Pre-Calculus Students' Mathematics and Art Cognitions in Making and Testing Conjectures About the Fibonacci Spiral and the Great Wave of Kanagawa

Tuto LopezGonzalez
San Francisco State University

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There are many forms of thinking and sense-making through artistic expression (e.g., painting, music, dance, and emotion). Similarly, there are many forms of thinking and sensemaking through mathematical expression (e.g., quantities, symbols, diagrams, logic). In mathematics, the Pirie-Kieren (P-K) framework has been useful for exploring the development of mathematical understanding (Pirie & Kieren, 1994). According to the P-K framework, a learner goes back and forth among eight cognitive layers when working on a mathematical task (primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, inventizing). No similar framework exists for the development of artistic understanding. Research has suggested that bringing arts into a mathematics classroom can be viewed as a rich way of engaging learners (Baird, 2015). A growing interest in cognitive development at the intersection of mathematics and arts offers an opportunity to explore how it might be modeled as a non-linear dynamical recursive process as in the P-K framework.

Thinking about art can inform/shape thinking about mathematics and vice versa. The poster presents research exploring the nature of pre-calculus students' mathematics and art cognitions in making and testing conjectures about recursive relationships. The work examines the potential of using the P-K model for documenting students' mathematics and art understanding and how this understanding develops while including attention to responses such as gesturing and body language. I developed and conducted interviews with four pairs of college precalculus students. The interview task used the Fibonacci spiral and an Edo-period woodblock print, the Great Wave by Hokusai, as spurs for student thinking and interaction about mathematics and art. Qualitative inductive analysis examined the audio- and video-recorded interviews using the P-K framework to identify and describe the mathematical and art-centered/visual thinking by students as they made sense of and responded to aspects of the task.

This poster will illustrate the findings on how the Pirie-Kieren framework is useful in unpacking the math-rich and diagram/visual-rich aspects of cognition by pre-calculus students. This poster aims to provide a space for RUME participants to share and discuss how art (song, painting, poem, etc) can support thinking about an idea/concept in mathematics. Additionally, it is a goal of this poster's graduate student author to talk with visitors about existing research about or near to mathematics and arts cognition, both to learn what they suggest as related readings and what more or different they would hope to see in a long report about this research.

The implications of the research include the value of (1) revisiting existing research at the intersection of art and mathematics with multiple P-K diagrams in hand (one for art, one for mathematics, and one for both) to make better/different sense of cognition involving both mathematics and art and (2) designing new curriculum and related research on how to combine art and mathematics that is better informed by the ways the combining may influence mathematical understanding.

References

Meel, D. E. (2003). Models and theories of mathematical understanding: Comparing Pirie and Kieren's model of the growth of mathematical understanding and APOS theory. In A. Selden, E. Dubinsky, G. Harel, & F. Hitt (Eds.), *Research in Collegiate Mathematics Education. IV. (pp. 132–181). American Mathematical Society.*

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26(2/3), 165–190.*

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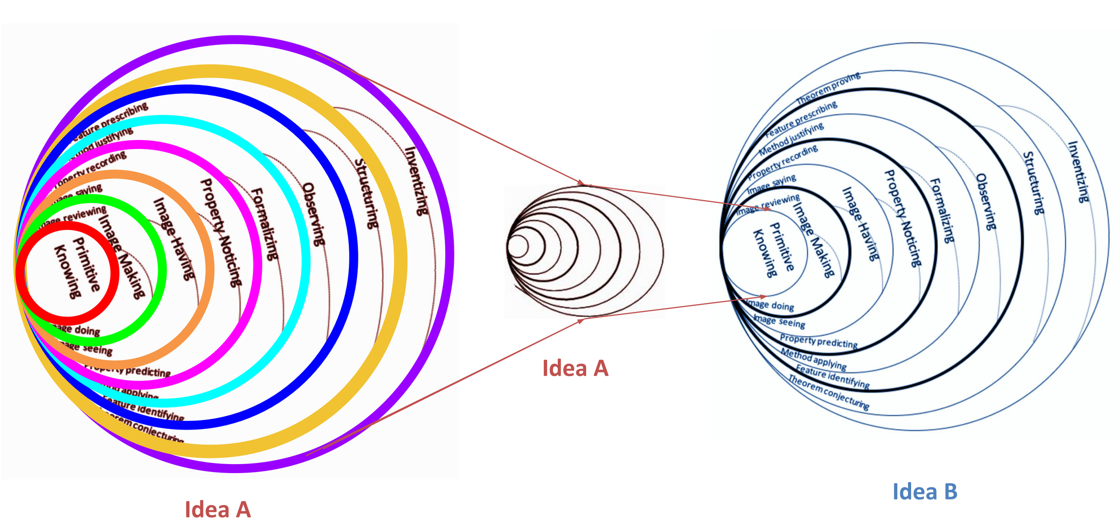
Project Overview

The Pirie-Kieren (P&K) framework

The Pirie and Kieren (P&K) framework (Pirie & Kieren, 1989; 1994) provides a way to explore the development of mathematical understanding by suggesting that a learner move through eight cognitive levels of sophistication as they go through a learning situation.

