Prioritizing Mathematical Topics for General Education Requirements

Jim Gleason	W. Gary Martin	Mariya Rosenhammer
The University of Alabama	Auburn University	Columbus State University

With increased availability of technology and data, the mathematical and statistical needs for many academic disciplines have likely changed over the past twenty years. In order to create appropriate general education courses in the mathematical sciences to meet these needs, a survey was conducted of individuals representing various academic programs in Alabama to determine the mathematical needs of students in their programs. The survey results indicate that traditional college algebra courses are not needed outside of the disciplines that require calculus. The needs of the non-calculus-based programs include linear and exponential functions; mathematical practices; statistical literacy; and the use of spreadsheets for data analysis.

Keywords: general education, data analysis, mathematical practices

Nearly all colleges and universities in the United States include at least one course in the mathematical sciences as part of the general education requirements for their bachelor's degree. Nearly half of the students enrolled in a post-secondary mathematics course in the United States are in courses focusing on polynomial, rational, exponential, logarithmic, and/or trigonometric functions (Blair et al., 2018, p. 213). While such topics are essential for students needing to understand calculus for their academic program, they may not be the best choices of topics for courses in the mathematical sciences for students who do not need an understanding of calculus.

A common alternative to the traditional algebra and function based undergraduate course is a course focusing on quantitative literacy. Beginning in the 1980's, the Consortium for Mathematics and Its Applications (COMAP) created curriculum for such courses (Campbell, 2007) and in the 1990's, the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (MAA) created a sub-committee to explore how quantitative literacy could be included as part of undergraduate general studies programs (Sons, 1995).

To improve all lower-division mathematics courses the MAA Committee on Curriculum Renewal Across the First Two Years (CRAFTY) started the Curriculum Foundations Project to better understand the mathematics needed by students in partner disciplines. This involved a series of workshops with faculty members from mathematics-intensive disciplines (1999-2001) and less mathematics-intensive disciplines (2005-2007) (Ganter & Haver, 2020) resulting in two documents (Ganter & Barker, 2004; Ganter & Haver, 2011) describing the mathematical needs of students in partner disciplines. This interdisciplinary work continues with projects like the Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) consortium (Ellington, 2020; Ganter et al., 2019a, 2019b).

In order to more holistically determine the mathematical needs of students from various non-STEM disciplines, the Arkansas Department of Higher Education partnered with the Dana Center to form the Arkansas Math Pathways Task Force (AMPT) to create a survey for departments to provide feedback about which topics in the mathematical sciences would be beneficial for their students (Korth et al., 2018) in order to provide guidance on what general education mathematics courses would be most useful for students in different majors. In considering these results, a similar working group in Alabama felt less confident that the current mathematics courses included in the general education program were appropriate. Thus, the current study expands upon the work of AMPT by creating a survey of academic programs at public post-secondary institutions in Alabama at a finer grain size and for a larger number of mathematical and statistical topics and practices than the AMPT survey in order to guide the redesign of mathematics courses included in the general education requirements for different majors.

Methods

A working group of faculty members from the mathematical sciences and institutional assessment from multiple higher education institutions commissioned by the Alabama Commission on Higher Education created a survey of mathematical and statistical topics and practices based upon syllabi from introductory post-secondary mathematics and statistics courses, standard secondary mathematics curriculum (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and post-secondary professional organization standards (American Association of Two-Year Colleges, 1995, 2006; GAISE College Report ASA Revision Committee, 2016; Mathematical Association of America, 2004, 2015). After the creation of the survey items, they were reviewed for content accuracy by 10 mathematics faculty and readability by four non-mathematics faculty that work in assessment divisions of their institutions.

Based on information provided by the chief academic officers from each of the public higher education institutions in the state, the survey was sent to individuals who oversee each academic program in the state. Calculus courses satisfy the general education courses for a range of academic programs, including Mathematics, Engineering, Physics, Chemistry, and Computer Science; these programs were not included in the study as they were outside its scope. Of the 548 eligible programs, 174 submitted complete responses for a 32% response rate.

