Visualizing Conceptual Change: Using Lakatosian Conflict Maps to Analyze Problem Solvers' Structures of Calculus Definite Integral Tasks

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The work of Imre Lakatos has been influential in the philosophy of science and mathematics. His writings on how science and mathematics progress as fields have been utilized in mathematics and science education. In this study, we apply Lakatosian theory in analyzing task-based interviews. We use conflict diagrams to center the discrepant events participants experience while solving calculus tasks. In so doing, we illustrate how problem context plays a vital role in the evolution of problem-solver's structures in problem situations.

Keywords: Lakatos, Conceptual Change, Data Analysis

Qualitative data is rich. Researchers can present complex, nuanced perspectives on mathematical reasoning with interview data. However, rich data is often also dense and difficult to understand and/or overwhelming in scope. We present conflict maps as a tool for analyzing task-based interview data rooted in the Lakatosian theory of scientific and mathematical progress that illustrates the rich problem-solving that evolves over the course of a task-based interview. As an analytic tool applied to interview data, conflict maps present problem-solving strategies as they emerge and how they change. This highlights and makes evident moments of cognitive conflict.

Lakatosian conflict maps exist independently as pedagogical tools in science education (Tsai, 2000). In utilizing such maps, lessons are designed so that learners engage with discrepant events, moments where they perceive and must respond to a discrepancy between expectations and results. Specific discrepant events are regarded as critical events when alternative conceptions of science are engaged and evolve into traditional scientific conceptions. These critical events are identified by the instructor ahead of the lesson and result from the experiences of instructor and/or research on student thinking. Our research goal is to explore the usefulness of Lakatosian conflict maps as analytic tools for task-based interviews. In the current study, we illustrate their use in analyzing how undergraduate students solved calculus accumulation tasks. Our research question is what discrepant events emerge in the solving processes of individuals while solving calculus definite integral tasks?

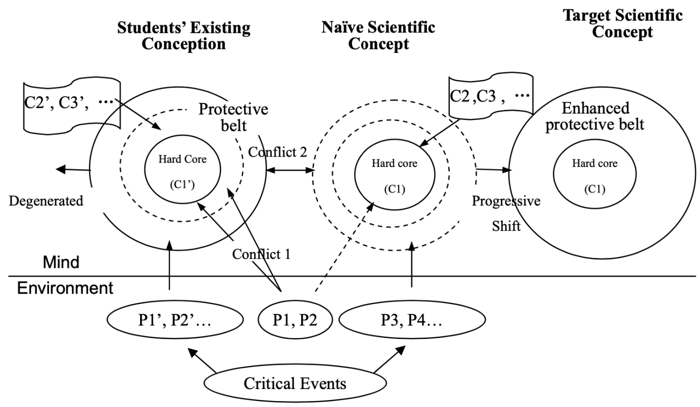
Literature Review

Lakatosian Theory

In this study, we draw from the perspectives of Lakatos (1970). Lakatos claims that scientific theories do not progress linearly matching objective reality better as time goes on. Instead, Lakatos states that theories either progress or degenerate over time. Theory falsification does not happen immediately upon identifying a contradiction for Lakatos. Lakatos describes scientific theories as having both a "hard core" of central assumptions and a "protective belt" of auxiliary hypotheses (p. 133). Seemingly contradictory evidence can be redirected towards the protective belt instead of instantly disproving the theory outright. The hard core can be altered, but only after accumulating significant contradictory evidence that leads to the development of a new hard core and protective belt that explains the observations and makes new predictions. Within the domain of mathematics education, there have been several works to emerge utilizing Lakatosian principles. Larsen and Zandieh (2008) explore guided reinvention in abstract algebra using processes rooted in Lakatos' Proofs and Refutations methodology. Nunokawa applied Lakatos' theory to mathematical problem solving in many of their studies, specifically examining the "solver's structures of a problem situation" (Nunokawa, 1996, p. 274).

Enhanced Conflict Maps

Several researchers in science education have made explicit use of Lakatosian principles in their work. Specifically, they have connected these ideas with students' conceptual change and developed what are called (enhanced) conflict maps (Tsai, 2000; Oh, 2011, 2014). These maps serve as pedagogical tools for building effective science lessons. See Figure 1 for the template for an enhanced conflict map.