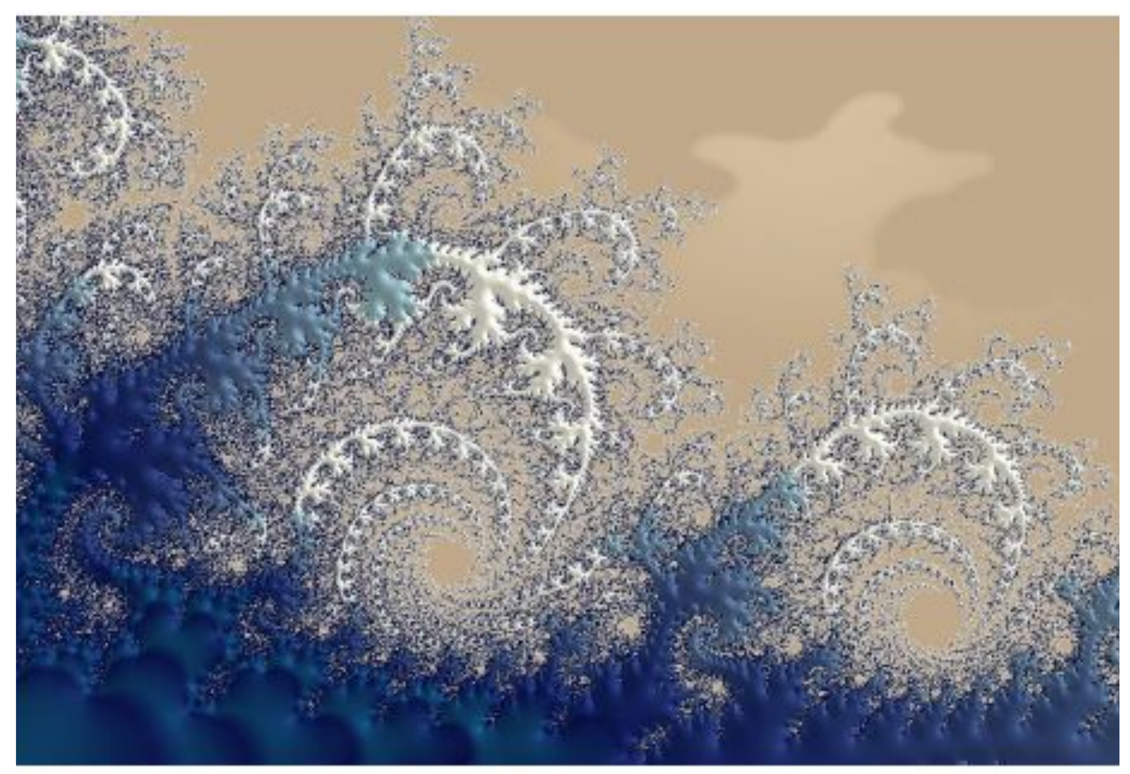
1. **Primitive knowing:** Learner's information brought to the learning situation.
2. **Image making:** The learner acts (mentally or physically) to gain an idea about a concept.
3. **Image having:** Single-activity associated images are replaced by a mental picture.
4. **Property noticing:** Notice properties and connections between mental pictures.
5. **Formalizing:** Description of class-like mental objects to produce full mathematical definitions.
6. **Observing:** Question formal statements verbalize cognitions about the formalized concept.
7. **Structuring:** Determine if formal observations are true by an axiomatic system (proof)
8. **Inventizing:** Complete understanding of a concept and pose questions that lead to new concepts



