Quantitative Reasoning Augments Scaffolding for Mathematical Model Construction

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In this report, we begin to explore the value of leveraging constructs from quantitative reasoning for informing scaffolding moves that would assist modelers in overcoming blockages to their mathematization of canonical real-world problems from a first course on differential equations. Our claims are based in qualitative retrospective analysis of a set of 7 design cycles used to develop a task trajectory and resulting learning environment responsive to STEM students' mathematical reasoning during mathematical modeling. We present a set of four scaffolding moves contingent upon and responsive to participants' in-the-moment quantitative reasoning that guided them towards a meaningful model for a predator-prey scenario.

Keywords: mathematical modeling, differential equations, quantitative reasoning, scaffolding

Mathematical modeling and quantitative reasoning are both key skills for successful interdisciplinary collaboration (National Research Council, 2012). In many undergraduate STEM programs, a course on differential equations serves the need for developing these skills. Doing so requires a consistent emphasis on how mathematical relationships are tied to conditions in and assumptions about the real world (Czocher, 2017). Thus, one of the most-studied research problems in the mathematical modeling (hereafter: modeling) genre regards mathematizing – the process of expressing important aspects of a real-world scenario in conventional mathematical notation. Mathematizing is critical, but it is difficult for learners to do and difficult for researchers to operationalize (Cevikbas et al., 2021; Stillman & Brown, 2014). These dual difficulties have challenged educators' capacity to effectively scaffold students' reasoning during modeling. Stender and Kaiser (2015) have called for educators to enact scaffolding based on a "student's current thinking in terms of the problem and the student's possible cognitive operations" (p. 1256). Meeting this challenge requires a cognitive account of mathematizing.

Recently, scholars have characterized mathematization during modeling in terms of constructs from quantitative reasoning. Several scholars have taken the position that constructs from quantitative reasoning afford an explanation of how modelers imbue real-world meanings into their mathematical inscriptions, thereby ensuring their inscriptions are representations of the modeler's conception of the real-world scenario (Czocher et al., 2022; Larson, 2013; Thompson, 2011). The open question regards uncovering how familiarity with students' quantitative operations during modeling may underpin effective scaffolding for extending students' capacity to mathematize. In this paper, we begin to address the question of how constructs from quantitative reasoning could be leveraged for designing effective, contingent scaffolding.

Theoretical Lenses

We adopt a Vygotskian view of scaffolding as an interactive process between educator and learner that supports the learner in performing a task she might not otherwise be able to accomplish (van de Pol et al., 2010, p. 274). One of the key characteristics of effective scaffolding is contingency, or adapting to the learner's current level of performance (van de Pol et al., 2010). Specifically relevant to mathematical modeling, Stender and Kaiser (2015) concluded that the moves constituting effective scaffolding may depend on which phase of the modeling process the learner is engaged in.

From a cognitive perspective, modeling is conceptualized as an iterative process transforming a phenomenon in the real world into a mathematical problem to solve. Within this perspective, it is common to accentuate the duality of a model as a person's conceptual system and as a representation using graphs, tables, or equations (Lesh et al., 2003). A mathematical model as a conceptual system refers to the constellation of knowledge the modeler has about how the world works (e.g., cats breed very quickly) and ideas about mathematical concepts (e.g., a derivative is the slope of a line tangent to a curve at a given point). In the case of equations, an expression of a mathematical model refers to a written representation that uses conventional notation like symbols for variables and arithmetic operations.

For an equation to carry meaning as a model of a scenario, it needs to hold mathematical meanings for the modeler and those mathematical meanings need to have counterparts in the real-world situation at hand. Theories of quantitative and covariational reasoning – how individuals conceive of quantities and relationships among them (Thompson & Carlson, 2017) – explain how meaning is given to the variables and arithmetic operations that constitute an equation (Czocher & Hardison, 2019). Quantities are created in the mind of the modeler when she conceptualizes a specified object in the real world as having a specified attribute that can be measured. Quantities can vary independently or interdependently with other quantities. For example, a modeler could imagine that the number (measure of an attribute) of a population of birds (object) in a backyard habitat waxes and wanes. She could also imagine that the number of birds increases as the number of feral cats in the habitat declines. Quantitative operations produce new quantities from already-conceived ones, usually through combination or composition (Ellis, 2007; Thompson, 2011). For example, a modeler could conceive change in bird population between two points in time by subtracting the latter population from the former.

