Connecting Instructor Mathematical Knowledge for Teaching, Intercultural Competence, and Undergraduate Learning

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We report on work to explore instructor intercultural competence and its relationship with the effectiveness of mathematics teaching in post-secondary settings. Results are from quantitative and qualitative inquiry into the mathematical knowledge for teaching and intercultural orientations of 15 instructors and 476 of their undergraduate students. Instructors participated to varying degrees in a professional short-course for faculty learning to teach mathematics for future grade school teachers. The greatest learning gains among undergraduates were in the classes of those instructor-participants who completed short-course activities about interculturally responsive mathematical knowledge for teaching future teachers.

Keywords: teacher preparation, intercultural competence, professional development, equity

Lee Shulman (1998), in reviewing the education of professionals (e.g., doctors, lawyers, teachers, clergy), noted six defining characteristics of a profession:

1. the obligations of *service* to others, as in a "calling";
2. *understanding* of a scholarly or theoretical kind [e.g., mathematics, pedagogy];
3. a domain of skilled performance or *practice*;
4. the exercise of *judgment* under conditions of unavoidable uncertainty [e.g., instructional decision-making in classroom contexts];
5. the need for *learning from experience* as theory and practice interact; and
6. a professional *community* to monitor quality and aggregate knowledge. (p. 516)

These characteristics play out in a variety of ways. Item #1 includes the fact that:

> The professional educator's challenge is to help future professionals develop and shape a robust moral vision that will guide their practice and provide a prism of justice, responsibility, and virtue through which to reflect on their actions. Medicine's Hippocratic Oath, therefore, is a central manifestation of the moral foundations of a profession. (Shulman, 1998, p. 516)

Now, consider the current state of the art in the professional education of college mathematics instructors. What is clearly and explicitly offered to instructors for how to make morally and contextually complex judgment calls? Not much, yet (Braley & Bookman, 2022). Certainly characteristic #2 is accepted as essential (Herman, 2011) while #3, and how to measure it, has had increasing attention in the recent past (e.g., Castro Superfine & Li, 2014). If characteristic #4 is not well addressed early on, then the alternative is #5: Learn it from experience. Learning from experience requires a multifaceted mirror of reflective practice, one that instructors can use to see encounters with others as opportunities for professional learning about mathematics teaching. Both #5 and #6 are emerging as areas for research and development (e.g., among graduate students and novice faculty in their early teaching experiences).

Theoretical Perspectives

Professional growth is shaped by professional and personal cultures: dynamic social systems of values, beliefs, behaviors, and norms for a group, organization, or other collectivity in which the shared values, beliefs, behaviors, and norms are learned, internalized, and changeable by

members of the group (McGrath et al., 1995). Many reports indicate that culture is a significant factor in the inequities of persistence and disparities in achievement in mathematics (e.g., Greer et al., 2009; Laursen & Rasmussen, 2019). University programs are responding (e.g., see Equity Alliance, equityalliance.stanford.edu). From anti-racism training to culturally responsive pedagogies, post-secondary instructor professional development efforts have emerged largely from the same arena as K-12 teacher education: psychology. Yet there is another area of the academy from which professional educators can draw insight: anthropology (Ladson-Billings, 2001). Psychology tackles professional education through an approach based in a teacher's *disposition,* attempting to change it through focused reflection on *behavior*; social anthropology offers the idea of movement along a developmental continuum of *orientation to differences* through focus on *relationships* and *communication*. In particular, healthcare and international relations professions offer theories of intercultural development and conflict resolution that are now being applied in education (e.g., Bennett, 2004; Hammer, 2009; Hauk et al., 2021; Hauk & Speer, 2022; Leininger, 2002; Wolfel, 2008). The core of intercultural orientation development is building skill at establishing and maintaining relationships in culturally complex situations.

