Instructor Perceptions of Student Example-Use

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*Keywords:* Proof, Example-Use, Mathematical Knowledge for Teaching

Researchers have commonly found students from K-16 struggle to understand the purpose of a proof and how to construct proofs (e.g., Harel & Sowder, 2007; Healy & Hoyles, 2000; Stylianides, 2007). One component leading to this struggle is the difficulty in understanding the purpose of examples in a proof context (Aricha-Metzer & Zaslavsky, 2019; Ellis et al., 2019; Epp, 2003). Few studies in the field of example-use in proof investigate the mathematical knowledge for teaching (MKT) that would support instructors of these proof classes to effectively build upon and communicate strategies of using examples to their students (Zaslavsky & Knuth, 2019). In particular, mathematical knowledge for teaching proof (MKT-P) is a promising framework to investigate instructors' knowledge of the example-use subdomain of proof. I will use MKT-P to further describe the sub-domains of knowledge for content and students (KCS) and knowledge for content and teaching (KCT). This study applies the MKT-P framework to answer the following research question:

1. How do instructors of introductory proof classes perceive students' understanding and use of examples?

This study is part of the author's dissertation. Instructors of introductory proof courses across the southeastern United States were interviewed from September 2022 to November 2022. During the interviews, participants were asked to respond to six sample student responses to two questions adapted from Balacheff (1987) regarding example-use in proofs. Later in the interview, the participants sorted and ranked the six responses. Interviews were coded according to the MKT framework outlined by Ball and colleagues (2008) and further coded with a lens of MKT-P outlined by Stylianides and Ball (2008), Steele and Rogers (2012), and Lesseig (2016).

One of the main findings of this study includes the variety in ways instructors value the role of example use in their students. Some instructors held a strong stance that any example used to aid understanding by the student should be fully excluded in the written product. Other instructors instead said they wanted to see more reasoning in the written product by their students and to include more examples. For instance, Steven noted when reflecting on a student submission from Gina, "I mean, it's reasoning by example, which is always rough." This starkly contrasts with Mark who responded to the same instance from Gina with, "She is arguing from the picture, which I appreciate." The field of example use in proof would benefit from more explanation as to the roles of examples for practicing instructors of introduction to proof classes. Aricha-Metzer and Zaslavsky (2019) argued productive instances of example-use aided student understanding and proof production. There is a potential disconnect between the state of the literature of example-use in proof and the practice of practitioners of proof courses. Presenting this in a poster format will allow me to share quotes from my interviews and detailed information about my participants. I hope to gather insight from attendees for future directions with this work and how to disseminate information to practitioners hesitant to include examples in their proving.

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## Introduction

- Understanding the purpose of examples in a proof context can help students develop proving skills (e.g., Harel & Sowder, 2007; Healy & Hoyles, 2000).
- Although work has been done understanding the student side of examples, far less work has been done on understanding how to aid instructors to help their students use examples more efficiently (Zaslavsky & Knuth, 2019).

## Research Question

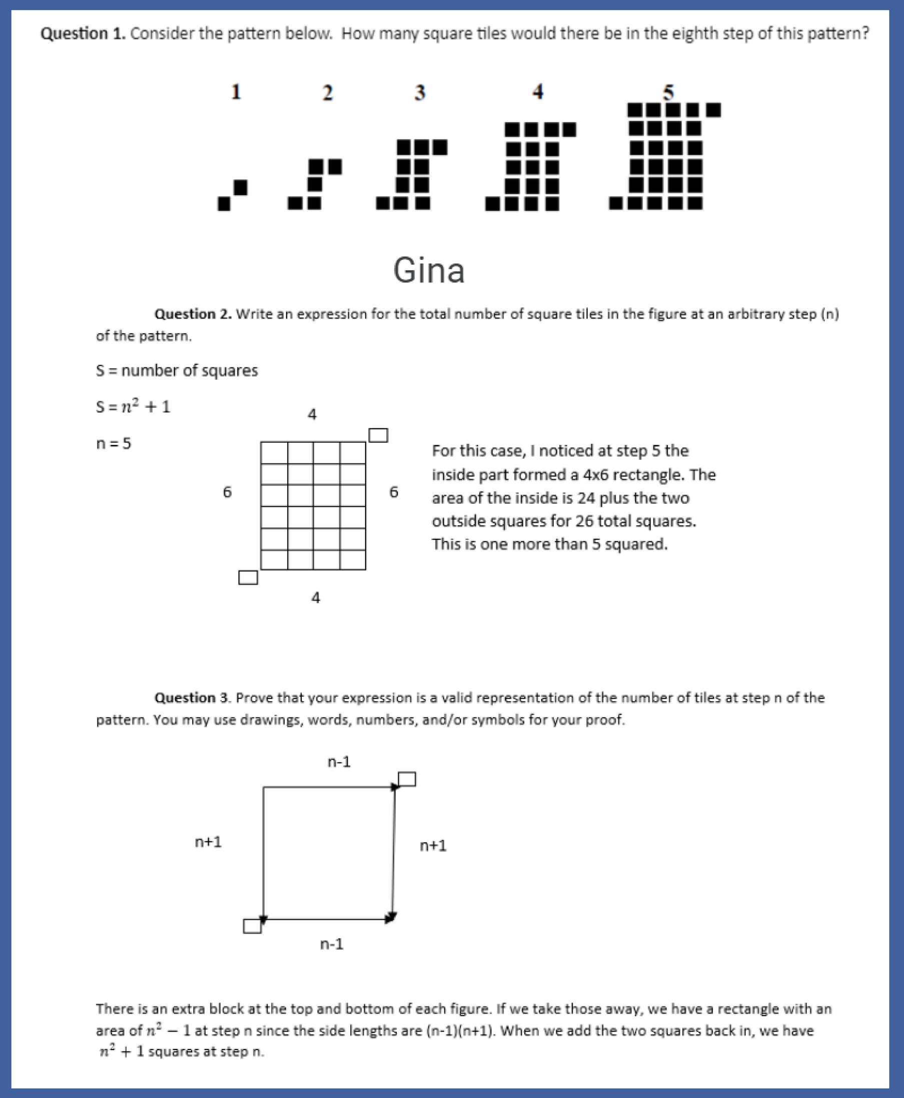
- How do instructors of Introduction to Proof courses perceive students' understanding and use of examples?

