Conceptions of Active Learning, Meaningful Applications, and Academic Success Skills Held by Undergraduate Mathematics Instructors Participating in a Statewide Faculty Development Project

Allison Dorko
Oklahoma State University

Michael Tallman
Oklahoma State University

John Paul Cook
Oklahoma State University

Michael Oehrtman
Oklahoma State University

Josiah Ireland
Oklahoma State University

William Jaco
Oklahoma State University

The Mathematical Inquiry Project engages mathematics faculty from all Oklahoma's public institutions of higher education in collaborative learning and resource development guided by three components of learning mathematics through inquiry: active learning, meaningful applications, and academic success skills. We report our analysis of participants' interpretations of these components and the social and cognitive mechanisms they entail. Our results (1) provide a descriptive analysis of beliefs and priorities about learning mathematics through inquiry held by post-secondary mathematics faculty and (2) contribute to research and outreach on faculty change using a model incorporating implicit and explicit learning theories, within which the professional activity and collaboration develop a community that supports productive aspects of learning mathematics.

Keywords: inquiry-based learning, entry-level math, professional development

The Mathematical Inquiry Project (MIP) and its Definition of Inquiry-Based Learning

The MIP is a collaboration among mathematics faculty from all 27 of Oklahoma's public higher education institutions. The project aims to improve learning in entry-level mathematics courses by engaging faculty in collaboration regarding inquiry-based learning, conceptualized to entail three components: *active learning*, *meaningful applications*, and *academic success skills*. The MIP defines these components through the lens of constructivist epistemology. The definitions were developed to guide project implementation and encourage participants to adopt a critical stance toward the nature of students' conceptions in their design of curricular materials that seek to engage students in the precise mental actions required to construct targeted mathematical meanings. The MIP defines the three components as follows:

> *Students engage in active learning* when they work to solve a problem whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned.
>
> *Applications are meaningfully incorporated* in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure.
>
> *Academic success skills* foster students' construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community.

In the summers of 2019-2021, the MIP leadership team (the authors) led five different week-long workshops during which faculty discussed key conceptual threads of entry-level mathematics courses and academic success skills in those courses. The leadership team aimed to support the participants' thinking about the social and cognitive mechanisms involved in developing productive ways of understanding foundational concepts in entry-level undergraduate mathematics, and how to design curricular materials that promote students' construction of those ways of understanding. For example, participants discussed the ideas of *conceptual analysis*

(Thompson, 2008) and *hypothetical learning trajectories* (Simon & Tzur, 2012), and critically examined curricular materials developed with these in mind. Participants also unpacked the MIP's definition of inquiry and its operationalizion in the design of sample curriculum materials. In later stages of the MIP, currently ongoing, participants are creating their own course materials and will share them via workshops and peer mentoring. The goal of engaging participants in curriculum development and dissemination is to build capacity through broad professional participation and leadership in an emerging community that values and supports reflective practice in learning mathematics through inquiry.

Active learning, meaningful applications, and academic success skills are of interest to the research in undergraduate mathematics education (RUME) community. In a review of the RUME 2022 proceedings (Karunakaran & Higgins, 2022), 12 of 82 contributed reports[1] contained the phrase 'active learning' in the abstract, keywords, or body. These reports tended to define or discuss active learning terms of pedagogical strategies (e.g., "think-pair-share," Cervello Rogers et al., 2022), its characteristics (e.g., student engagement and collaboration, Johnson et al., 2022), or affordances (e.g., equity; Rios, 2022). Two papers framed active learning with reference to cognitive theories (e.g., Kerrigan et al., 2022; Rogers et al., 2022). Some definitions of active learning in the proceedings mention engaging students in meaningful mathematical tasks or projects (e.g., Broley et al., 2022; Chowdhury et al., 2022; Fukawa-Connelly et al., 2022; Johnson et al., 2022). Moreover, the proceedings also indicate interest in academic success skills such as students' mathematical identity (e.g., Guglielmo et al., 2022; Voigt et al., 2022) and problem-solving (e.g., Corey, Weinberg, & Tallman, 2022). We generally observe, however, that much extant literature focuses on students' activity when engaged in such tasks and less on instructors' views of what meaningful mathematics might entail and how it might be productively realized in the classroom. We seek to address this gap.