Due to the sample size from our survey responses we are not able to draw conclusions at the level of a single major. However, by combining majors into meta-majors based upon assumed mathematical needs, we can attain more generalizable conclusions. Academic programs that include a significant focus on using large data sets and/or computer programming as a key component of their field are likely to have a significant need for mathematical and statistical knowledge outside of calculus. For this reason, we group together majors such as geographic information systems, computer forensics, econometrics, and computer and information sciences into a meta-major that we will call **Data Sciences**. We group academic programs such as biology, geography, environmental science, and nursing together into a meta-major that we will call **Natural Sciences**, although this grouping does not include physics or chemistry since they require calculus. Majors related to education, psychology, human development, political science, and social work all incorporate the use of data and interpretation of statistical studies in their fields, so we incorporate these disciplines into a meta-major that we will call **Social Sciences**. Academic programs with a focus on business and/or management are grouped into a **Business** meta-major. Although such programs often include "business calculus" for the general education requirement, calculus is not inherent to the work of those fields and so we included this in this study. Finally, the **Humanities** meta-major includes majors that focus on studying and/or contributing to human cultures, including languages, arts, communication, philosophy, religion, and anthropology. Number of academic programs and response rates in each of these meta-majors is included in Table 1.

Table 1. Response rates by meta-major.

Meta-major	Eligible Programs	Responses	Response Rate
Data Sciences	34	13	38%
Natural Sciences	115	38	33%
Social Sciences	164	48	29%
Business	106	29	27%
Humanities	129	46	36%
Totals	548	174	32%

Mathematical topics were arranged into five major categories commonly included in general education mathematics courses, while statistical topics were organized into eight general categories. In order to facilitate the completion of the survey while still getting the desired level of detail, the survey included items addressing broader topics of mathematics within each of these categories, for which respondents denoted that their students “definitely needed”, “somewhat needed”, or “not needed”. The “definitely needed” and “somewhat needed” responses were compressed into a “needed” category and these respondents were asked one or two follow-up questions to determine the need of students at a finer grain analysis. For example, respondents were asked “What best describes the importance of the following statement about students in your degree (or program)?” with the statement regarding functions being, “Students will understand properties of functions (domain, range, compositions, inverses) and use algebraic, symbolic, graphical, and numerical techniques to model related contexts and solve related equations.” The respondents who denoted this statement as needed for their students received follow-up questions about the types of functions needed and the properties of those functions that their students would need to understand.

For each of the items in the survey, the need for that topic or practice for each meta-major was determined by the proportion of programs denoting a need for the topic. If the proportion was above half to a statistically significant level ($p<0.05$), we considered that topic as needed for the meta-major. If the proportion was below 50% at a statistically significant level ($p<0.05$), we considered the topic as not needed for the meta-major. If there was no significant difference from 50%, we did not draw a conclusion about that topic for that meta-major.

Results

The primary result of the study is that most of the meta-majors did not need most of the mathematical content traditionally included in introductory college mathematics courses. However, all the meta-majors, other than Humanities, reported a need for all the general statistical topics and practices from introductory college statistics courses, although not all the subtopics.

Mathematical Topics and Practices

Of the students enrolled in introductory mathematics courses in the fall of 2015, 20% were enrolled in precollege level (courses not receiving college credit), 40% were enrolled in a course focusing on functions and equations that involve polynomial, exponential, logarithmic, and/or trigonometric functions, and 20% were enrolled in liberal arts or finite math courses focusing on some logic, set theory, financial mathematics, and probability (Blaire et al., 2018, p. 213). The respondents to our survey reported that these topics are not the ones that are most needed for students in their majors (Table 2). None of the meta-majors denoted a need for polynomial and trigonometric functions, core components of the most common introductory courses, and instead denoted a need for linear functions, a topic usually included in the precollege level course. Furthermore, the topics related to functions denoted as needed by the largest number of programs was the ability of students to find numerical approximations of solutions using technology, a topic often not a major focus of college courses.

Table 2. Mathematical needs by meta-major.

