Ongoing Efforts to Develop an Assessment Tool for Students' Participation in Defining

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*There has been a push to develop curricula that engages students in various mathematical practices (e.g., defining). As more curriculums of this type arise, there is a greater need for assessment tools for students participating in these practices. We see Modeling Eliciting Activities (MEAs) as one answer to this need. In this study we discuss how the six design principles of MEAs can be used to (re)design an assessment tool for students' defining activity.*

*Keywords:* Modeling Eliciting Activities, Assessment, Defining, Classroom Practices

One common goal of many curriculum designers is to engage students in mathematical practices such as defining, conjecturing, and proving. This engagement can be a means to support students in learning about mathematical concepts (Larsen, 2013; Lockwood & Purdy, 2019; Oehrtman et al., 2014; Swinyard, 2011) and/or as a means to increase students' participation in these practices themselves (Larsen et al., 2022; Rasmussen et al., 2005; Zandieh & Rasmussen, 2010). We have engaged students in mathematical practices for both reasons in our ongoing efforts to design inquiry-oriented curriculum for Introduction to Proof courses.

We have faced a practical problem in our efforts: the lack of an appropriate assessment tool to reveal students' participating in mathematical practices. While there are relevant concept inventories (e.g., Melhuish, 2019; Mejía-Ramos et al., 2017), these take an acquisition lens on learning (Sfard, 1998) to measure students' understandings of important concepts or skills rather than a participation lens. We also found relevant observational tools (Melhuish et al., 2021; Melhuish & Thanhesier, 2017); yet, using these would require one to analyze classroom data to get a sense of whether students' participation changed overtime.

An assessment tool that measures students' participation in mathematical practices would be useful for us as researchers to measure the efficacy of our materials on student participatory learning. Additionally, the need for an assessment tool has been echoed by the instructors who use our curriculum materials; they want their tests and quizzes to reflect what they do/teach in class. This study aims to make progress on this goal for the case of assessing students' participation in defining - a practice that our curriculum materials are specifically engineered to promote. In doing so, we hope to open the discussion about how such an assessment could be adapted to assess students' participation in other classroom mathematical practices (Cobb & Yackel, 1996; Rasmussen et al., 2015). In what follows, we introduce Model Eliciting Activities (MEAs), which informed our design and revisions of the assessment.

## Conceptual Framework

Lesh and his colleagues (2000) describe MEAs as tasks designed so that students' work "reveal explicitly the development of constructs (conceptual models) that are significant from a mathematical point of view" (p. 362). These tasks emphasize communication and reflection by asking students to explain their reasoning, thinking, and strategies including revisions and refinements. Teachers and researchers have used these artifacts to assess students' conceptual understanding (e.g., Carlson et al., 2003), problem-solving strategies (e.g., Chamberlin & Moon, 2005), and metacognition (e.g., Kim et al., 2013). In our study, we aim to add to this literature by using MEAs as a tool to assess students' participation in classroom practices, and specifically, we are designing an assessment tool that elicits a model of students' defining activity.

The six principles to guide the creation of productive MEAs with guiding questions (adapted from Carlson et al., 2003, pp. 472-473 to fit our defining context) are: (1) *Model Construction* (Does the task immerse students in a situation in which they are likely to confront the need to develop their defining activity?), (2) *Construct Documentation* (Will the task require students to reveal their defining activity?, (3) *Reality* (Will students make sense of the situation by extending their own knowledge and experiences?), (4) *Model Shareability and Reusability* (Does the model provide a general model for analyzing this type of defining activity?) (5) *Effective Prototype* (Is the situation simple?), (6) *Self-Assessment* (Does the task promote self-evaluation on the part of the students?). This study aims to answer: How can the MEA principles guide the design of an assessment tool for students' participation in defining?

## Methods

Data for this study comes from an NSF-funded projected (ASPIRE in Math IUSE #1916490) that aims to create modular inquiry-oriented introduction to proof curricular and instructor support materials. One goal of the curriculum is to support students in participating in defining. A typical defining task sequence first engages students in a situation in which an informal idea of a concept emerges. Then students create and negotiate (non-)examples of the concept and then the teacher and students work together to refine their informal idea into a formal definition (see Vroom, 2020). We expected that our course would support students to strategically use (non-)examples when creating and testing their definitions, and to use those (non-)examples to strengthen their understanding of the concept to support them in formalizing their informal ideas.

