

Learning from Lesson Study in the College Geometry Classroom

Steven Boyce
Portland State University
Kwadwo Oppong-Wadie
Elmhurst University

Tuyin An
Georgia Southern University
Michael Ion
University of Michigan

Laura J. Pyzdrowski
West Virginia University
Julia St. Goar
Merrimack College

Authors of this proposal are members of an inter-institutional working group focused on the teaching and learning of transformations in college geometry courses taken by prospective secondary teachers. After exploring axioms and definitions for transformational geometry in our courses, we decided to shift to identifying not just what, but how students were learning about transformations in our courses. To explore this, we began a lesson study (Boyce et al., 2021). In this report, we discuss our engagement in the lesson study, its outcomes, and new directions.

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The authors of this proposal are members of an inter-institutional working group focused on the teaching and learning of transformations in college geometry courses taken by prospective secondary teachers. As a newly formed group of educators with members representing a diversity of professional backgrounds in various higher education institutions from across the United States, it is not surprising that our preliminary work often focused on how different our courses and needs were. It was when we decided to move forward by focusing on our commonalities that the idea of conducting a lesson study took hold. We chose the goal of creating a lesson that could be used in all of our classrooms with a content focus on transformational geometry. As we refined that goal, we decided to craft a lesson that focused on definitions and axiomatic systems. We also wanted the lesson to engage students in learning from each other while comparing definitions in a way that was connected to secondary geometry objectives. Thus, we began our journey into a lesson study with a focus on transformational geometry.

Theoretical Framework

Lesson study, a teaching improvement and knowledge-building method with roots in Japanese elementary education, enables instructors to collaborate in small groups to prepare, teach, observe, and refine individual class lessons. It has been used extensively for professional development in K-12 mathematics settings; its use in tertiary settings is more limited but growing (Applegate et al., 2020; Kamen et al., 2011). The benefits of lesson study include building community and deepening mathematical and pedagogical knowledge (Fernandez, 2020). A guiding framework for the lesson study project can be found in the work of Cerbin (2011, p. 11).

The major phases of a lesson study cycle are:

1. finding a focus for the lesson study;
2. planning the research lesson;
3. planning the study;
4. teaching the lesson, observing, and gathering evidence;
5. debriefing, analyzing, and revising the lesson;
6. repeating the research cycle; and
7. documenting and disseminating lesson study findings.

In our case, the specific content focus for the lesson was the concept of isometry. In our planning, we chose to adapt a lesson successfully used by one of our group members, finding that it allowed us to embed all of our targeted goals for the lesson. As we focused on planning the study, we determined which of our group members would teach the lesson, whose classroom would be used, and how we would observe and record data related to student learning and their engagement with the lesson. Once the lesson was taught and the data and evidence were collected, we were able to debrief and organize the findings so that we could use what we learned to revise the lesson and begin again. It is important to point out that the data presented in this dissemination is from the debriefing sessions and represents the perceptions of the faculty involved. We used thematic coding (Gibbs, 2007) to categorize discussions in our debriefing sessions according to (1) our thoughts on the previous lesson and (2) what we planned to accomplish in future implementations of the lessons; and within each of these codes, we identified course content topics. In this report, we describe how engaging in lesson study has led us in new directions for attending to learning goals for our geometry students. We begin by providing some background on the lesson we created and implemented.

Lesson Background

At the onset we set some goals for the design of a lesson we could implement at our different institutions. We wanted students to learn from each other and talk about geometric concepts, definitions, and axiomatic systems in productive ways. We also wanted a lesson that would allow students to compare definitions and do so in a way that could be connected to secondary geometry from a transformation perspective. We explored several ideas for topics, such as symmetries and frieze patterns, and decided to design a lesson focused on Adinkra symbols. There are hundreds of Adinkra symbols, exhibiting a rich variety of symmetries and meanings (Adapo, 2010). Adinkra symbols originate in Ghana, and they have cultural significance, especially in Black urban communities within the United States (Oppong-Wadie, 2020). Introducing them to the prospective teachers in our geometry courses also had the potential to help them connect to this culture, which they could then build upon in their teaching.