Research on faculty change highlights that supporting strategies should seek to alter individual's beliefs as opposed to mandating "effective" curricular resources or enacting top-down policy initiatives to impact teaching quality (Henderson et al., 2011). The literature establishes a need to characterize faculty's conceptions of mathematical learning through inquiry and describe how particular professional development experiences contributed to their evolution (Williams et al., 2022). The MIP is one such professional development project. This paper contributes to research on faculty change by constructing models of participants' explicit and implicit learning theories in relation to their conceptions of the three components of learning through mathematical inquiry. We focus on the following research question: Following engagement in a professional development initiative centered on the components of mathematical inquiry detailed above, what are participants' images of (a) active learning, (b) meaningful applications, and (c) academic success skills?

Theoretical Perspective

The MIP definitions of active learning and meaningful applications are based on Piaget's genetic epistemology. We find this a useful way to characterize mathematical inquiry because (1) it focuses on an individual's cognition, which provides a means for MIP participants to conceptualize and think carefully about students' thinking and learning, and (2) the field, as noted in the section above, has typically not dealt with issues of active learning and inquiry from an explicit epistemological perspective. In Piaget's view, conceptual development is a process of constructing and modifying cognitive structures to enable individuals to establish a fit, or

[1] These numbers exclude our 2022 conference proceedings regarding the MIP.

equilibrium, between their conceptual model of reality and the reality they experience (von Glasersfeld, 1995). Piaget regarded cognitive development as a process that inclines toward a balance, or equilibrium, via *assimilation* and *accommodation*. Assimilation is the process whereby a subject incorporates experiences into existing cognitive structures, and accommodation entails the modification of an individual's cognitive structures to enable their assimilation of novel experiences. Assimilation and accommodation, and thus equilibration, rely heavily on the notion of *abstraction*, of which Piaget distinguished four varieties (*empirical*, *pseudo-empirical*, *reflecting*, and *reflected*). Piaget (2001) explained that higher forms of knowledge (such as logico-mathematical structures) derive from abstractions of the subject's actions and the results of applying them in specific contexts. The nature of mathematics knowledge structures, or *operative schemes*, are organizations of internalized (mental) actions constructed through the process of pseudo-empirical or reflecting abstraction. The *action of the subject* is thus the basis from which mathematical knowledge structures are constructed.

Commensurate with this epistemological framing, the MIP definitions of active learning and meaningful applications are established on the premise that supporting students' construction of mathematical knowledge is a process of engaging them in the specific *actions*—and engendering abstraction of either the products of these actions (*pseudo-empirical abstraction*) or the actions themselves (*reflecting abstraction*)—that reflect the structure of the internalized actions organized within the schemes that an instructor expects students to construct. Notice that the MIP definitions of active learning and meaningful applications specify the conceptual mechanisms engendered. The relation of our definitions to an explicit learning *process* differs from characterizations of learning mathematics through inquiry that specify the environmental, pedagogical, or social conditions that must be established for students to be actively learning or for a teacher to incorporate meaningful applications in their instruction.

The MIP definition of academic success skills is similarly informed by constructivist epistemology, while being supplemented by theories of emotion (Ortony, Clore, and Collins, 1988), identity (Blumer, 1986), and goal structures (Middleton, Jansen, & Goldin, 2017). See Tallman and Uscanga (2020) for a synthesis of these theories and their relation to students' affective experiences.

The MIP afforded opportunities for faculty members to engage with the definitions of the three components. Hence faculty experienced opportunities to engage in reflecting and reflected abstractions (Piaget, 2001) regarding the characteristics and mechanisms entailed in active learning, meaningful applications, and academic success skills. Our study was designed to investigate their images of these three components following this professional development.

Methods

We conducted semi-structured interviews on Zoom with 15 MIP participants in spring 2020. Interviewees had participated in at least one workshop. Interview questions included:

1. Please describe your image of active learning in entry-level college math courses.
   a. Why is this important for entry-level math courses?
   b. Can you describe a specific example of active learning in an entry-level math course, yours, or someone else's? What made this example effective? What could have been better?
   c. Has your participation in the MIP activities changed your thinking about active learning? How?
2. Here is the MIP's definition of active learning. [Participants were shown the definition].
   a. Are there parts of this that you think are important but haven't discussed yet?

b. Do you particularly agree or disagree with emphasizing any aspect of the MIP definition for improving instruction in entry-level college mathematics?

The questions were repeated twice more with ‘active learning’ replaced by ‘meaningful applications’ and ‘academic success skills.’ The interviews were audio recorded and transcribed for analysis.

