Designing Activities to Support Preservice Teachers' in Promoting Generalizations

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The practice of generalizing, identifying a relationship to describe multiple examples or instances of a phenomenon, engages students in algebraic thinking and allows them to identify, investigate, and represent relationships (Blanton et al., 2011; Kieran et al., 2016). Generalizing is fundamental to learning mathematics (Carraher & Schliemann, 2002; Kaput, 1999), yet students often have trouble developing (Lannin, 2005) and justifying generalizations (Breiteig & Grevholm, 2006). To first build their understanding of generalizing as a mathematical practice, we introduced a cohort of preservice secondary teachers (PSTs) to two frameworks for generalizing and asked them to use these frameworks to analyze others' teaching. Then we designed a teaching activity embedded in their methods course to help them promote generalizing among their grades 7-12 students. Ellis's (2011) generalizing promoting actions (GPAs) are teacher actions that foster the development or refinement of a generalization. Strachota's (2020) priming actions (PAs) usually precede GPAs and set the stage for more explicit attention to generalizing and prepare students to build on an idea.

We provided an outline for a lesson focused on generalizing and asked three PSTs to customize the lesson for a high school audience and to plan questions to elicit student generalizations. All PSTs used a pattern task, a visual representation of a series of objects where the number of objects changes based on a rule, as the focus of their lesson. The goal of a pattern task is to use the structure of the presented images (e.g., objects organized in arrays) to develop multiple ways of representing the rule. We chose pattern task as the focus for the potential of the tasks to promote generalizing because we believed it served as an accessible entry point for our novice PSTs.

We video-recorded each PST's teaching and asked them to watch their video and analyze the effectiveness of their practice. We coded video data for GPAs (Ellis, 2011), PAs (Strachota, 2020), and student generalizations to identify teacher actions that promoted and hindered students in generalizing. Through our analysis, we sought to understand the patterns of generalizing actions that supported students in articulating the generalizations and determine how teacher educators can support PSTs in promoting student-created generalizations.

Patterns of interactions that promoted generalizing included using multiple GPAs in a short span of time and following student-created generalizations with a new GPA to prompt them to expand or deepen their contribution. Additionally, using PAs and GPAs together helped support students in generalizing more so than using GPAs alone. The PA of naming a critical tool proved especially useful when students invented and described their own strategies. Patterns of interaction that hindered students in generalizing included using GPAs when the actions were centered on a teacher strategy instead of a student strategy or when the teacher was not able to use repeated GPAs in a series. There was not a one-to-one link between GPAs or PAs and students' generalizations, which demonstrates the complexity of generalizing. Enacting the pattern task revealed much about the shortcomings and strengths of teachers' understandings of generalizations; future research will explore how to better support preservice teachers in promoting generalizing.

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BACKGROUND

- *Generalizing*: identifying a relationship to describe multiple examples or instances of a phenomenon
- Engages students in algebraic thinking and allows them to identify, investigate, and represent relationships (Kieran et al., 2016)
- Generalizing is fundamental to mathematics (Carraher & Schliemann, 2002)
- Students have trouble developing (Lannin, 2005) and justifying generalizations (Breiteig & Grevholm, 2006)

METHODS

Participants: 3 teaching fellows in one-year master's program

Learning about Generalizing: Fellows completed pattern tasks as learners and analyzed teaching videos for promoting generalizing

Data Collection: Implement pattern task activity in field placement

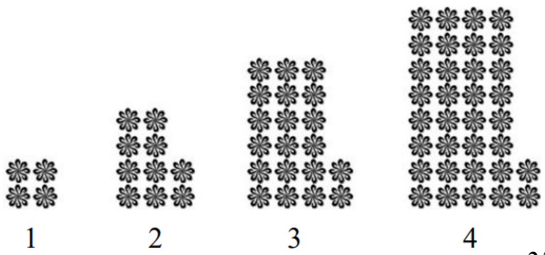
- Choose pattern task & solve in many ways
- Plan for questioning to elicit generalizations using five practices approach (Stein and Smith, 2011)

Data Analysis: Video data analyzed in 15-second segments for generalizing promoting actions and student contributions

RESEARCH QUESTIONS

1. What patterns of generalizing actions best supported students in articulating generalizations?
2. How can teacher educators support preservice teachers in promoting student-created generalizations?

SAMPLE PATTERN TASK



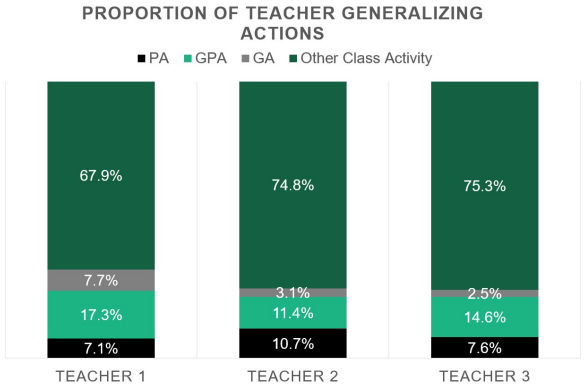
Nguyen, (2020)

FRAMEWORK

Priming Actions (PAs)[1]	
Naming a phenomenon, clarifying critical terms and tools	"Offering a common way to reference a phenomenon or emphasizing the meaning of a critical term or tool."
Constructing or encouraging constructing searchable and relatable situations	"Creating or identifying situations or objects that can be used for searching or relating. Situations that can be used for searching or relating involve particular instances or objects that students can identify as similar."
Constructing extendable situations	"Identifying situations or objects that can be used for extending. Extending involves applying a phenomenon to a larger range of cases than from which it originated."
Generalizing Promoting Actions (GPAs)[2]	
Encouraging Relating and Searching	"Prompting the formation of an association between two or more entities; prompting the search for a pattern or stable relationship."
Encouraging Extending	"Prompting the expansion beyond the case at hand."
Encouraging Reflection	"Prompting the creation of a verbal or algebraic description of a pattern, rule, or phenomenon."
Encouraging Justification	"Encouraging a student to reflect more deeply on a generalization or an idea by requesting an explanation or a justification. Includes asking students to clarify a generalization, describe its origins, or explain why it makes sense."

[1]Categories and descriptions from Strachota (2020, p. 7).
[2]Categories and descriptions from Ellis (2011, p. 316). Adapted encouraging relating and searching, following Strachota (2020), by combining these into one category.

RESULTS & DISCUSSION



Conditions that Supported Students' Generalizing:

- Teacher used multiple GPAs in short amount of time
- Teacher followed student generalizations with GPA
- Teacher used PA and GPA together to elicit and make public student thinking and connect it to multiple representations

Conditions that Hindered Students' Generalizing:

- GPA centered around a teacher strategy, not a student strategy
- Pattern task not at appropriate level for students

Other Findings

- Generalizing is complex
- There is not a one-to-one link between GPA, PA, and student-created generalizations

FUTURE RESEARCH

- Examine influence of teacher questioning on GPA and student-created generalizations
- Examine quality of PSTs' reflections on pattern tasks
- Track use of generalizing promoting actions across student teaching year

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