Methods

Our data are drawn from the Cats & Birds task, which appeared as part of a larger study of effective scaffolding for promoting modeling competencies.

Cats, our most popular pet, are becoming our most embattled. A national debate has simmered since a 2013 study by the Smithsonian's Migratory Bird Center and the U.S. Fish and Wildlife Service concluded that cats kill up to 3.7 billion birds and 20.7 billion small mammals annually in the United States. The study blamed feral "unowned" cats but noted that their domestic peers "still cause substantial wildlife mortality." In this problem, we will build a model (step-by-step) that predicts the species' population dynamics, considering the interaction of the two species.

We intended for participants to arrive at the following model (using their own symbol choices), which are a version of the Lotka-Volterra equations:

$$\frac{dB}{dt} = \gamma B(t) - [\lambda B(t) + \alpha\beta B(t)C(t)]$$

$$\frac{dC}{dt} = \kappa[\alpha\beta B(t)C(t)] - \rho C(t)$$

We worked with 19 undergraduate STEM majors who were enrolled in or had completed a course on differential equations. They participated in a series of 10 hour-long clinical task-based interviews. Cats & Birds featured structural scaffolding in the form of Subtasks to help participants learn to justify the quantities and relationships chosen for inclusion in a model. Seven iterative rounds of design research (beyond the scope of this report) aided in determining which conceptual steps needed to be addressed as subtasks and how big those steps needed to be. Relevant to this report, participants regularly exhibited difficulty handling the complexity surrounding the facts that the number of cats and birds can vary, not all birds meet all cats, and cats are imperfect hunters. Thus, the first three Subtasks were developed to support participants' thinking about how to mathematize the number of cat-bird interactions at time t $(B(t) \times C(t))$ and accounting for encounter rates (α) and successful hunting rates (β). We then followed with Subtask #4:

4. Consider the decrease in magnitude of bird population due only to cat predation during a short interval of time Δt. Write an expression modeling this decrease, in terms of the size of cat and bird populations present at time t.

Goal: Produce $\left[\alpha \times \beta \times \left(B(t) \times C(t)\right)\right] \times \Delta t$ and interpret it as the change in bird population due only to cat predation during a small, fixed, but arbitrary duration of time

The contingent interview protocol targeting mathematizing was designed to explicitly focus on aspects of quantities and quantitative reasoning. For example, follow-ups requested justification for choices of arithmetic operations (e.g., Can you say why you used × here?) or to explain how they were thinking of specific quantities (e.g., do you see ΔB as a change in bird population or as the number of surviving birds after some are killed? What units does B(t)×C(t) have?) in order to inform our interviewer-proffered scaffolding. We used cross fertilization (Brown, 1992) to adapt what we learned about student reasoning from one participant for use in another participant's sessions. Our retrospective analysis of the design cycles focused on participants' quantifications and quantitative reasoning that aided in mathematization.

Results

Among those who did not autonomously produce the target expression for Subtask #4, we found four scaffolding moves that effectively moved the participant towards the Lotka-Volterra equations as a meaningful model for the interaction of species in the backyard habitat.

Contingent scaffolding 1: attend to the object and/or attribute represented by the symbol for a variable. For example, some participants conceived *decrease in population as time passes* to mean a record of the population size at different points in time. Sensibly, they produced a model representing the remaining population, $B\left(t_f\right) = B(t_i) -$ Birds Killed. We asked the participant to reason through what quantity each symbol represented and then followed up with clarifying questions like: *Great! Do you see the computation on the right-hand side as yielding the number of birds remaining or the number of birds that died?* We then asked the participant to identify (and then focus on) the "change-by" amount and used small numerical values such as $B(t_i) = 10$ and Birds Killed $= 3$ to solidify the distinction. When answering the former, we asked the participant to identify (and then focus on) the "change-by" amount.

Guiding the participant to attend to object/attribute/symbol alignment also meant suggesting changes to their notation to better align their intended meanings with conventionally correct meanings. For example, some participants represented decrease in population as time passes using the symbols $B(\Delta t) = B\left(t_f\right) - B(t_i)$. We found that participants conceived the value of the change in population as dependent upon the length of time interval and strongly desired their model to reflect that. Introducing the notation $\Delta B(t, \Delta t)$ to signify the fact that population change depends on duration of time as well as when the duration began resolved the conflict between student meaning and conventional meaning for the symbols.