We employed the constant comparative method (Strauss & Corbin, 1994) to identify themes in the data. For each of the three components of inquiry, one author read all the transcripts, highlighting phrases that characterized participants’ images of that component. When a new phrase was added to the list, the author reread previous transcripts seeking instances of it. The author then grouped similar items into themes and described each theme using the list of phrases. That author applied the themes to re-code all the data, adjusting the themes’ descriptions until they adequately described the span of participants’ images. To ensure consistency amongst coders, a different author independently coded the data, employing the first coder’s themes and seeking excerpts that were not described by the themes. The two authors discussed differences until they agreed on a final set of themes. The final themes for each component are described in the results. The themes for active learning refine those presented in Ireland et al. (2022). Themes for a component are not mutually exclusive; for instance, ‘motivating students with real-world examples’ was coded as both ‘real world examples’ and ‘affective affordance.’

Results

We present the answers to our research question in terms of themes we identified in participants’ interpretations of (a) active learning (b) meaningful applications and (c) academic success skills. Participants focused primarily on characteristics, affordances, and requirements of the three components with some attention to learning mechanisms. While each of the themes we explore below emerged in our analysis from a multitude of supporting transcript excerpts, here we illustrate a proper subset of the themes we identified and support them with one or two illustrative, prototypical excerpts.

Active Learning

Student engagement. All participants described student engagement as part of active learning. Jude, for example, defined active learning as “involvement of [students’] brains” in contrast to instruction in which only the “instructor’s brain that is running.” Generally, participants’ comments associating active learning to student engagement suggested that they viewed engagement (and related ideas of interest and involvement) as the mechanism by which mathematics learning occurs.

Format in which students engage with the content. All participants discussed active learning in terms of the format in which students would engage with the content, such as small-group work, projects, class discussions, guided or scaffolded problem sets, or using manipulatives or calculators. For example, Jack said, “I incorporate a lot of collaborative project learning so that they can see the math in action… the less I talk… the better.”

Participants’ descriptions within this theme focus on characteristics of active learning, that is, what active learning might “look like” to a classroom observer. This contrasts with our view of the MIP’s three components of inquiry, which all involve descriptions of cognitive activity that is not observable in the same way. From our theoretical perspective, active learning might occur when students work in small groups, but working in small groups is not a sufficient condition for students’ construction of a desired mathematical meaning.

Nature of the mathematical tasks. Some participants described characteristics of the mathematical tasks they present to students, such as novel problems (not exercises) that would support understanding over simple memorization. Participants believed active learning occurred when problems required students to struggle productively and problem-solve. Some participants believed real-world examples were useful for engaging students in active learning. The attention to these characteristics is notable because it indicates attention to a key consideration for curriculum design: what students are engaged in and how they are engaged in it is consequential for the mathematical meanings they construct.

Participants expressed their belief that productive struggle is a mechanism of learning. For example, Quinn described,

> I have come to see [active learning] as asking students to truly problem-solve in a way that, that they may or may not have seen before…. Instead of having me say "here are the tools that we're going to use, here is exactly how we're going to use them and exactly the order that I want you to use them… it's set up more like "okay here are the tools, now here's a problem and maybe here are some guiding questions."

Quinn was unique among participants in that she not only described productive struggle, but hinted at the idea of students needing to engage in particular mental activity (selecting "tools").

Reveals student thinking. Two participants valued active learning for revealing student thinking, allowing the instructor to modify instruction and address individual or critical issues. Reagan[2] said:

> active learning [makes it] easier for the instructor to know… where the students struggle… the instructor can… feed additional example or deep explain certain concept to help them better retain the information… my hope is through repetition they get the structure.

We infer that Reagan thought repetition was a learning mechanism that helped students "retain" information and "get" general structure.

Affective requirements and benefits of active learning. Participants' images of active learning included what they saw as affective requirements for it and affective benefits of it. Per the former, participants described that active learning would only be successful if students were interested in the problems, motivated to work on them, possessed perseverance, and if the classroom community felt safe. Participants felt students' having growth mindsets was important, and that a positive classroom community could help minimize students' math anxiety. For example, Finn said, "you have to have an environment where everyone's safe to throw out answers and get responses back." Participants also believed active learning could help students develop all these characteristics, noting that active learning sometimes served to increase students' interest, motivation, and perseverance. Eden, for instance, described active learning as "making them do the work, it's [building] self-efficacy."

Meaningful Applications

Format in which students engage with the content. Like their definition of academic success skills, participants described their images of meaningful applications as problems embedded in formats like small-group work or class discussions. One difference is that participants felt presenting a real-world example in a lecture constituted a meaningful application.

[2] English is not Reagan's first language.

Nature of the mathematical tasks. While the MIP characterization of meaningful applications hinges on applications that are mathematically meaningful, participants seemed to think of a meaningful application as one that was relevant to students' daily lives. Some participants mentioned that an application would be meaningful if it were relevant to other mathematics. For instance, Quinn said

> I also think of meaningful applications as in what can, what can they do with a definition within mathematics…And if it, and if it helps them with their problem-solving skill then it is extremely meaningful.

