

How Do Postsecondary Linear Algebra Instructors Implementing Inquiry-Oriented Approaches Address Goals of Instruction in an Online Work Group?

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Postsecondary instructors interested in inquiry-oriented instruction of Linear Algebra participated in a sequence of eight one-hour online work group meetings with other experienced inquiry-oriented linear algebra facilitators and teachers. Recordings from three meetings were analyzed for how two participants referenced goals of instruction in preparation for teaching a new instructional unit on subspaces. We identified four goals of instruction of teaching subspaces. We discuss the intersections of several goals of instruction and possible implications for those who want to transition to inquiry oriented instructional approaches.

Keywords: goals of instruction, inquiry-oriented instruction, online work group

Linear Algebra is a postsecondary mathematics course that is typically required of all mathematics and many STEM (Science, Technology, Engineering, and Mathematics) majors. The topics learned in this course are foundational for many other math courses. Tucker (1993) and Carlson et al. (1993) point out the importance of linear algebra in addressing not only the theoretical and practical aspects but also its applicability in modern computation in industrial science, technology, computer science, engineering, and economics.

The number of students required to take Linear Algebra provides motivation to search for ways to improve instruction. Active learning in undergraduate STEM education is linked to increased student performance, and some literature suggests it may be linked to more equitable outcomes (Burke et al., 2020; Freeman et al., 2014; Haak et al., 2011; Kogan & Larsen, 2014). Inquiry-oriented Linear Algebra (IOLA) is one approach to active learning in undergraduate mathematics education. Inquiry-oriented instruction derives from Realistic Mathematics Education (RME) with the intent to support students in reinventing key mathematical concepts (Freudenthal, 1991; Kelley & Johnson, 2022). To support the students in these courses with an engaging learning experience, teachers need to be supported (Andrews-Larson et al., 2019) because changing instructional approaches is difficult and long-lasting change requires shifts in instructional practice (Cohen, 1996; Henderson et al., 2011). Therefore, there is “a need for professional development programs that foster the development of undergraduate mathematics instructors’ pedagogical reasoning” (Andrews-Larson et al., 2019, p. 129)

Thus, an online work group (OWG) created as part of a larger project to support college mathematics instructors teaching Linear Algebra, gave instructors an opportunity to collaborate with others interested in continuing their pursuit to enact reform-oriented instructional practices. In the OWG of this study, participants with inquiry-oriented instruction (IOI) experience work on implementation of a newly developed set of IOLA tasks and discuss their teaching practices with researchers and facilitators of the OWG.

In this analysis, we focus on the two undergraduate instructors who are not researchers or facilitators of the larger project because this enables us to examine how experienced IOI instructors discuss content and related pedagogical decisions they make before and during instruction. Their discussions can highlight how teachers’ knowledge shapes teachers’ practices (Borko & Putnam, 1996), in particular practices related to the preparation of a lesson. One of those practices is attending to goals of instruction (Wagner et al., 2007) which is the focus of our

study. The practical goal (Maxwell, 2013) of this research is to continue the conversation initiated by Wagner and colleagues, whose focus was on a novice instructor's experience with the inquiry-oriented instructional tools, and extend this to a discussion including experienced IOLA instructors. This research will aid in understanding the reasoning of experienced IOLA instructors, particularly regarding their instructional goals and the challenges of IOLA curriculum implementation. Our research question is "What is the nature of the goals of instruction articulated by experienced Inquiry-Oriented Linear Algebra (IOLA) instructors as they work to implement a new instructional IOLA unit in the context of the Online Work Group (OWG)?"

