Zone Theory Exploration of Teaching to Foster Mathematical Creativity in a Coordinated Calculus

Gulden Karakok	Houssein El Turkey	Miloš Savić
University of Northern Colorado	University of New Haven	University of Oklahoma
V. Rani Satyam	Emily Cilli-Turner	Gail Tang
Virginia Commonwealth University	University of San Diego	University of La Verne

We report on experiences of two Calculus 1 instructors who implemented teaching practices geared towards fostering students' mathematical creativity. While teaching a coordinated Calculus 1 course, these instructors participated in weekly professional development sessions on such teaching practices. Our preliminary analysis of participants' experiences describes how they navigated between the coordinated course structure and the added teaching practices discussed in the professional development. We utilize a zone theory lens to gain insight into participants' navigations in their professional environments, observing tensions between existing zone of proximal development and the zones of free movement and promoted action complex.

Keywords: Calculus, coordination, instructional practices, mathematical creativity.

The inclusion of mathematical creativity as a learning outcome in mathematics courses and programs has been discussed in numerous research studies and curriculum-standard documents (e.g., Askew, 2013; Levenson, 2013; Schumacher & Siegel, 2015; Silver, 1997; Sriraman, 2009). Despite these calls, the growth of explicit curricular work on creativity in mathematics courses, and professional development on teaching to include mathematical creativity at the tertiary level, has been slow. Recent work at the tertiary level has been limited to pre-service teachers (e.g., Bicer, 2021), mathematics majors in mathematics seminars (e.g., Mayes-Tang, 2020) or upper-division courses beyond Calculus (e.g., Omar et al., 2019). Thus, our overarching goal is to challenge dominant ways of teaching undergraduate mathematics by adding a focus on students' mathematical creativity and explicit teaching strategies to foster it.

In this preliminary report, we first describe our work with Calculus instructors. We discussed strategies to implement Calculus tasks geared towards fostering mathematical creativity in a semester-long online Professional Development (we refer to this professional development as C-PD here). Our project focused on Calculus 1 as this course plays a critical role in students' persistence in science, technology, engineering, and mathematics (STEM) programs (Rasmussen et al., 2019) and creativity is often overlooked in this course (Ryals & Keene, 2017). We also share results of preliminary analysis of data from the qualitative case study designed to explore the lived experiences of Calculus 1 instructors' implementation of creativity-based tasks (El Turkey et al., 2020) and teaching strategies to foster their students' mathematical creativity within a coordinated Calculus course. Our focus on coordinated Calculus 1 course was intentional - course coordination is considered a driver for instructional change (Rasmussen et al., 2019), and we believe it is timely to explore individual instructors' experiences within such structures. The research question of this study is: What are the experiences of Calculus 1 instructors in a coordinated Calculus course when utilizing teaching strategies to foster mathematical creativity?

Conceptual Framework

We situate our work on fostering students' mathematical creativity within the Developmental theoretical perspective (Kozbelt et al., 2010). The primary assertion of this theory is that creativity develops over time, and the main focus of investigation is a person's process and actions as opposed to the product created by them. Within this theoretical framing, we consider mathematical creativity as a process of offering new solutions or insights that are unexpected for the student with respect to their mathematics background or the problems they could have seen before (Savic et al., 2017). Fostering mathematical creativity with this conceptualization requires instructors to take intentional actions to engage students in the creative process of problem solving (Sriraman, 2005, Cilli-Turner et al., 2019).

In Tang et al. (2022), we examined teaching actions that students reported as contributing to their sense of creativity in their Calculus 1 course during which the instructors participated in C-PD. The student-reported teaching actions focused on designing and implementing creativity-based tasks using inquiry-oriented actions, in addition to holistic actions attending to students' affect. These teaching actions centered around students' lived experiences in these Calculus 1 courses, whereas in this paper, we shift our focus to the instructors' lived experiences implementing creativity-based tasks and teaching strategies in a coordinated setting.