## Methods

- 1-hour semi-structured interviews with 11 professors of Introduction to proof classes across the SE US.
- Instructors were presented with sample student answers to basic proof tasks.
- Instructors responded to these tasks and gave feedback about their understanding of the student and how they would respond to the student in their class.
- Then, instructors were asked to organize the responses in whatever manner they deemed necessary.
- Finally, the 6 student work samples were ranked from "best progress toward a proof to least progress toward a proof."

| | R1/R2 | Masters | Baccalaureate |
|---|---|---|---|
| **Math** | 2 | 2 | 2 |
| **Math Ed** | 2 | 1 | 2 |

# Do it, but don't show me



**Question 1.** Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?

Gina

**Question 2.** Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

S = number of squares

S = $n^2 + 1$

n = 5

For this case, I noticed at step 5 the inside part formed a 4x6 rectangle. The area of the inside is 24 plus the two outside squares for 26 total squares. This is one more than 5 squared.

**Question 3.** Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.

There is an extra block at the top and bottom of each figure. If we take those away, we have a rectangle with an area of $n^2 - 1$ at step n since the side lengths are (n-1)(n+1). When we add the two squares back in, we have $n^2 + 1$ squares at step n.

Jacob

**Question 1.** Given the number of vertices in a polygon, find a method to calculate the number of diagonals of the polygon. Explain why your answer is correct.

I started form a polygon with 7 vertices. The number of diagonals to each vertex is n – 3.

For a polygon with 7 vertices, each point has two neighbors. So:

7 – (2 + 1)   The 7 is the number of vertices, the 2 is the number of neighbors, and the 1 is the point itself

In general, n – (2 neighbors + itself)

Then take this and multiply by the number of vertices. This counts each diagonal twice, so divide by 2 for the answer.

**Evan (Assistant Professor at Private Baccalaureate/ Math Ed/ 4 years experience/Lecture):**
"So she has a nice little example here, of when if n is equal to five, so for this case, you notice the step five, the inside part formed a four by six rectangle, the area inside is 24, plus the two outside squares for 26 Total squares. This is one more than five squared. Okay, that's great for that particular example. So it kind of shows me how she came up with that formula."

**Steven (Professor at Public R1/ Math/ 22 years experience/Lecture):**
"I mean, it's, it's reasoning by example, which is, which is always rough."
"I would say that the n minus one n plus one could have helped me early on."

**Barry (Associate Professor at Public M1, Math, 7 years experience/ Lecture):**
"So, I do like the, I guess, kind of started out with, you know, thinking of a fixed number, just to kind of get an idea of what to do with it."

**Lisa (Assistant Professor at Public R1, Math Ed, 4 years experience/ IBL):**
"Okay, so it's interesting, because like, I think Jacob is thinking about seven-sided figure, right? And he's taking it as like a generic example. But then the seven is actually never important. Like in his work. Like, he could have, you know, thought about seven and then gone through and replaced all the seven with an M, and now think would have been enough."

**Howard (Professor at Public M1, Math Ed, 17 years experience, lecture)**
"But as an educator in a first course, I want to see the example, right? And if you're leaving the example out, you're actually leaving out a step in the learning."

## Data Analysis

- Data was first coded by episode looking for instances of knowledge of content and teaching (KCT) or knowledge of content and students (KCS) using the mathematical knowledge for teaching framework (Ball et al. 2008).
- Then, a thematic analysis was conducted across episodes looking for themes within KCS and KCT.

## Results

- Almost even split of instructors who wanted to see the students written example in their final product (HW).
- No difference based on PhD, time at university, self-identified teaching style.
- Slight difference on if the instructor was at an R1/R2 institution versus anything else.
- Almost all participants asked students to show their thinking through examples when asked how to respond to students.

## Discussion

- As a field, researchers have argued that examples, when used productively, aid student understanding and production of proofs (Aricha-Metzer & Zaslavsky, 2019).
- There seems to be a disconnect between what researchers believe, and what we believe in practice.
- Should students be expected to write textbook proofs in introduction to proof classes for all assignments?

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