality may leave novice readers of proof-based mathematical texts wondering whether remarks can be skipped or not.

Moving to the mini purposes (or sub-purposes) of mathematical remarks, I offer the following synthesized list of mini purposes from steps 1, 2, and 6 of my genre analysis:

1. justify the author’s choices on a meta-level (e.g., why some hypotheses are necessary or natural, why one cannot hope to get a stronger result, why one took a seemingly circuitous route, why one made a particular choice);
2. respond to a possible reader assumption or reaction (e.g., pointing out a pitfall, addressing a possible distraction in the reader’s mind);
3. point out something the reader may have overlooked (e.g., pointing out a border case, pointing out an additional implication, pointing out that B is a special case of A);
4. introduce new or different notation and terminology;
5. point out an alternative (e.g., to a definition, an equation, a proof);
6. give advice; and
7. share background or additional context (e.g., historical anecdote, practical application, related concept).

As the step 6 analysis showed, these purposes are not mutually exclusive and there may be some that combine more frequently (e.g., purpose 2 followed by purpose 7). As aforementioned, I also hypothesize that some purposes (e.g., sharing background or additional context) may lead to remarks which authors consider more ignorable than remarks with other purposes (e.g., responding to a possible reader assumption or reaction).

During the analysis, I also observed three differences between the mini communicative purposes identified in steps 2 and 6. First, the discussion threads seemed to emphasize purpose 1—a purpose only seen once in the lecture notes. Second, purpose 6 appeared in the lecture notes but did not explicitly appear in the discussion threads. Third, purpose 3 appeared in the notes but not in the discussion threads. (This last observation is partially in line with my suspicion voiced in pre-analysis step 1.) I am left wondering to what extent these differences are due to a difference in intended audience.

Compiling the corpus, I encountered a large variety of labeled exposition: “advice,” “aside,” “comments,” “digression,” “notation,” “note,” “observation,” and “warning.” Comparing these to the mini purposes, some similarities appear: warning (purpose 2), observation (purpose 3), notation (purpose 4), advice (purpose 6), and digression and aside (purpose 7).

Finally, I wish to share with the reader two of my wonderings: (a) What are the pedagogical ramifications of the mini purposes and the duality of the central purpose(s) of remarks? and (b) What possibilities would applying genre analysis to other mathematical genres offer?

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