Note: P1', P2'… : perceptions protecting existing conception
P1, P2 : perceptions involving a discrepant event
P3 , P4 , … : perceptions supporting Naïve scientific concept
C1….. : the Hard Core of Students' Existing Conception
C1'….: the Hard core of Naïve Scientific Concept
C2', C3'… : related scientific concepts supporting existing conception C2 , C3, … :

Figure 1. Enhanced Lakatosian conflict map Oh (2014)

Conflict maps have been utilized as pedagogical tools rather than analytical ones. These approaches map out how students are expected to move from one naïve or alternate conceptions to a scientific or mathematical one and include the planned events that (hypothetically) move students from one conception to the other. Ogbonnaya & Dimitriou-Hadjichristou (2022) illustrate such as approach in mathematics education, specifically for a student learning about surface area of a cone. We claim these diagrams can be used for the purpose of data analysis in research. Whereas discrepant and critical events have been presented intentionally by the instructor to move students through the various conceptions, we map these events as they occurred naturally during a task-based interview. This approach offers an alternative way to analyze qualitative data and provides an opportunity to reflect on analytic techniques in qualitative research based in visualization of data at several levels simultaneously, which may serve a researcher with neurodivergence better than traditional text-based techniques.

Theoretical Perspective

Problem Solving

Nunokawa (1994) defines problem solvers' structures as "structures given by the solver to the problem situation" and that "a solving process is described as a sequence of changes in the problem solver's representation structures" (p. 276). Nunokawa (1992) illustrated that the "sense" a problem-solver generates for a problem situation impacts the problem-solver's thinking during problem-solving. We use problem situation as Nunokawa does, to describe the in situ problem-solving environment as perceived by the participant.

Knowledge as an Adaptive Function

A view of knowledge as an adaptive function played a key role in the data analysis in this study. von Glasersfeld (1982) claims "knowledge for Piaget is never (and can never be) a 'representation' of the real world. Instead it is the collection of conceptual structures that turn out to be adapted, or as I would say, viable within the knowing subject's range of experiences" (p. 4). Viability is the crucial idea. Just as with the evolution of an organism in an ecosystem, what students learn is not driven by matching some objectively true reality, but what the student finds viable. For this study, the personal ecosystem is what Nunokawa calls the problem situation**,** and the problem-solver's structures exist (and thrive or perish) within that ecosystem. von Glasersfeld states that "in the sphere of cognition, though indirectly linked to survival, equilibrium refers to a state in which an epistemic agent's cognitive structures have yielded and continue to yield expected results, without bringing to the surface conceptual conflicts or contradictions" (p. 5). This is the heart of the concept of viability in constructivism, that learning is the development and application of stable cognitive structures.

Methods

This study concerns data from a broader study of the ways students reason in calculus accumulation tasks. In the broader study, twelve (12) undergraduate students participated in task-based interviews lasting approximately one hour. Participants in the study were undergraduate students from a large research university in the United States. All had majors or intended majors in the biological or life sciences and were solicited from across various upper and lower division undergraduate courses. Participants solved five calculus tasks concerning accumulation and rate of change set within contexts relevant for biological and life science students. In this study, we focus on one of the tasks and how students processed moments of conflict in their problem-solving work.

> Interview Task: The greenhouse effect is the rise in temperature that the Earth experiences because certain gases prevent heat from escaping the atmosphere. According to one study, the temperature is rising at the rate of $R(t)=0.014t^{0.4}$ degrees Fahrenheit per year, where t is the number of years since 2000. Given that the average surface temperature of the Earth was 57.8 degrees Fahrenheit in the year 2000, predict the temperature in 2200.

Diagramming the evolution of problem structures

Diagrams were created to visualize the evolution of problem solvers' structures during their work. Each diagram represents a student's full solution strategy for one interview task. Each diagram illustrates the central assumptions problem solvers made about the given rate of change function, this is referred to as the hard core of their problem structure. The hard core is surrounded by a belt of supporting concepts, assumptions, and calculations that the participant accepts as viable given their current assumptions. When contradictions arise, these are visualized externally to the current structure in the diagram. These are discrepant events for the participant. Each idea within the belt is presented chronologically how it appeared within the interview, read from top to bottom. The diagrams were first sketched by hand and then they were transferred to PowerPoint. The diagrams were validated using external checks, other expert researchers reviewing and providing feedback comparing the diagrams to the interview transcripts.

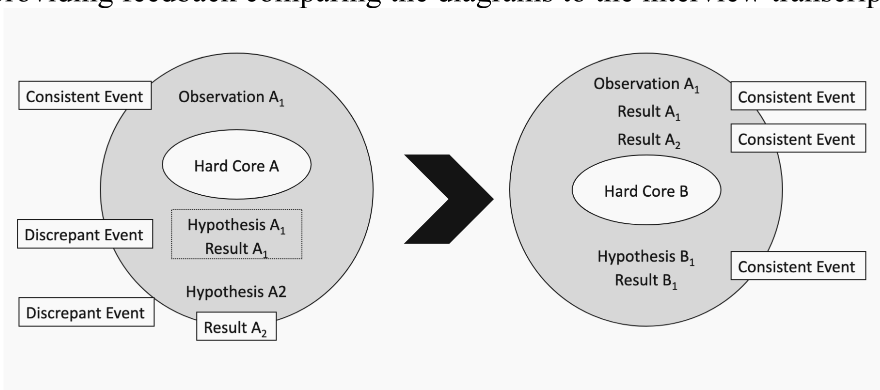


Figure 2. Theoretical problem structure diagram

In this report, we describe the discrepant events for eight of the participants. We focus on only those diagrams with discrepant events and identify the nature of those events and whether/how they were resolved. Figure 3 shows a theoretical problem structure diagram.