Self-similar nature - Encapsulated Idea A becomes Primitive Knowing for Idea B

Developing a framework for the development of artistic fulfillment/understanding

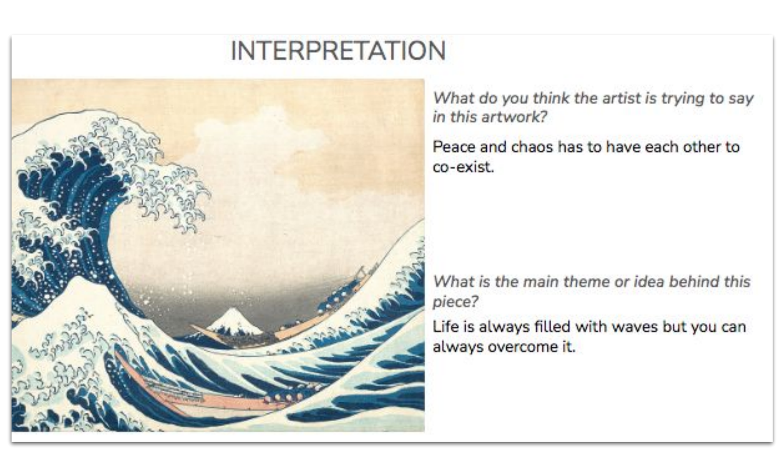
Research has suggested that bringing arts into a mathematics classroom can be viewed as a rich way of engaging learners (Baird, 2015). A growing interest in cognitive development at the intersection of mathematics and arts offers an opportunity to explore how it might be modeled as a non-linear dynamical recursive process as in the P&K framework.



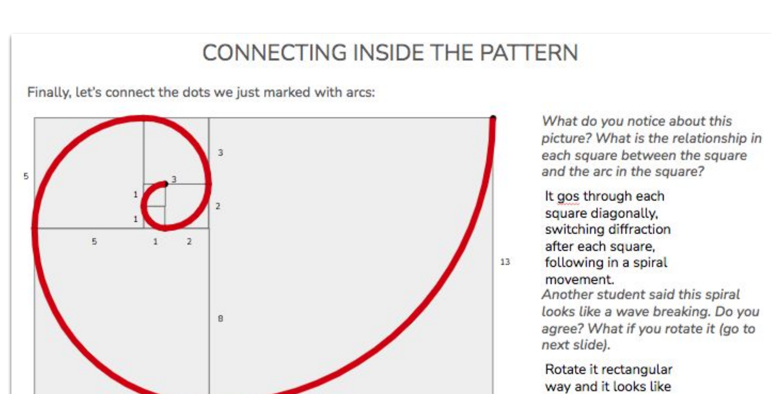
I created an experience completed by 3 pairs of pre-calculus students. Each pair interacted with ***The Great Wave off Kanagawa (GWK)*** (1831) by artist Katsushika Hokusai. Participants were asked to analyze the GWK. Then, pairs completed a mathematical task with the **Fibonacci Spiral (FS)**. Finally they interacted with the GWK using the FS.

The Experience: Interacting with the Great Wave off Kanagawa

Part 1: Analyzing The GWK using the the 4-step art analysis framework i.e., **Description**, **Analysis**, **Interpretation**, and **Evaluation**.



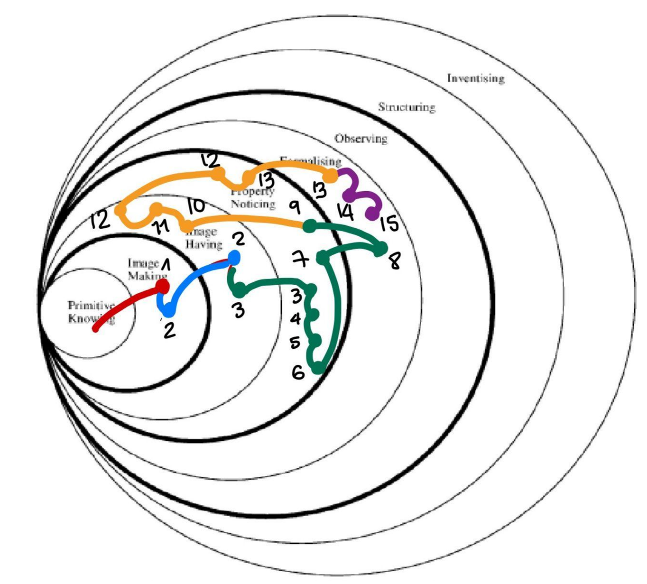
Part 2: Building the Fibonacci Spiral by making and testing conjectures about its patterns.