Theoretical Framework

Wagner et al. (2007) identified four categories related to instructional goals in the context of inquiry-oriented instruction at the undergraduate level: classroom orchestration goals, cognitive/learning goals, assessment goals, and content goals. The four goals emerged as a result of continuous work on studying teacher knowledge, such as pedagogical content knowledge (Shulman, 1986) done by Wagner et al. (2007) that focused on teacher knowledge shaping teacher practices through analyzing data from a novice inquiry-oriented instructor. The goals of instruction are signified through questions that instructors ask in their "transition" to teaching in reform-oriented ways. While instructors are in transition, each goal is evidenced by the, but not limited to, following questions:

1. Classroom orchestration goals: "How does mathematics emerge in the classroom? What is the instructor's role or students' role?" (p. 257)
2. Cognitive/Learning goals: "What supports learning? Is there a correspondence between what students are learning and what the teachers want them to learn?" (p. 259)
3. Assessment goals: "What is evidence of learning? What determines pace?" (p. 261)
4. Content goals: "Specifically what mathematics ought to be covered?" (p. 263)

Using the work from Wagner and colleagues as a priori scheme, this proposal will work to identify the ways in which these kinds of goals were conceptualized by experienced IOLA instructors in the online work group.

Study Context: Inquiry-Oriented Linear Algebra and Online Work Group

Inquiry-Oriented Linear Algebra (IOLA) is a design-based research project in linear algebra education as one approach to active learning (Freeman et al., 2014) and inquiry-based mathematics education (Laursen & Rasmussen, 2019). Laursen and Rasmussen (2019) describe inquiry-based mathematics education as "student engagement in meaningful mathematics, student collaboration for sensemaking, instructor inquiry into student thinking, and equitable instructional practice to include all in rigorous mathematical learning and mathematical identity-building" (p. 140). Specifically, the IOLA-X project focuses on developing student materials composed of challenging and coherent task sequences that facilitate an inquiry-oriented approach to the teaching and learning of linear algebra (Wawro et al., 2013). There are five main phases in the Design Research Spiral: *Design*, *Paired Teaching Experiment (PTE)*, *Classroom Teaching Experiment (CTE)*, *Online Work Group (OWG)*, and *Web* (Wawro et al., in press). The participants of our study come from the OWG in phase 4 of the research project.

The main purpose of the OWG for the IOLA research team is to learn from instructors how IOLA is implemented in various classrooms with various student populations and to gain insights to develop instructor notes and revise tasks (Wawro et al., in press). At the center of the

OWG for this study was the discussion of a unit of tasks about subspaces; the tasks were contextualized in a problem about students walking in one-way hallways past cameras monitoring their traffic (See Figure 1). To draw out the feedback from the instructors, the facilitators manage mathematical discussions about the tasks as well as facilitate discourse about the preparation and implementation of the tasks. Through examining discussion and input from the experienced undergraduate instructors participating in the OWG, questions and thoughts about the goals of instruction and challenges with implementation naturally arise.