Our primary goal with the assessment was to measure how students' participated in the sort of defining that our curriculum materials supported. The assessment had two main parts. First, the students were asked to define two concepts in a similar manner as our curriculum materials. To do so, students were asked to read an informal description of a concept, come up with (non-)examples, and write a formal definition. The two made-up concepts on the assessment were called Mirror-point Functions (described as "functions defined on a domain *D* that have at least one input-value that is the same as the corresponding output-value") and Repeat Functions (described as "functions where no matter what input you consider, that input maps to the same output value as the input that is one unit to the left of it"). We had no expectations for a "correct" definition as our focus was on the students' defining activity. The second part of the assessment was a "strategy guide" task that asked students to reflect on and articulate how they engaged in the previous defining tasks. We anticipated that the students' responses would provide us insights into how/if the students engaged in defining in such a way that was supported by the curriculum.

Creating and refining the assessment was a cyclic process in which we designed (or redesigned) the assessment, tested it with students, and analyzed the students' assessment responses and conducted follow-up interviews. We administered our assessment in three community college courses over the course of a year. For each course we administered the assessment during the first week of the course and then again during the last week. After each administration, we invited students to a follow-up semi-structured interview to discuss their responses. This resulted in a total of 18 pre-assessments with 6 follow-up interviews and 16 post-assessments with 7 follow-up interviews. Our data analysis focused on how, if at all, the assessment satisfied the six MEA principles based on what the students expressed on the assessment and follow-up interviews. We used this analysis to redesign the assessment before the next administration. In this report, we primarily focus on the second part of the assessment (the strategy guide task) since it provided us the most insight into how students participated in defining activity. The most updated version of the task is given in Figure 1. We will refer to this figure (and it's labeled parts) in the next section.

> **Strategy guide.** Notice that in the previous two sections you were given a name for a concept along with an informal description of the concept. Then in part B you were asked to write a definition. Now your job is to write a strategy guide that would help other students do what you just did on their homework! This strategy guide should explain exactly what the students should do (and how) to create a definition. Imagine they are given some NEW concept and an informal description and then they are asked to write a definition. Fill in the bulleted list below (add more bullets if needed) to write this general strategy guide. b. c.
>
> *After creating examples and non-examples of a given concept, you will be asked to write a formal definition of the new concept on part B. In order to write this definition, you should…* a.

*Figure 1. Latest version of "how to write definitions" strategy guide prompt*

## Results

In this section we discuss how our assessment meets each of the six MEA principles and describe refinements that were made to address issues we found during our implementations.

### Model Construction Principle

Given that we wanted to assess students' defining practice, designing a task in which they would create a model of that practice was critical. We addressed that need (and this principle) with the strategy guide task, where students were asked to create a guide for writing a formal definition for any concept. To do so, the students had to generalize their defining activity and thus, construct a model of their practice. In our first version of this task, students were informed that the strategy guide would be used to prepare students to write definitions on their exam and that "*They will not know what they will be asked to define [...] in advance.*" We found that this prompt was confusing for students, especially about what we meant by a "strategy guide". Some students opted to skip the task altogether while one student attempted the task but adding the note "I'm still a little unclear about this portion". To be more explicit about the desired model, we refined the task by adding a sentence starter (see Figure 1a).

### Construct Documentation Principle

Presenting students with the opportunity to create a model of their activity (by creating a strategy guide to address the previous principle) was only part of the battle. We also needed to design the strategy guide task in such a way that the students explicitly revealed their defining activity. We wrote the strategy guide prompt in such a way that would encourage students to reflect back on their activity on the previous tasks to not only generalize it (as part of the Model Construction Principle) but also to document that generalization (as part of the Construct Documentation Principle). For instance, the first version of our prompt stated "*This strategy guide should explain what exactly the students should do (and how) to create the definition requested in part B of the example exam items.*" In the previous tasks, "Part B" was where students were asked to create a formal definition of the concept. By directly pointing to that part of their tasks, we hoped students would reflect on and document that aspect of their activity.

As part of the response to the students' initial confusion described in the previous subsection, we edited the prompt to make it clearer that we wanted students to create a guide that reflected their activity in the previous tasks by expliciting stating to reflect on what they did as they defined the two given concepts on the assignment, see Figure 1b. Not only did we see the refined prompt as supporting students in creating a general model of defining activity (addressing the

Model Construction Principle) but it also made it clearer that the model should reflect their activity when completing the previous tasks. This edit appeared successful. For instance, one student wrote for their first two steps in their guide “1) Look at the differences between examples and non-examples, and 2) Look at the similarities between examples.” During the student’s follow-up interview, they confirmed that this was reflected in what they did to create definitions, saying “it did yeah, that's what I use here.”