Results

We present the problem structure diagrams for two participants, Jake and Brenda. These visualizations capture the participants' full work on the task and what they recognized as viable and what discrepant events they experienced.

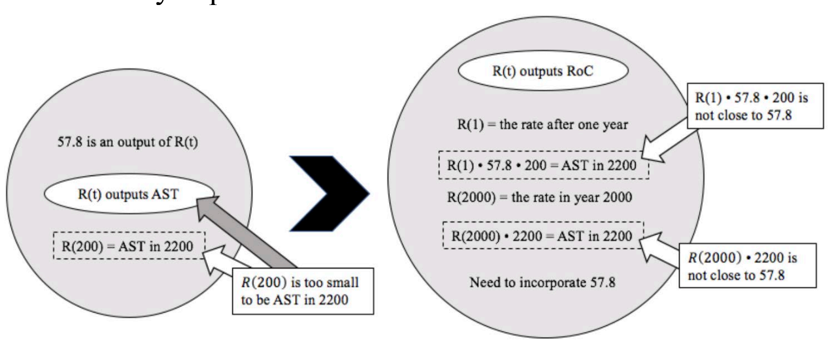


Figure 3. Problem structures diagram for Jake

In Jake's first problem structure, he claims the output of the function for t=200 will be the average surface temperature for the Earth in the year 2200. Jake's observation that R(200) is too small to the be average surface temperature is a discrepant event, similar to the case of Brenda. Jake formulates a new assumption about the values of the function R(t), namely that the outputs are rates of change. While this is accurate, his calculations lead him astray and new discrepant events result.

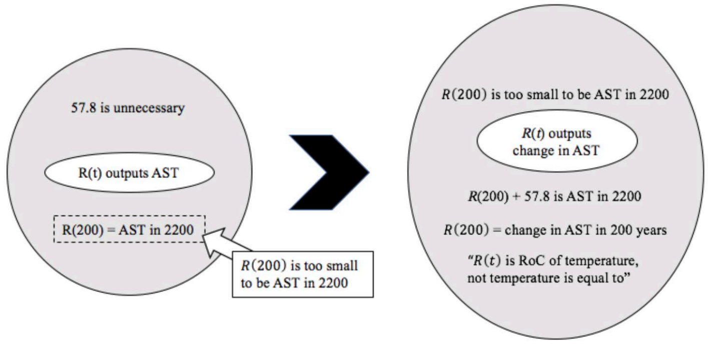


Figure 4. Problem structures diagram for Brenda

Table 1 contains a summary of the discrepant events observed in the eight participants under consideration. Each of the participants reasoned about the size of the value they calculated in relation to their expectations for climate change and the initial given temperature. Several others utilized the setting of the interview in reflecting on discrepant events or their work on other task.

Table 1. Summary of discrepant events

Participant	Discrepant event description	
Jane	Value is too low for AST(2200)	
	R(0) not 57.3	R(2000) very small
Anne	R(200) too small for AST(2200)	
	Interviewer direction	Work on Task 5
Shauna	R(200) too small for AST(2200)	
Andy	R(200) too small for change in temperature	
	Computed value too large for AST(2200)	Computed value too small for AST(2200)
Ron	R(200) too small for AST(2200)	
	R(200) too small for change in temperature	Work on Task 5
Jake	R(200) too small for AST(2200)	
	Computed value not close to 57.3	
Brenda	R(200) too small to be AST(2200)	
Gina	R(200) too small to be AST(2200)	

Discussion and Questions

Our research question is: what discrepant events emerge in the solving processes of individuals while solving calculus definite integral tasks? Most of the discrepant events observed involved the context of the task. Participants assessed the viability of their assumptions by using their expectations and knowledge of the context. The calculated values being either unreasonably low or high for the context of average surface temperature of the Earth was often a key indicator that assumptions were not viable. Questions for discussion: What are the benefits of adopting this analytical approach in mathematics education research? How can Lakatosian conflict maps benefit us in analyzing student thinking as they work through a task or broader concept? We feel visualizing interview data on problem solving math tasks leads to greater accessibility. We also believe the diagrams preserve the participants' voice, their authentic ideas as opposed to solely the researchers' perspective of the work.

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