We utilize a theoretical model proposed by Goos and colleagues (Goos, 2009, Goos et al., 2007; Goos & Bennison, 2019) to examine teacher learning and development, which applied and adapted concepts from Valsiner's (1997) zone theory. As this model centers its constructs in socio-cultural theory of teaching, it provides opportunities to examine our participants' experiences in their context (i.e., one institution's culture) and through their interactions with other C-PD participants while trying to utilize the C-PD materials.

Goos et al. (2007) describe three zones, each of which focuses on different aspects of teacher learning and development: the *Zone of Proximal Development*, the *Zone of Free Movement*, and the *Zone of Promoted Action*. Adapted from Vygotsky's (1978) work, the *Zone of Proximal Development* (ZPD) refers to the possibilities for developing new teacher knowledge and beliefs on content and pedagogy. The *Zone of Free Movement* (ZFM) focuses on "constraints and affordance within the professional context" (Goos et al., 2007, p. 26). It explores the environmental and contextual factors that allow or hinder certain actions. For example, teachers' views of curriculum design and assessment methods, and their connections to institutional culture and so forth are part of the ZFM. The third zone, the *Zone of Promoted Action* (ZPA), "represents the activities, objects, or areas of the environment in respect of which an individual's actions are promoted" (Goos & Bennison, 2019, p. 408). For example, in a department that focuses on active-learning strategies (i.e., institutional norm), an instructor may utilize teaching practices learned in a professional development on inquiry-based learning, which represents ZPA.

Overall, Goos et al. (2007) observed that these zones complement each other in the effort to describe teacher's learning and development. Goos et al. state, "For teacher learning to occur, professional development strategies [ZPA] must engage with teachers' knowledge and beliefs [ZPD] and promote teaching approaches that the individual believes to be feasible with their professional context [ZFM]" (p. 26). Also, the ZFM and ZPA are stated to be "dynamic and inter-related and form a ZFM/ZPA complex" (Goos & Bennison, 2019, p. 408) which plays an important role on learning and development of teaching strategies. For our study, this zone theory approach provides a structure to better understand the experiences of participants' teaching to foster mathematical creativity in a coordinated course environment.

Methods

We report on two instructor-participants at a university in the Northeastern United States. This works falls within a larger research project investigating students' mathematical creativity and the ways in which it can be fostered through teaching of Calculus I. Even though the larger research project consisted of 3 cohorts with 14 instructors total, we focused on only two participants, as they were in the same cohort and at the same institution in which the course was coordinated.

The two instructors chose their pseudonyms, and we also report their self-identified pronouns, genders and race: Farah Nurhalizah (They, Male, Asian) and On the 1s and 2s (She, Female, Black). These instructors formed the case for this study. At the time of the C-PD, Farah was a full-time instructor and On the 1s and 2s was an assistant professor, and both were in their first year teaching at this particular institution. Both participants had taught Calculus 1 more than 5 times previously with 6-10 years of teaching experiences.

Both instructor participants attended weekly online professional development with members of the research team to discuss the different components of the project: creativity-based task design and implementation; implementation of a reflection tool for formative assessment (Karakok et al., 2020); pedagogical actions; and attending to students' affect. All participants were asked to implement two common creativity-based tasks in their courses and design at least four additional tasks to implement. Participants could decide how to implement these tasks according to their setting. During the C-PD, they discussed implementation strategies with the research team and other instructor participants. We collected various data: Calculus teaching artifacts from prior semesters, video recordings of all C-PD sessions, participants' entrance tickets to weekly C-PD, video recordings of teaching of creativity-based tasks, and end-of-semester interview with participants. In this report, we share some of our initial analysis of video-recorded interview data from each participant. We started our data analysis with the interview data to gain insights into participants' experiences to identify critical moments or experiences before engaging with other data sources. For this initial coding (Saldaña, 2013) as the first cycle in our analysis, the first two authors read through the interview transcripts to get analytic leads to explore the lived experiences of teaching Calculus while attending to the C-PD. From this coding, we started our second cycle with a deductive approach to structure the observed experiences through a zone theory with ZPD, ZFM, and ZPA constructs. By the time of the conference, we aim to complete interview data analysis and analyze participants' entrance tickets and conversations during C-PD sessions with ZPD, ZFM, and ZPA constructs to describe their experiences.