Though some instructors have largely monocultural classrooms – in the sense that most students share experience of a particular set of cultural-general norms and practices – the nature of diversity in the U.S. is shifting from such segregated monocultural circumstances to cultural heterogeneity (Aud et al., 2010). We argue that intercultural competence is an essential component in the shift from teacher-centered to student-centered approaches to instruction. Knowing one's orientation, or the normative orientation of a group, can inform instruction. In particular, the authors are researchers in a project that has created professional learning for faculty new to teaching future K-8 teachers. Here we report on a small exploration into the results of infusing intercultural competence development into professional development.

Research Question: What evidence is there that intercultural orientation is part of the mathematical knowledge for teaching needed by post-secondary instructors to be effective in teaching future teachers?

Mathematical Knowledge for Teaching (MKT) and teaching future K-8 teachers

There are particular understandings and skills associated with effectively teaching children, a sociological synergy of mathematics and mathematics education called *mathematical knowledge for teaching* that encompasses types of *subject matter knowledge* and *pedagogical content knowledge* (MKT; Ball et al., 2008). Subareas of these include knowledge of the intended curriculum (Herbel-Eisenmann, 2007), "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill et al., 2008, p. 375); and knowledge about teaching actions and decision-making (e.g., productive ways to respond in-the-moment to students to support learning). All of the components of MKT are situated in a seventh aspect, *knowledge of discourses,* about communication in/with/through mathematics among people in and outside of the classroom (Hauk et al., 2014; Scheiner, 2019).

A related idea at the post-secondary level is *mathematical knowledge for teaching future teachers* (MKT-FT; Castro Superfine et al., 2013, 2014). A deep MKT-FT is vital in the student-centered and inquiry-oriented approaches to teaching shown to improve learning, increase persistence, and reduce inequities (Bressoud et al., 2015; Laursen et al., 2014). Generally, instructors build MKT-FT by learning from experience and professional community: grading, examining their own learning, observing and interacting with students or colleagues, reflecting on and discussing their own practice *and* the practices known to be effective in teaching (Kung, 2010; Speer & Wagner, 2009). Those with advanced degrees in mathematics who teach future

teachers are faced with a need to connect mathematical learning by their future-teacher-students to the special mathematical knowledge needed for working with children as well (Castro Superfine & Li, 2014).

Model of Intercultural Competence Development

Our effort in applying the ideas of intercultural competence development to mathematics teaching and learning is based in the developmental model of intercultural sensitivity (Bennett & Bennett, 2004). The model ranges from a monocultural, ethno-centric view to an ethno-relative, intercultural view. Figure 1 is a schematic of the five benchmark orientations and developmental processes that support intermediate transitions.

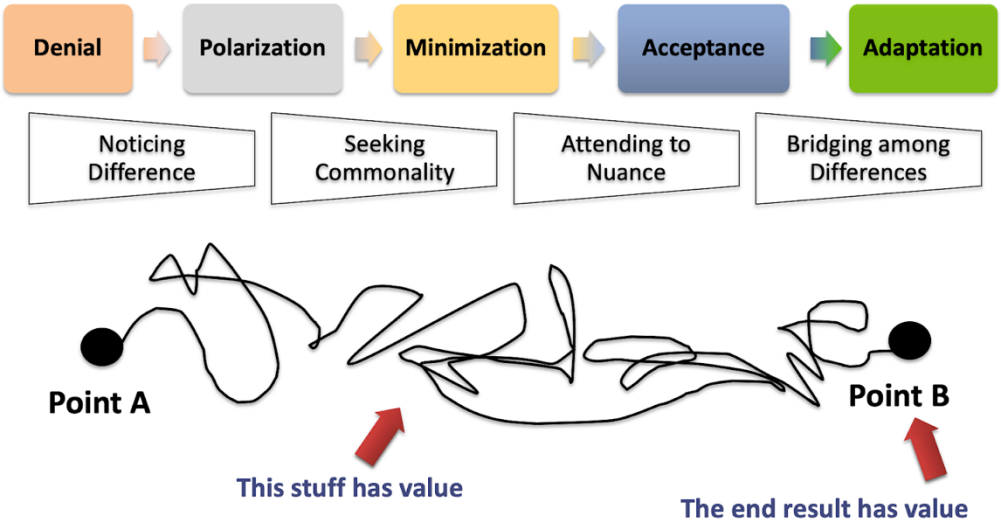

Figure 1. Milestones and processes for the intercultural development continuum.