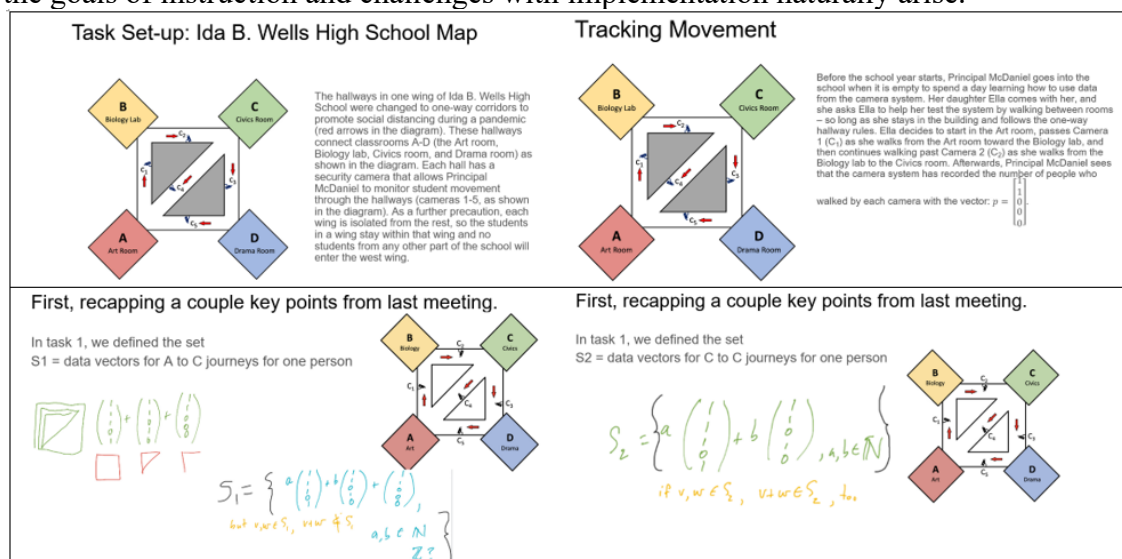


Figure 1. The inquiry-oriented linear algebra tasks in the subspaces unit

Methods

Our primary data source was the recorded videos of the OWG meetings (held and recorded via Zoom); group artifacts such as Google Slides and Jamboards served as secondary data sources. The OWG meetings were held in the Spring 2022 semester. During the first three meetings, participants worked through and discussed the mathematical progression of a new IOLA unit comprised of three tasks. The subspaces unit focused primarily on notions of closure of sets of vectors under vector addition and scalar multiplication, and on null and column spaces. Participants worked on the mathematical problems as a group, then discussed mathematical goals, approaches, and links to other ideas and topics. The remainder of the meetings took place throughout participants' implementation of the sequence, with each participant reporting on how the implementation went, what they liked, how their students reasoned about tasks, what they would change, and what they would do differently.

In the OWG meetings, there were six members. In this proposal, the members are coded as F1 and F2 (facilitators), R1 and R2 (IOLA researchers), and I1 and I2 ("pure" participants who are experienced inquiry-based instructors but not IOLA researchers). This team involved one graduate student (F2), one instructor (I2), three associate professors (F1, R1, and I1), and one full professor (R2). I2 taught linear algebra at a large public university in the Northeastern United States at the time of the OWG; and I1 taught the same course at a small private college in the Northwestern United States. The participation and contributions of I1 and I2 serve as the focus of our study.

Eight hours total of OWG meetings were conducted and recorded over several days. We analyzed transcripts of the first three videos. Narrowing these three videos allowed us to examine

the ways in which participants worked through the mathematical tasks and identify the instructional issues that participants anticipated in their implementation. The videos were transcribed by Otter, an online artificially intelligent transcription application. Both authors separately coded all three videos for the four goals of instruction for all the participants of the OWG using Nvivo software. While coding, we assigned four codes, which mean four goals, at the level of a single turn of talk. Then, we compared codes to reach agreements to build inter-rater reliability. We identified common themes within each code in terms of broad questions and agendas that the set of codes seemed to address. Our findings highlight the themes in which each goal of instruction emerged from the two “pure” participants in our data set. Figure 2 summarizes these findings. Each goal of instruction is, not necessarily, but preferably described in question form in order to follow the original form of goals of our theoretical framework.

Findings

Generally, in the OWG meetings, the pure participants discussed classroom orchestration goals involving how to manage discussions of contextualized tasks about “closure”. Also, cognitive/learning goals discussed in the OWG meetings are contextualized goals in subspaces and communication in engaging in IOLA tasks. Assessment goals are discussed in terms of pacing, timing, and grading of inquiry-based teaching. Lastly, content goals include curricular trajectories and mathematical content relevant to subspaces reorganized by instructors.

The participants talked about how to orchestrate the discussion of closure under scalar multiplication, considering irrational and complex scalars in the context of the IOLA task.

I1: We had a conversation we were talking about... here's a bunch of examples, what's the span of each of these sets? And, and I thought it was superfluous, but it turned out not to be. The last one I asked was span of the vector, $\langle e, \pi^2 \rangle$. And that really freaked them out. And I was glad that I included [it] because we had already talked about magic carpet ride. Any real scalars are fine. If you want complex scalars, that's a different class, but any real numbers, great. And they seemed okay with that. But no, it was clear, we're going to have that conversation again.

From this excerpt, the instructor is unpacking a class interaction they have had before this OWG meeting where they introduce complex components of vectors in another IOLA task (magic carpet ride). I1 determined that interaction was relevant to the future implementation of the subspaces task because the idea of complex scalars will spark “that conversation again”. That demonstrates the instructor’s reference to orchestration goals, but also content and cognitive/learning goals. The instructor is connecting mathematical concepts across the IOLA tasks and using the tasks as an opportunity for elaboration beyond the initial exploration.