Preliminary Results

We first note that both participants implemented only two creativity-based tasks, due to interruptions in teaching modality caused by the COVID-19 pandemic in the middle of the semester.

Farah Nurhalizah (They, Male, Asian)

Farah described their teaching Calculus 1 experience through its "highly coordinated" structure. They referred to this structure as a new experience for them in which "you have the same exams, the same homework, and the same worksheets even." Coordination was the most significant feature of their environment (ZFM) and Farah seemed to believe that it hindered their access to their already-established ways of teaching Calculus. In addition, Farah thought "[i]t took a while for me to get used to it… And I think students could feel that I wasn't like totally buying into the structure."

As we examined Farah's reported experience, we noticed that their negotiation between the recommended focus on conceptual understanding from the coordinator (ZFM) and the newly added focus on creativity from C-PD (ZPA) created a tension with their existing beliefs on teaching Calculus (ZPD). When asked how they could have adjusted pre-designed worksheets to incorporate fostering creativity tasks, Farah stated they would have "underemphasized the conceptual understanding as much…And…sort of do more creativity instead of conceptual." For Farah, conceptual understanding questions on worksheets required "theoretical foundation," definitions, and some proofs, which the course did not design to deliver them. For their section of the course, Farah "snuck in…the epsilon-delta definition," that they thought they were not supposed to do. The inclusion of definitions and some proofs for Farah was about being "intellectually honest," and relates to their existing beliefs of mathematics and teaching mathematics (ZPD).

As Farah tried to reconcile their existing teaching Calculus practices with these pre-designed worksheets and the coordination structure, they identified some aspects of the worksheets that could potentially foster mathematical creativity (ZPA). Incorporating their experience with C-PD on features of creativity-based tasks (El Turkey et al., 2020), they mentioned "some of the worksheet problems…had like some open-endedness" to them. Farah further noted that not all students got to these questions nor was there an emphasis to discuss or do these questions in class (ZFM). However, Farah thought that there was a possibility to foster creativity if they had a chance to discuss these questions. These observations highlight the dynamic and inter-related relationship of ZFM and ZPA in Farah's experience (ZFM/ZPA complex) (Goos, 2009). Furthermore, Farah's proposed method of using such worksheet problems through more flexible class discussion speaks to their developing ZPD on teaching Calculus 1 to foster mathematical creativity and navigation through the tension they experienced between their existing ZPD and the professional environment (ZFM).

On the 1s and 2s (She, Female, Black)

On the 1s and 2s's experience teaching Calculus 1 in a coordinated Calculus to foster creativity highlights explicit aspects of the ZFM/ZPA complex. Speaking about her experience teaching coordinated calculus (ZFM) and participating in C-PD (ZPA), she thought her teaching did not necessarily change in the current course, but this C-PD experience changed "how I think about calculus and encouraged me to include more…creativity tasks" in her future courses.

For On the 1s and 2s, one of the important differences between the coordinated worksheets (ZFM) and the creativity-based tasks (ZPA) was the goals of these tasks. She identified one of the goals of the creativity-based tasks as "what different ways can you think about this problem" or "what different ways can we as a group think about this problem" whereas the worksheet problems tended to have a goal of "can you get this answer?" With this noticing, and her interactions with Farah and other participants of the C-PD (ZPA), she was able to change some elements of the professional environment (ZFM). For example, they (all Calculus instructors, including Farah and the course coordinator) used one of the creativity-based tasks in their weekly check-in assessment. This task asked students to create a function where the limit exists at -1, but not continuous. Even though all instructors did this task in the same format on an assessment, On the 1s and 2s followed up with students, after the assessment, with a whole-class discussion on their solutions. As she believed her class "is a life that grows and changes and you have to adjust," explicit discussion in C-PD on how to design and implement creativity-based tasks provided her opportunities to envision how to navigate in her environment (ZFM).