**Reality Principle**

In our original design of the assessment, we framed the strategy guide as a guide for a friend who would be taking an exam in which they would define some new concept. Students at this stage of their mathematics career have substantial experience studying for and taking mathematics exams. Thus, we felt that students could “make sense of the situation based on extensions of their own personal knowledge and experience” (Lesh et al., 2000, p. 367). While the revisions of the assessment described above showed promise, we kept running into a problem that could often be tied back to the context of the task: students provided general test taking strategies rather than strategies that reflected their defining activity. For instance, one students’ guide listed “1. remember the difference between for all there exists and there exists for all. 2. practice defining familiar terms such as linear function. 3. become comfortable with making mistakes, you probably won’t come up with your formal definition on your first try.” The fact that this guide was written about preparing for an exam rather than defining was confirmed in a follow-up interview where this student said in reference to his second point “That isn't so much a thing I used here. It's a thing that I’d use if I was getting ready for a test….”

After encountering several responses similar to the one above, we reflect on how the exam context failed to meet the reality principle and how we could refine it. We determined that students might struggle to see the exam context as realistic for their defining activity since they most likely have never been put in a position during an exam where they are asked to engage in such an exploratory task. Instead, we realized that the task would be better suited in a “homework” context as it aligns more closely with the situation in which students would be doing this kind of activity. We reflected this context in Figure 1b as it suggests students will be in a similar situation as them.

**Model Shareability and Reusability Principle**

In order to meet this principle, the model students are asked to develop needs to be general enough that it can be used by someone other than the writer itself (i.e., shareable) for other scenarios than the one that motivated its creation (i.e., reusable). Our assessment works to meet these requirements by having students write a strategy guide *for another student* and by asking that the strategy guide be written so that it can be used for *any* concept. We found that framing the task as writing a guide for a peer seemed natural for students. This was perhaps unsurprising since other scholars have successfully written Model Elicit Activities with a similar framing (e.g., Carlson et al., 2003; Vroom, 2020). Additionally, we found that our previously discussed edits to the prompt also were effective in supporting students to create a general model that can be reused when defining other concepts (see Figure 1b and 1c).

**Effective Prototype Principle**

As mentioned, the assessments first engaged students in formally defining two concepts: mirror-point functions and repeat functions. We saw these two tasks as “effective prototypes” of defining activity since students would have knowledge of various function properties at this point and thus should be able to make sense of a new property. Additionally, there is no one "right answer"; we acknowledge students would likely have different interpretations of the given informal descriptions. At the same time, we aimed to write the descriptions so that the different interpretations would be rich enough that students would be able to come up with various (non-)examples to support their defining activity. We encountered a problem when it came to our initial description of mirror-point functions. Initially, we had not clarified the "at least one" property in the description, this led to various students interpreting the statement to mean *all* input-values needed to equal the corresponding output value. With this interpretation only one type of function fits the description (i.e., $f(x) = x$ for all $x$ in the domain) making the task trivial. As such, we added the "at least one" note to enrich the context and encourage a wider variety of (non-)examples.

### Self-Assessment Principle

To create a rich model that reflects the descriptiveness and transparency that we are searching for in an assessment, students need to be able to assess themselves whether their model was adequate. One way we addressed this need was by asking students to create a strategy guide for *any* concept. This provided criteria for which students could assess their model: can this guide be used to any concept a student might encounter? More explicitly, we addressed this principle in the follow-up interviews by asking students "How does your strategy guide reflect what you did when you wrote your claims about mirror point functions and repeat functions?" With this question, the students were encouraged to assess how/if their strategy guide reflected their defining activity (another criteria of their guide). We found this question to be particularly revealing as it often identified when there were discrepancies between the students actual defining activity and what they had written in the guide. Given its revealing nature, we hope to incorporate a similar question in future versions of the guide in hopes that it supports students to refine their guide to more accurately reflect their activity.

## Discussion

Through working with students we have found that MEAs seem to be a productive first step for developing an assessment tool that documents the extent to which students participate in the sort of mathematical practices that our curriculum materials aim to support. We found that the MEA principles supported some effective refinements, including more explicitly articulating what kind of model we expected the students to create and document in the strategy guide (i.e., with the use of sentence starters, emphasizing the need for generality, and being explicit about reflecting on their activity) and by changing the context of the task from an exam to homework so that the activity was more realistic to the students.

Our experience also generated a number of questions that we will pose to the audience for discussion. Specifically, we encountered that one limitation of using MEAs is that rather than assessing students' activity directly, we are assessing their ability to reflect on and articulate their activity, which is a higher-order task (Wheatley, 1992). With that, we wonder: 1) What kind of insights might we gain from a "high bar" assessment of this type? What are ways we can mitigate this challenging aspect of the assessment?

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