The starting point of the continuum of orientations towards cultural difference, *denial*, is a lens for perceiving the world based on denying difference and assuming "Everybody is like me." A person with this orientation will avoid or express disinterest in cultural difference. The transition to the next benchmark, *polarization*, comes with the recognition of self as distinct from "other" through a noticing of difference. A polarization orientation can be driven by the assimilative assumption "Everybody *should* be like me/my group" where cultural differences is viewed in terms of "us" and "them." Transitioning *minimization* involves noticing commonalities beneath the surface differences, in particular a growing awareness of norms. This middle orientation is a minimization of difference, a lens for experience based on the idea, "Despite some differences, we really are all the same, deep down" and focuses on similarity and universals (e.g., biological similarities – we all have to eat and sleep; and presumed universal values). A minimization orientation will, however, ignore recognition and appreciation of subtle difference. Through increased attention to nuance in the differences that exist within noticed commonalities, one begins the transition to an *acceptance* orientation. Here, "accept" is used in its socio-cultural sense – the action or process of consenting to receive (rather than its psychological one – believe or come to recognize as valid or correct). Someone with an acceptance orientation has both some mindfulness of self as having a culture and awareness of moving among multiple cultures (plural). However, the importance of relative context, how to respond and what to respond in-the-moment of interaction with others is underdeveloped. The transition to *adaptation* involves developing culture-general frameworks for perception and

behavioral shifts that are responsive to a full spectrum of detail in an intercultural interaction along with a concomitant awareness that one's own perceptions are limited and the whole picture is bigger than what is perceived. Adaptation is an orientation wherein one bridges cultural perspectives, without losing or violating one's authentic self, and adjusts communication and behavior in ways viewed as appropriate by each cultural group involved.

In the particular context of post-secondary mathematics teaching and learning, the continuum from ethno-centric to ethno-relative orientation may begin in a denial of difference (e.g., mathematics is acultural). Further development will go through a phase characterized by a tendency to polarize (e.g., there is one best or right way to solve every problem). From there, development can progress to a minimizing search for universals (e.g., there are "objective" or "mastery based" ways that are universally applicable to assess *all* students, and grades become the essential element of interaction for instructor and student, disconnected from students' mathematical funds of knowledge). With time and intentional development, one can learn more about mathematical ideas, contexts, and human interactions and reflect on teaching with greater attention to relational details (e.g., learning about implicit bias and suddenly noticing it in every word problem in the text), but how to use this knowledge to improve opportunities to learn, classroom climate, and interactions with others remains elusive. At its most developed, adaptation in mathematics teaching and learning involves networks of people and ideas; interactions can be anticipated (enough) that teaching responds to the needs of those involved (Hauk et al., 2014). Development along the continuum is not direct or linear. Folding back to previous orientations (particularly in times of stress) is common. Also, the time spent in learning about self and others during transitions and folding back hold value in developing more variety in the ways in which one perceives, makes sense of, and enacts culturally-informed relationships.

Methods

The *Professional Resources & Inquiry in Mathematics Education* (PRIMED) *for K-8 Teacher Education* project field-tested a 15-hour, 10-week online short-course for college instructors new to teaching future K-8 teachers. This online professional development (PD) experience covered topics such as mathematical knowledge for teaching, inquiry- and task-based learning, and tools for equitable instruction. The PRIMED experience was hybrid: Modules 2 and 5 included both asynchronous and live online activity-based work; the other three modules were sets of asynchronous web-based activities completed by instructors individually and in teams (see Figure 2). Participant teams rarely met in real-time and tended to check-in with each other by email or through online discussion boards.