Mathematical Topics	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Function	Linear and Exponential	Linear and Exponential	Linear		Not Needed
Equations and Inequalities	Linear	Linear			Not Needed
Geometry and Trigonometry			Not Needed	Not Needed	Not Needed
Logic and Set Theory		Not Needed	Not Needed	Not Needed	Not Needed
Financial Mathematics		Not Needed	Not Needed		Not Needed

The respondents did denote that their students need experience with the mathematical practices that come from general education mathematics courses:

1. Interpreting quantitative information (needed by all but Humanities),
2. Strategically evaluating, inferring and reasoning (needed by all),
3. Manipulating mathematical expressions and computing quantities (needed by Data Science; not needed by Social Sciences or Humanities),
4. Communicating mathematical ideas in various forms (needed by Business, not needed by Humanities),
5. Make sense of quantitative problems and persevere in solving them (needed by Data Science and Business; not needed by Humanities),
6. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace (needed by all), and
7. Look for patterns and relationships and make generalizations (needed by all).

Combining the information of topics and practices, the survey communicates a need for introductory mathematics in the general education curriculum with a course focusing on the mathematical practices within the context of linear and exponential functions and equations. Such content could easily be incorporated into a data analysis course that includes linear and exponential regression.

Statistical Topics and Practices

Using the goals for introductory statistics courses established by the American Statistical Association (GAISE College Report ASA Revision Committee, 2016), the survey asked the respondents about the following statistical topics and practices:

1. Students should become critical consumers of statistically-based results reported (needed by all),
2. Students should be able to recognize statistical questions and design appropriate statistical studies (needed by all but Humanities),
3. Students should be able to produce and interpret data visualizations, numerical summaries, and statistical models (needed by all but Humanities),
4. Students should recognize and be able to explain the central role of variability and randomness in the field of statistics (needed by all but Humanities),
5. Students should demonstrate an understanding of, and ability to use, basic ideas of statistical inference, both hypothesis tests and interval estimation, in a variety of settings (needed by all but Humanities),
6. Students should be able to interpret and draw conclusions from standard output from mathematical and statistical software packages (needed by all but Humanities, not needed by Humanities),
7. Students should demonstrate an awareness of ethical issues associated with sound statistical practice (needed by all), and
8. Students can use tools and techniques involving the theory of probability to understand the nature of chance and to quantify variation (needed by all but Humanities, not needed by Humanities).

We note that outside of the Humanities meta-major, all these topics and practices were denoted as necessary for students as part of their academic program. For the Natural Sciences, Social Sciences, and Business meta-majors, the only topics denoted as not needed by there students are creating and computing statistical representations by hand; using any technology other than spreadsheets, SPSS, or R; and details of the theory of probability involving probability distribution functions and the Central Limit Theorem. All the other sub-topics were either denoted as needed or it was inconclusive about the need.

Conclusions and Implications

The primary conclusion from the survey results is that there is a much greater need among undergraduate students for courses in statistics and data analysis than the traditional courses of college algebra and precalculus. The results also corroborate the goals for students in introductory statistics courses of the ASA GAISE Report (GAISE College Report ASA Revision Committee, 2016, p. 8), and its recommendations of what to teach in such a course: “Teach statistical thinking” and “Focus on conceptual understanding” (p. 6).

One implication is for colleges and universities to consider the role of the mathematical sciences in the general studies curriculum at their institution. Mathematical ways of thinking have always been a core component of a liberal arts education, and the current study confirms that academic disciplines across the spectrum still value students having those ways of thinking. What they do not seem to value is the current menu of topics that have been standard for introductory mathematics courses. This should cause departments in the mathematical sciences to reflect on the goals of their current course offerings and to perhaps create new courses for this broad constituency of students. As a result of the survey, the Alabama Commission on Higher Education that initiated the study changed the corresponding category from a Mathematics requirement to a Mathematical Sciences requirement and changed from not allowing statistics courses to satisfy the requirement to encouraging such courses to be used to satisfy the new requirement.

Many of the academic programs surveyed in this study have program requirements that include mathematics courses based on topics that they denoted as not needed by their students, either explicitly or implicitly as a prerequisite. The results of this study should motivate such departments to collaborate with the mathematical sciences departments to determine the most appropriate courses for their students: if such courses do not exist, they should work together to develop new courses.

Acknowledgments

We would like to thank the members of the Alabama Commission on Higher Education working group on options for gateway mathematics for their feedback and assistance throughout this project.

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