This scaffolding move was also supportive for participants who used t to represent an arbitrary duration of time (rather than Δt). For these participants, we found that explicitly appealing to convention as a rationale for swapping the symbol t for Δt endorsed the participant's reasoning while making a correction in a way that did not disrupt their thinking.

Contingent scaffolding 2: Use precise and detailed language to refer to referents. Returning to the example of participants conceiving *decrease in population as time passes* to mean a record of the population size at different points in time, we found that keeping careful distinction in language between *change* (for them, the result of a change) and *change-by* (the amount by which a quantity changes) was effective. Introducing the additional vocabulary was fruitful for these students who already associated a quantitative meaning with *change as result*. Another important area for precision of language was when discussing time, in particular, carefully and consistently using phrases like *at time* t and *during a fixed but arbitrary duration of time* Δt. This precision aided participants who inconsistently used $B(t)$ to refer to *population since time 0*. We purposefully avoided the phrase *over time* and instead used *as time passes*, to avoid signaling division in cases where it would be inappropriate. In all cases, we avoided referring to quantities by symbols, instead speaking what quantity the symbol represented.

Contingent scaffolding 3: Support participant in quantification. Some participants conceived *decrease in population as time passes* to mean *accumulation of dead birds since the beginning of time*. These participants intentionally used t to measure time lapsed from 0. We took care to acknowledge their model was correct *under the assumption* that intervals of time were always measured from time 0. Building on their own understanding, we asked: "Now what if we wanted to consider a duration of time that did not begin at 0?" We built on participants' demonstrated knowledge that $\Delta t = t_f - t_i$ to develop a brief intervention using a numberline and concrete values for an initial time t_i and a final time t_f. We emphasized that given Δt as an unknown, but fixed value, it could correspond to infinitely many specific time intervals with different starting (t_i) or ending $\left(t_f\right)$ points, illustrated in the exchange below:

Int: Now what is the delta t between t equals 2 and t equals 5?
Peet: It would be 3.
Int: How did you get that?
Peet: Because I just subtracted 5 minus 2.
Int: So you found the distance between, in terms of time anyways. But you found the length of time, which is represented by a distance on the page, between 2 and 5. And you got delta t equals to 3. Can you find another pair of points corresponding to delta t equals 3?
Peet: Ok, yeah. I would do something along the lines of 0 and 3. Or 1 and 4. 2 and 5. 3 and 6.

In this excerpt, the interviewer aided Peet in *quantification*, the mental act of conceiving a measurement process for a specified attribute associated with a specific object (Thompson, 2011). The exchange continued:

Int: I'm going to say initial is 1.
Peet: OK.
Int: Delta t is 2.
Peet: OK.
Int: Where is t initial plus delta t?
Peet: It would be t final, which is 3.
Int: 3. Very good. And if t initial is 1 and delta t is 1.5, where is t initial plus delta t?
Peet: It's going to be 2.5.
Int: It would be at 2.5. So of independently we can set how long the duration of time is that we're looking at. And independently from that, we can set where does the duration of time begin.
Peet: OK.

Int: So going back up to your equation, we can think about tau as being the place where delta tau begins-- the moment that delta tau begins.

Peet: Right. OK. So in other words, you're saying the tau-- these two taus are the exact same.

Int: They don't have the same value. They have the same role.

Peet: Yes, yes. I understand.

Int: So tau 0 minus tau-- so just subtracting those two times-- should give us the delta tau.

Peet: OK.

Int: Did that help?

Peet: Right. Yes, it did. I'm trying to cement it into my mind

As a result of the above exchanges, Peet conceived Δt as a quantity that could vary independently of its composition from two values for the quantity *time*.

Some participants needed help conceiving a quantity as agnostic to usual units. For example, some participants arrived at Subtask #4 with the idea that $[\alpha \times \beta \times (B(t) \times C(t))]$ represented a count for bird deaths. For these participants, because the output units did not include a "per time" element, it did not make sense to them to multiply the expression by Δt. We found two approaches to support these students into developing a quantification of time that was agnostic to usual units like hours, months, years, etc.