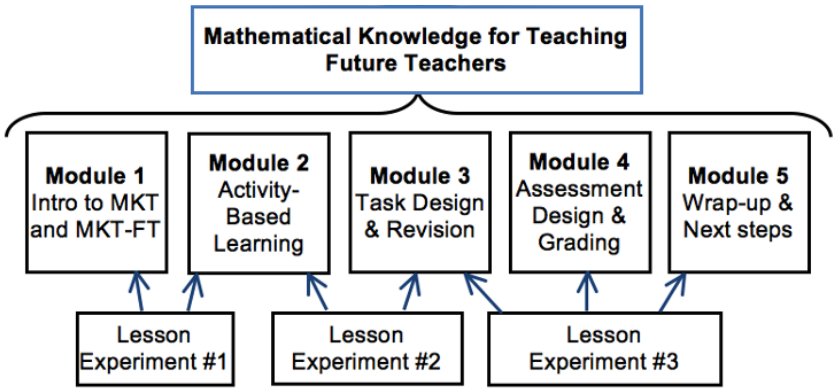

Figure 2. Overview of the PRIMED short-course for instructors new to teaching future teachers.

The short-course included consideration of MKT and MKT-FT in interculturally rich discourse contexts (e.g., examining a scenario where students are learning mathematics while also learning English as an additional language). Instructors had opportunities to learn about, witness (though video-clip-based activities) and practice (through lesson experiments) how to deal with the realities of negotiating the multiple experiences and relationships in a math-for-teachers classroom (see Jackson et al., 2020 for more on short-course development).

Each participating instructor agreed to complete research tasks and to include assignments in their course through which undergraduate students completed assessments of MKT. Instructors (n=15) and undergraduates (n=476) providing data for the study were from 12 different institutions of higher education (2 public community colleges, 7 master's granting universities [6 public, 1 private], and 3 doctorate-granting universities [2 public, 1 private]).

One of the project's main measures was the valid and reliable elementary grades number and operations *Learning Mathematics for Teaching* (LMT) assessment (Hill et al., 2008). Items on the LMT capture whether respondents correctly solve primary school mathematics problems as well as how they address teaching tasks (e.g., evaluating unusual solution methods, representing content to students, identifying adequate mathematical explanation). Instructors and their future-teacher-students completed the LMT at both the beginning and end of the semester in which instructors participated in PRIMED. Matched pre-post data were available for 12 instructors and 226 undergraduates. All also completed the Intercultural Development Inventory (IDI), a validated and reliable measure of intercultural development (instructor-participants did the IDI pre- and post-short-course, the undergraduates once, at the beginning). Also, each instructor had a PD involvement index (low, substantial, high) based on three characteristics: count of contributions to course online discussions, number of course tasks completed, and self-evaluation of effort. Though limited, these data allowed initial investigation of the relationships among PD, instructor and undergraduate MKT, and intercultural competence development.

Results

According to the LMT, faculty and their students gained MKT. The overall mean gain for instructors on the LMT was 0.5 standard deviations (sd) and for undergraduates was 0.29sd (see Figure 3, next page). Other research on prospective teacher learning has indicated that one semester of instruction in a course carefully designed and taught by an experienced teacher educator can lead to gains of up to 1sd (Castro Superfine et al., 2013). However, far smaller changes are usual (Phelps et al., 2016). When faculty have PD to strengthen their teaching, LMT gains among their students have been reported at about 0.25sd (Laursen et al., 2016). In the PRIMED study, when involvement-index is considered, average undergraduate learning gains were highest in the classes taught by instructors who participated most consistently in the short-course (student gain of 0.35sd), lower for those who skipped lesson experiments or modules (gain of 0.28sd for substantial participation group), and lowest for those who did only a few activities (0.13sd for this group).

The distribution of intercultural orientations for students was similar to that in previous research among pre- and in-service K-8 teachers (the majority at or before polarization). Figure 4 illustrates this along with the fact that the distribution of orientations for instructors was different from that of students (the majority at or after minimization) – which also is similar to results in other studies, (see Hauk et al., 2017 and references therein). That is, the distribution for students is rooted in polarization (seeing difference) while that for instructors is heaviest in minimization (minimizing difference and focusing on similarity). We have reported elsewhere on the challenges posed by the distinction (e.g., Hauk et al., 2015).