Even though she and Farah were teaching their own sections of the same coordinated Calculus 1 course, her experience with the worksheets or the coordinated structure (environmental constraints) mostly interacted with the ZPA to form a ZFM/ZPA complex that informed her evolving ZPD. These ZFM/ZPA negotiations played a role in On the 1s and 2s's teaching experience for mathematical creativity.

Conclusion and Questions

Coordinated courses have opportunities to create coherent experience for students taking Calculus 1 and subsequent courses at the same institution. For instructors, coordinated structure could provide several opportunities: collaboration among faculty with shared objectives, professional development, and building of common tools and resources. Furthermore, Williams et al. (2022) claim that course coordination can be "a vehicle for shifting departmental culture" (p. 126). However, some instructors may feel loss of autonomy and flexibility in their teaching. This loss of autonomy relates to 'coordinated independence' construct, which is "intended to embrace how in-step elements of a calculus program work together with elements that allow for individual autonomy" (Rasmussen & Ellis, 2015 p. 114). As many institutions use or aim to use coordination for Calculus courses, it is important to explore the coordinated independence construct. This construct could allow us to understand the barriers and opportunities to create a coherent course design for students.

In this study, we examined instructor's experience of implementing teaching practices from a professional development in a coordinated structure. Our preliminary analysis results relate to how two instructors experienced "coordinated independence" in slightly different ways through a ZFM/ZPA complex. For Farah, this complex created a tension with their existing ZPD. Farah's existing ZPD was more aligned with the ideas promoted in C-PD (ZPA) than the coordinated calculus structure (ZFM). On the other hand, On the 1s and 2s identified certain aspects of the C-PD (ZPA) to make changes in her professional environment (ZFM) and had a vision to do more in her teaching practices (developing ZPD). These experiences, with their differing tensions and negotiations, may explain formally why it is difficult for teachers to incorporate creativity into their classrooms, particularly in a coordinated setting. We must acknowledge the unique nature of the setting in our analysis – COVID-19 forced many to use pedagogical tools that were not in their ZPD at all. In our future work, we will continue to explore these areas as they relate to the coordinated settings and instructors' developing ZPD.

We are interested to learn from audience:

1- What strategies do instructors use when they experience tensions between their zones of development?
2- What strategies do professional development facilitators need to adopt when instructor participants experience such tensions?

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References

Askew, M. (2013). Issues in teaching for and assessment of creativity in mathematics and science. In D. Corrigan, R. F. Gunstone, & A. Jones (Eds.), Valuing assessment in science education: Pedagogy, curriculum, policy (pp. 169-182). Dordrecht, the Netherlands: Springer.

Bicer, A. (2021). Multiple representations and mathematical creativity. *Thinking Skills and Creativity, 42*, 100960.

Cilli-Turner, E., Savic, M., El Turkey, H., & Karakok, G. (2019). An Initial Investigation into Teacher Actions that Specifically Foster Mathematical Creativity. In M. Nolte, editor, *Proceedings of the 11th International Conference on Mathematical Creativity and Giftedness* (pp. 130-135). Hamburg, Germany.

El Turkey, H., Karakok, G., Tang, G., Regier, P., Savic, M., & Cilli-Turner, E. (2020). Tasks to Foster Mathematical Creativity in Calculus I. In S. Karunakaran, Z. Reed, & A. Higgins (Eds.), *Proceedings of the 23rd Annual Conference on Research in Undergraduate Mathematics Education* (pp. 579-588). Boston, MA, 2020.

Goos, M. (2009). A sociocultural framework for understanding technology integration in secondary school mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 113-120). Thessaloniki, Greece: PME.

Goos, M., & Bennison, A. (2019). A zone theory approach to analysing identity formation in mathematics education. *ZDM*, *51*(3), 405-418.

Goos, M., Dole, S., & Makar, K. (2007). Designing professional development to support teachers' learning in complex environments. *Mathematics Teacher Education and Development*, *8*, 23-47.