1. All these participants agreed that "for something to have happened, time must have passed." We then noted that the birds must have been killed by cats during *some* finite block of time where the exact duration was unimportant. We asked them to consider $[\alpha \times \beta \times (B(t) \times C(t))]$ as the number of successful hunts (or birds killed) during one Block of time.
2. We leveraged participants' natural inclination to justify multiplication through repeated addition. To do this, we asked participants to successively iterate the expression: if $[\alpha \times \beta \times (B(t) \times C(t))]$ is the number of birds killed by cats during one Block of time, how many birds would be killed during two Blocks? During three Blocks? During five Blocks? During half a Block? During 2.7 Blocks? During a fixed, but arbitrary length of time Δt?

This approach had two advantages. First, it allowed participants to generalize to $\times\, \Delta t$ by building on a simple, observable pattern with concrete numbers. The exchange followed a pattern similar to development of units coordination among children learning proportional reasoning (e.g., Hackenberg, 2007). Second, it built for participants a conceptual object called a *multiplicative object* (Thompson & Carlson, 2017). A multiplicative object is the result of a mental act that couples quantities and their attributes, much like an ordered tuple, where each component of the tuple can vary independently and the resulting tuple is recognized as a quantity as well. Here, the participants formed a conception of "birds killed during an arbitrary time interval" by coupling $[\alpha \times \beta \times (B(t) \times C(t))]$ with Δt. For these participants, the units of the coupling were more similar to "light-years" than to "kills per unit time times time".

Contingent scaffolding 4: Guiding participant to re-quantify a quantity. Some participants realized that the cat and bird populations would vary as time progresses. These participants interpreted $\alpha \times \beta \times (B(t) \times C(t)$ as a per-unit-time rate that also varied as time progresses. They formulated an integral expression for the number of birds killed during $\Delta t = t_f - t_i$, $\int_{t_i}^{t_f} [\alpha \times \beta \times (B(t) \times C(t))] \cdot dt$. While this is a correct model for the question asked, it is not useful for moving toward a *differential equation* to model the interaction of the species.

We found two ways to help participants conceive $\alpha \times \beta \times (B(t) \times C(t)$ as (nearly) constant during a small Δt, a critical conceptual step for passing from $\Delta B/\Delta t$ to dB/dt. The first way was to suggest that $B(t)$ was large enough such that the fluctuation of a few birds would not matter so much. Many participants were surprised that "it was okay" to make this kind of simplifying assumption, since students often have strong impulses to preserve precision in their models (Czocher, 2019). We then directed the participants to consider a small duration of time, on the order of a day or an hour, rather than a month or a season. With these conditions in place, the participants were able to write $[\alpha \times \beta \times \left(B(t) \times C(t)\right) \times \Delta t]$. The second way was to encourage participants to consider an average value for $B(t)$ and $C(t)$ during Δt. Some students naturally thought of an arithmetic mean using the time interval's end points $B(t)$ and $B(t + \Delta t)$, some took the value $B\left(t + \frac{\Delta t}{2}\right)$, and others introduced concrete numbers to communicate the idea of a weighted daily average. For example, Niali used the notation $B_{\bar{x}}(\Delta t)$ to signify the average value of the bird population during the interval of time Δt and obtained the following formula to represent the quantity "bird decline during Δt": $P_k \times \left[P_e \times [B_{\bar{x}}(\Delta t) \times C_{\bar{x}}(\Delta t)]\right] \times \Delta t$, where P_k and P_e notated the percent of encounters that end in a bird's death and the percent of encounters that happened per unit time, respectively. Through replication with other participants, we found that Niali's method was viable for those who had completed a course in probability or statistics, and the first method was sensible to a broader range of participants.

In Subtask #4, the same set of symbols can represent multiple aspects of the scenario. The expression $[\alpha \times \beta \times \left(B(t) \times C(t)\right) \times \Delta t]$ represented "successful hunts," "birds killed," and "change in bird population." Shifting interpretations for this expression is instrumental to making progress in the model's development, because eventually the expression needs to be associated with ΔB as its output. These equivalences are interchangeable for instructors, and also to many students, but for some undergraduate STEM majors, shifting interpretations of this expression is nontrivial. For these participants, these three interpretations correspond to three distinct quantities (mental constructs) because their referents in the scenario (objects and attributes) are distinct, as shown in Table 1. To scaffold these participants, we appealed to the tripartite nature of quantities to guide participants' attention towards the object attribute pairings forming the referent for each interpretation of the expression.