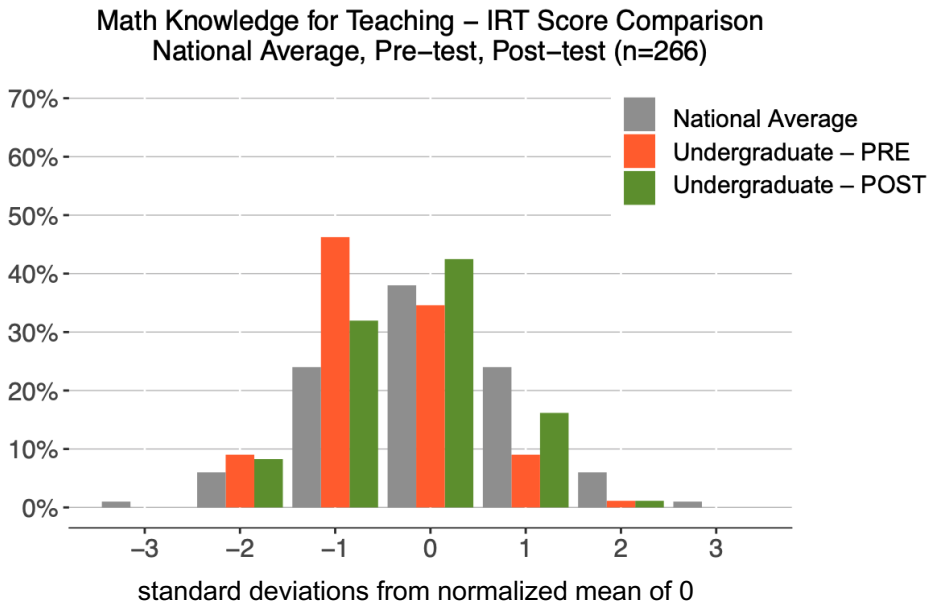

Figure 3. Comparison of LMT scores among future teachers

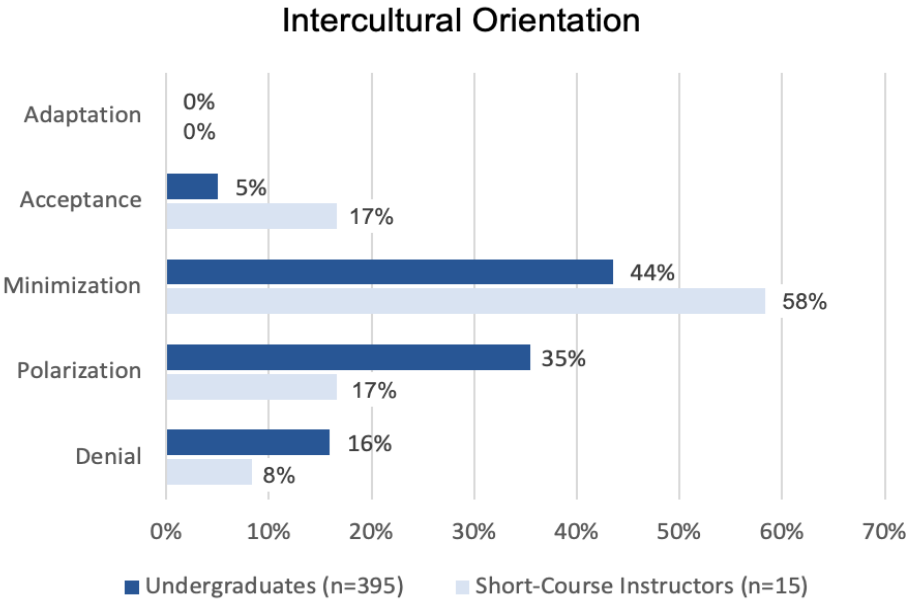

Figure 4. Pre-course IDI results as % of group for undergraduate students and instructor-participants.