Karakok, G., El Turkey, H., Savic, M., Tang, G., Cilli-Turner, E. & Regier, P. (2020). Creativity-in-progress rubric on problem solving at the post-secondary level. In A.I. Sacristán, J.C. Cortés-Zavala, & P.M. Ruiz-Arias (Eds.), *Proceedings of the forty-second annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* (pp. 986–990). Mexico.

Kozbelt, A., Beghetto, R. A., & Runco, M. A. (2010). Theories of creativity. In J. C. Kaufman & R. J. Sternberg (Eds.), *The Cambridge handbook of creativity* (pp. 20–47). Cambridge University Press.

Levenson, E. (2013). Tasks that may occasion mathematical creativity: Teacher's choices. Journal of Mathematics Teacher Education, 16, 269-291

Mayes-Tang, S. (2020). Designing Opportunities for Mathematical Creativity: Three Ways to Modify an Existing Course. PRIMUS, 30(3), 261-273. DOI: 10.1080/10511970.2019.1566184

Omar, M., Karakok, G., Savić, M., Turkey, H. E., & Tang, G. (2019). I felt like a mathematician: Problems and assessment to promote creative effort. *Primus*, *29*(1), 82-102.

Rasmussen, C., & Ellis, J. (2015). Calculus coordination at PhD-granting universities: More than just using the same syllabus, textbook, and final exam. In D. Bressoud, V. Mesa, & C. Rasmussen (Eds.). *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 107–115). Washington, DC: The Mathematical Association of America.

Rasmussen, C., Apkarian, N., Hagman, J. E., Johnson, E., Larsen, S., & Bressoud, D. (2019). Characteristics of Precalculus Through Calculus 2 Programs: Insights From a National Census Survey. Journal for Research in Mathematics Education, 50(1), 98-112.

Ryals, M., & Keene, K. (2017). A success factor model for calculus: The relative impact of and connections between factors affecting student success in college calculus. In *Proceedings of the 20th annual conference on Research in Undergraduate Mathematics Education* (pp. 871-878). San Diego, CA.

Saldaña, J. (2013). *The coding manual for qualitative researchers*. Los Angeles: SAGE Publications.

Savic, M., Karakok, G., Tang, G., El Turkey, H., & Naccarato, E. (2017). Formative assessment of creativity in undergraduate mathematics: Using a creativity-in-progress rubric (CPR) on proving. In R. Leikin & B. Sririman (Eds.), *Creativity and Giftedness* (pp. 23-46). Springer.

Schumacher, C. S., & Siegel, M. J. (2015). *2015 CUPM curriculum guide to majors in the mathematical sciences.* Washington, DC: Mathematical Association of America.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. ZDM Mathematical Education, 3, 75–80.

Sriraman, B. (2005). Are Giftedness and Creativity Synonyms in Mathematics? *Journal of Secondary Gifted Education, 17*(1), 20–36. https://doi.org/10.4219/jsge-2005-389

Sriraman, B. (2009). The characteristics of mathematical creativity. ZDM Mathematics Education, 41, 13-27

Tang, G., Savić, M., Satyam, V. R., El Turkey, H., & Karakok, G. (2022). "The reason why I didn't like [math] before is because I never felt creative": Affective outcomes from teaching actions to foster mathematical creativity in Calculus 1. In S. Karunakaran, & A. Higgins (Eds.), *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education (*pp. 630-638). Boston, MA.

Williams, M., Apkarian, N., Uhing, K. et al. (2022). In the driver's seat: Course coordinators as change agents for active learning in university Precalculus to Calculus 2. *International Journal of Research in Undergraduate Mathematics Education, 8*, 121–148. https://doi.org/10.1007/s40753-021-00153-w

Valsiner, J. (1997). Culture and the development of children's action*: A theory of human development*. (2nd ed.) New York: John Wiley & Sons.

Vygotsky, L. S. (1978). Socio-cultural theory. *Mind in society*, *6*(3), 23-43.