Furthermore, the participants shared their thoughts on student reasoning that may happen in class discussions and the contextualized goals in subspaces and sets of scalars, coded as cognitive/learning goals.

I2: And, but I think S2 is closed. [...] Is it closed under scalars? Scalar multiplication of non-integer or negative scalars? I guess [it] depends on, do we require this to be walkable by number of persons? Right? Or does it just have to fit set description?

As Figure 1 illustrates, S2 is a set of camera vectors that pass through hallways from room C to room C. In this context, scalars mean the number of loops one person creates through one big square loop (through rooms A-B-C-D) and one small triangle loop (through rooms A-B-C). In the contextualized ways, S2 is closed under scalar multiplication but as I2 mentioned, the scalars depend on how much we address that real-life context. The non-integers or negative scalars cannot be defined in the contextualized goals, but this should be clarified to move on to the

purely abstract mathematical definition of closure. During this conversation, participants discussed how they manage their trajectories of teaching linear algebra, how to address contextualized mathematical goals with IOLA tasks, and how they orchestrate discussions.

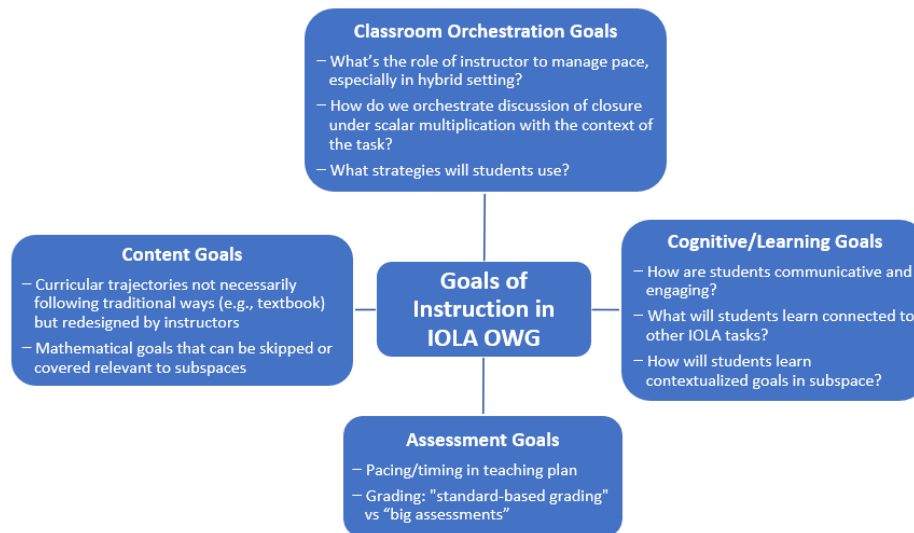


Figure 2. Goals of instruction in inquiry-oriented linear algebra online work group

Discussion

The two participants of our study asked questions and made comments in reference to all four goals of instruction introduced by Wagner et al. (2007). However, some dialogue from the participants were coded as referencing two or more goals. That made clear a separation of assessment goals from the other three goals. The results from the three goals - content, classroom orchestration, and cognitive/learning goals - show an overlap exposed by the instructors. The intersection of those goals may exist due to the contextualized nature of the IOLA tasks that the instructors work through in the OWG. In the task, scalars are constrained due to the context. Therefore, the scalars in the task would not be defined as scalars abstractly because other real numbers (like negative integers) could not be included. The instructors recognized the scope of the tasks and made plans to use the tasks as an opportunity to launch into more (abstract?) concepts chosen by the instructor and guided by student contributions.

On top of referencing goals of instruction as the novice IO instructor did in Wagner and colleagues' (2007) investigation, the experienced IOLA instructors of the OWG were able to expose tensions in instruction and mathematics that goes beyond attending to goals of instruction. The IOLA instructors addressed challenges of teasing through the tension between the aim of Realistic Mathematics Education (RME) and what mathematics is pursued in traditional linear algebra classrooms. This tension can be something to consider for new and transitioning inquiry oriented mathematical instructors.

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