Juxtaposing the two data sets, Figure 5 (next page) is a visualization of undergraduate student LMT average gains (horizontal) plotted according to the difference between the instructor and class average IDI orientations (vertical). The central boxed values indicate where the instructor and class (on average) had statistically insignificant differences in IDI score. Away from the central box the differences in IDI were statistically significant (p<0.5). For example, the dot furthest to the lower right is for a high-participation instructor (blue dot) whose class' polarization orientation was more interculturally developed than his denial orientation.

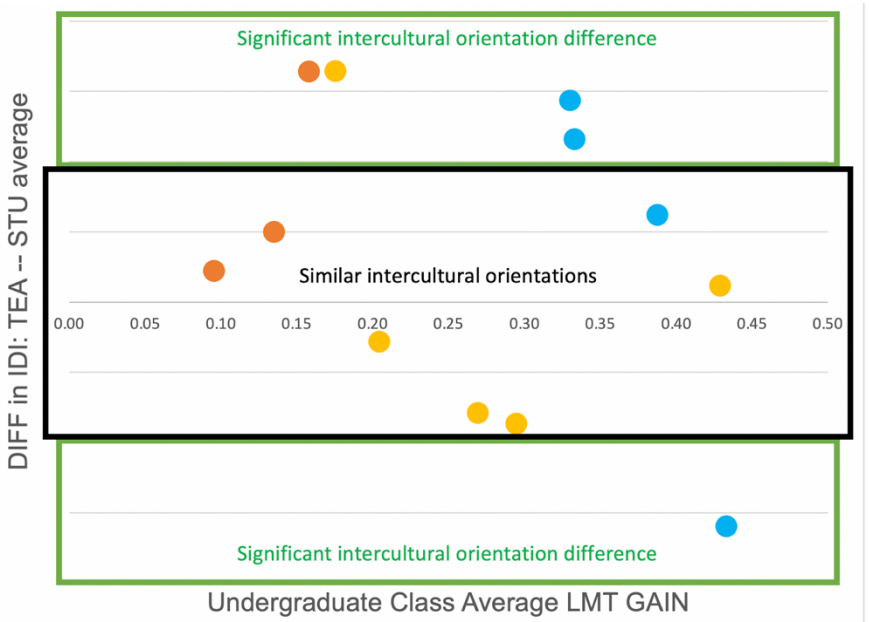

Figure 5. Scatterplot of student LMT gain by difference in IDI score. Dot colors indicate instructor level of participation in PRIMED (blue=high, yellow=substantial, orange=partial).

The research question driving the analysis sought insight into whether and how faculty intercultural competence development and MKT-FT development might be related to changes in undergraduate MKT. As indicated in Figure 5, instructors with high participation in the interculturally explicit PRIMED project had classes with the largest measured learning gains, even when there were significant differences between their own intercultural orientations and those of students. Figure 5 also suggests that PRIMED supported at least one instructor with a highly ethno-centric view to be effective with a diverse class group (rightmost dot in Figure 5).

Discussion and Conclusion

This study echoes, on a small scale, recent work on PD for inquiry-based learning in college settings that demonstrated an instructors' perceptions of their ability to enact behavior which they can control was positively associated with instructional practice aligned with PD aims (Archie et al., 2022). Our group continues to explore the relationships among intercultural competence, professional learning, and classroom practice (Hauk et al., 2017; Jackson et al., 2020). Finally, on a practical note, at the end of the short-course semester, three common goals for future professional learning emerged from participants, each related to interconnecting MKT and orientation:

(1) build *awareness of self as having a cultural lens for viewing the world* in a post-secondary mathematics/department culture that is distinct from the culture of K-12 teaching;
(2) find guidance in the transitions from minimization and into acceptance, particularly how to *be mindful of one's cultural filter(s) for interacting with the world*;
(3) engage in *building a knowledge base about equity*, including knowledge about culturally normative values and distinguishing these from essentializing or stereotyping approaches.

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