Table 1 Three interpretations for the expression$[\alpha \times \beta \times \left(B(t) \times C(t)\right) \times \Delta t]$ *with explicit reference to objects and their attributers in the scenario.*

Object	Attribute	Unit
Set of encounters	Successful hunts	Count
Bird population	Dead birds	Count
Bird population	Change during Δt	Count

Discussion & Conclusions

In this paper we reported four contingent scaffolding moves with quantities and quantitative reasoning at their foundation that supported undergraduate STEM majors to mathematize aspects of a canonical modeling problem from differential equations The interviewer questioning, which contemporaneously focused on students' quantitative meanings for their equations simultaneously revealed participants' conceptual difficulties and provided avenues for overcoming them. Contingent scaffolding move 1 targeted participants meanings for their notation. Guiding participants to attend to the object and/or attribute their notation represented helped them communicate their intended meaning with conventional notation, and in some cases, helped them modify their notation to correspond with their intended meaning. For example, asking the participant what quantity each symbol in $B(t_f) = B(t_i) - \text{Birds Killed}$ represented typically led to participants realizing $B(t_i) - \text{Birds Killed}$ represented "number of birds remaining" instead of the intended "decrease in the bird population." An appreciable subset of these participants then modified their model to $B(t_f) - B(t_i) = -\text{Birds Killed}$. It was then a natural next step for participants to associate $B(t_f) - B(t_i)$ with "change in bird population" via the expression $\Delta B = -\text{Birds Killed}$. Contingent scaffolding move 2 targeted clear and concise communication between participant and interviewer. This aided in mathematization by clarifying which aspect of the real-world situation the modeler intended to model. For example, the language "change in birds over time" is imprecise. It is reasonably interpreted as either "absolute change in the bird population during an arbitrary duration of time" or "change in the bird population per change in time." Contingent scaffolding move 3 aided in mathematization by helping participants quantify an important aspect of the real-world scenario that could reasonably (from their perspective) further operated upon. For example, Peet conceived Δt as a quantity that could vary independently of two specific values of time. Only then could he combine it with the already-constructed quantity *frequency of encounters that result in death* to mathematize *decrease in the bird population*. Similarly, scaffolding move 4 aided in mathematization by helping a participant quantify an important aspect of the real-world scenario so that they could combine that quantity with other quantities. However, the goal of move 4 is not to help them construct a new quantity, but to guide them into re-quantifying an already constructed quantity. In both examples, we helped participants conceive $\alpha \times \beta \times (\mathrm{B(t)} \times \mathrm{C}(t))$ in such a way that the modeler found it reasonable to combine $\alpha \times \beta \times (\mathrm{B(t)} \times \mathrm{C(t)})$ and Δt.

Some of the moves we implemented may seem counterintuitive. For example, typically curricular materials across grade bands assume that learners need to comprehensively study one variable functions before encountering multi-variable functions. The evidence we considered here suggests that some cognitive conflicts encountered during modeling could be resolved if learners had access to notational conventions and increased capacity for analyzing multivariable relationships. Participants clearly recognized complex multivariable interdependences, an observation echoed in the covariational reasoning literature (Jones, 2020). One troublesome implication is that the standard curriculum may be depriving learners of important mathematical tools for use in modeling that would allow them to identify and represent mathematical aspects of real-world considerations.

We wish to emphasize that at any moment during the interview sessions, we could have directly remediated errors or misconceptions, shown the correct equation, and explained why it was correct. However, our purpose was to understand how the participants were thinking and how educators could respond in student-centered ways that would build on or change their thinking. Hence, the nature of our claims take the form: "When a participant exhibited X under Y conditions, doing Z was supportive." While the numbers in our sample were small and the number of participants ripe for each contingency even smaller, the examples we shared are emblematic of the ways that quantitative reasoning, quantification, and quantitative operations can be both a source of cognitive dissonance for students as well as the basis for effective scaffolding in modeling that still respects learners' intuitive ways of reasoning.

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