In addition to real-world examples, participants described that meaningful tasks were usually formed when students were given open-ended questions or novel problems. One participant noted that a task that supported students in generalizing would be a meaningful task, and several participants said tasks that connected to other mathematics were meaningful applications.

Affective requirements and benefits of meaningful applications. Participants' images of meaningful applications included the affective requirement of managing students' fear and dislike of applications. Affective benefits included piquing students' interest (which in turn supported student engagement and motivation) and increasing their mathematics self-efficacy.

Academic Success Skills

Instructor agency. Several participants described academic success skills in terms of their own agency as instructors in supporting students' development of academic success skills. This theme is notable because it indicates that some participants saw the intended connection between the three definitions: that active engagement with meaningful applications will build students' academic success skills. Participants mentioned that it was part of their duty as instructors to help students develop academic success skills (e.g., Quinn gave an example of developing time-management multiple times in the semester; Reagan described training students to include detail in their solutions). Others described how they used content to support students in developing affective academic success skills. For example, Ellison said,

> we want to be presenting students with problems that are at the right level… an interesting problem to the student but they still feel like they've got what it takes to be able to solve it, then it's going to keep them engaged.

Part of what participants saw as their agency as instructors was the necessity of their developing classroom community. Anna saw this as important because it enabled students to engage and participate productively:

> I can provide them opportunities to fail until they feel safe… [know] that they're not going to be laughed at… it's developing trust.

Characteristics of students. Participants' images of academic success skills included many skills they felt students needed to possess or develop. Participants' images of academic success skills included perseverance, knowing how to learn, knowing how to manage math anxiety, having an identity as a learner, having basic math knowledge, and having behavioral skills. For example, Ellison's response discusses many of these characteristics:

> If you have [academic success skills] it means that you have some knowledge about how to tackle different problems that you're given. You also have some positive motivation happening that's contributing to your desire and ability to be able to complete problems… Has an inherent sense that they can do it. And, and they also need to have the, the content knowledge skills to be able to, to solve a problem as well.

Most participants also mentioned that academic success skills also entailed behavioral skills such as time management, doing homework, taking notes, attending class, and doing homework.

Discussion

Participants' characterizations of the components of inquiry primarily emphasized environmental conditions or pedagogical strategies that would need to be in place for students to be engaged in active learning. Others have noted these sorts of emphases as common in faculty members' conceptions of active learning (e.g., Williams et al., 2022). The MIP participants attended somewhat to mechanisms of learning, seeming to believe that student engagement, repetition, generalizing, and productive struggle were learning mechanisms. These appeared to be rational, stable beliefs that the participants had developed over the course of their teaching careers. That faculty members hold these beliefs is not inherently problematic, but what students are engaged in (or repeating, or struggling with) is consequential for the mathematical meanings they develop. The participants' images of meaningful applications centered primarily around real-life relevance over mathematical meaning. Taking the images of active learning and meaningful applications together, we see a need for perturbing participants' images of these components in ways which go beyond students engaging with problems with relevant contexts. How to perturb participants such that they engage in the reflective abstraction that their current images of the components may be inadequate for supporting the development of mathematical meaning is an area for future research. We note work with provided definitions may perturb participants somewhat, and our evidence for this is Gemma's inclusion of generalizing as a learning mechanism because generalizing was specifically mentioned in the definitions. More broadly, we see participants' mention of engagement, repetition productive struggle as learning mechanisms as evidence of some attention to the idea of what a learning mechanism is, and both offer inroads to discussing how what students engage or struggle with as consequential. Thinking of the three pillars in terms of characteristics, rather than mechanisms, can be problematic because focusing on characteristics makes it harder to design and implement resources to support effective active learning. Without an understanding of why those characteristics may produce the conditions for learning and what concepts may be supported, an instructor cannot make instructional decisions to optimize the potential for students to construct desired concepts and reasoning patterns.

Our results broadly reinforce the literature on instructional change, which emphasizes that instructors' initial conceptions and beliefs about teaching are resistant to change and require continual, ongoing professional development. We propose that the themes we have identified nevertheless paint a clear image of how undergraduate mathematics instructors conceive of active learning, how they attempt to realize it in their classrooms, and what they value about it. These findings therefore provide important information for other researchers and change agents. More specifically, our results build on this prior research by describing the conceptions of undergraduate instructors across several institutions and after professional development has taken place.

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