Part 3: Fitting the FS within the GWK and communicating art and math with math key words

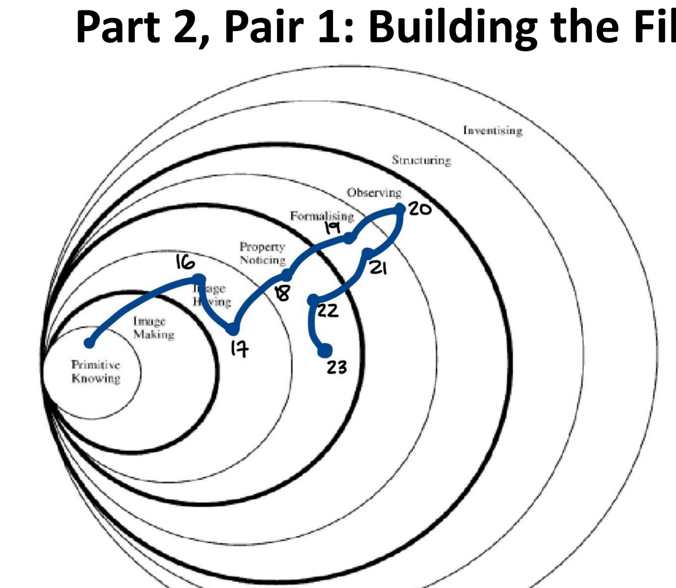
Maps of Development of Understanding

Part 1, Pair 1: Analyzing the GWK.



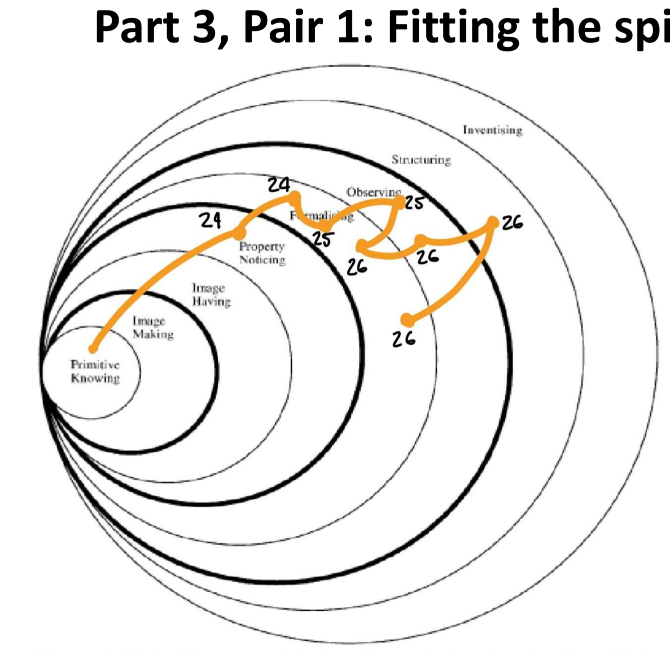
The P&K framework was extended to track the learner's development of fulfillment/understanding of the GWK.
(# indicates prompt of 27 questions total)

Part 2, Pair 1: Building the Fibonacci



The P&K framework for the development of mathematical understanding was used to draft the map on the second part of the experience
(# indicates prompt)

Part 3, Pair 1: Fitting the spiral



The P&K framework was extended to track the learner's development of fulfillment/understanding of both the GWK and the FS
(# indicates prompt)

Sharing

Let's talk arts and math in research :)

What more or different would you hope to see in a long report about this research?

How do you think arts can support thinking about mathematical concepts?

What do you recommend reading?

Post-its notes area

Findings

In studying the nature of pre-calculus students' mathematics and art cognitions based on the experience, the following observations were done:

- P&K is useful for understanding mathematical cognition about the FS and documenting student's sense making.
- P&K is useful for exploring student's sense making and fulfillment of arts.
- A multi dimensional version of the P&K framework is proving useful in documenting how students use visual and mathematical information together to make sense of mathematical ideas related to the FS.
  - In analyzing art (Part 1) learners come up to the boundary (Meel, 2003) of formalizing.
  - When interacting with **both** the GWK and the FS (Part 3), learners crossed the boundary and entered mathematical formalizing.
- Preliminary analysis indicates gesturing when interacting with artwork is a form of embodied cognition that supports formalizing concepts beyond noticing.

Future Work

- Testing different Machine Learning algorithms such as decision trees, regressions and neural networks to help identify the cognitive level of sophistication a learner is in for a given mathematical task. Such algorithms can be useful for assessment, personalized learning, curriculum development, and future research.
- Extending the P&K map to a 3-dimensional map where the axis represent art fulfillment growing, mathematics understanding growing and time.

References

- Baird, D. (2015). Integrating the arts in mathematics teaching. (Master's thesis). University of Toronto. https://bit.ly/3xET4hT
- Meel, D. E. (2003). Models and theories of mathematical understanding: Comparing Pirie and Kieren's model of the growth of mathematical understanding and APOS theory. In A. Selden, E. Dubinsky, G. Harel, & F. Hitt (Eds.), *Research in Collegiate Mathematics Education. IV.* (pp. 132–181). American Mathematical Society.
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