The lesson we created consists of seven components. In the first component, students individually sort twelve Adinkra symbols and label their groupings. The second component involved students comparing and contrasting the ways their classmates created and labeled their groupings of the symbols (cf. Larsen, 2009). Did they identify symmetries or do something else? What mathematical language did they use in their labels of groupings? Here, the goals were to (a) establish a common vocabulary about symmetries and (b) provide opportunities for diverse mathematical thinking about the Adinkra symbols. During the planning stage of the lesson study, each member of the working group sorted the symbols themselves. They then compared and contrasted their groupings with the goal of anticipating the variety of student responses.

The third and fourth components of the lesson focused on a particular Adinkra symbol, Boa Me Na Me Mmoa Wo. Its name in English is, “Help me, and let me help you.” Boa Me Na

Me Mmoa Wo has reflectional symmetry (a vertical line of symmetry) and “nearly” has horizontal reflectional symmetry. For the third component, we asked students to identify the aesthetics of Boa Me Na Me Mmoa Wo and create another figure exhibiting those qualities. The fourth component involved them sharing and attempting to reach a consensus on a description of the aesthetic exhibited by Boa Me Na Me Mmoa Wo.

In the fifth component, the instructor provides additional foci for *mutuality*, which Eglash (2021) identified as the “near” symmetry aesthetic that Boa Me Na Me Mmoa Wo exhibits. Students read a one-page excerpt from Eglash’s website that explains the concept of mutuality and calls for its definition as a mathematical property. The sixth lesson component involves students attempting to create such a definition. Working group members created and shared their own definitions in the lesson planning stage to help anticipate student responses. The final lesson component involves students reflecting on their learning experiences throughout the lesson.

The initial lesson was created with the intention of being repeated in a variety of settings. So far, it has been taught four times at three different universities. Due to differences in class time, some implementations moved initial or final lesson components to pre/post-class assignments. Each teaching session was delivered remotely, via Microsoft Teams or Zoom. Several (free-to-use) technologies were used to allow students to share their work (Flipgrid, Google Jamboard, Google Slides, and Padlet).

Research Questions

When considering the goals and results of a lesson study, there are two types of learning outcomes to investigate: the students’ and the instructors’. Our purpose in this report is the latter; thus our analytical focus is on debriefing sessions of observers (including instructors) reflecting on lesson implementations. Our research questions are 1) to what did working group members attend in their debriefing sessions and 2) how did working group members’ attentions shift over the course of the lesson study.

Methods

Data Sources

We primarily used Google Docs to create, document, and share our ideas, beginning with the group’s initial lesson brainstorming and lesson planning. After the lesson was first taught, we incorporated (anonymized) versions of students’ work as references for the observations that were written in the same shared document. In addition to screenshots of “final” student creations, we included screenshots of intermediate products (such as when students were working in breakout rooms). We incorporated transcriptions for all Flipgrid-completed video responses. Following the teaching of a lesson, we met over Zoom (recorded and transcribed) for an hour of debriefing, with at least one working group member taking notes. In addition to inputting their own observations, working group members added comments to the Google Doc to highlight others’ ideas or to propose connections or questions for discussion. The initial lesson plan was modified between teaching sessions to accommodate differences in the class meeting duration as well as suggestions for modification from the working group participants. This updated copy of the lesson plan was used to enter copies of student work, observations and reflections, comments, and questions for the following teaching session.

Members of the working group were present as observers in two of the four teaching instances, the first and fourth. We focus on data from these two teaching sessions in the current study. The first observed setting was a 210 minute class taught in April 2021. The second

observed setting was a 75-minute class taught in April 2022. We focus on the analysis of the post-teaching debriefing sessions for these two teaching sessions. The comparison of the two debriefing sessions aims to clarify how the teaching of the lesson has evolved from the observers' perspectives, how these outcomes inform lesson revisions, and how these changes reflect instructor learning.

Methodology

For the analysis of post-teaching debriefing sessions, the focus was on observers' (including the instructors') reflections on how the teaching went and what aspects could be continued or improved. The session transcriptions were open coded using qualitative data analysis software (NVivo 12). The second author (who was not involved in either debrief sessions or classroom observations) manually coded the data and compared it with the results of NVivo's automated insights to capture elements within the data that may have been overlooked. Multiple iterations of inductive/deductive thematic analysis (Braun & Clarke, 2006; Gibbs, 2007) were conducted to identify and refine the themes emerging from the data. The fifth author (who was involved in both debrief sessions and one of the observations) reviewed these themes and re-coded the data from each of the two debriefing sessions into two categories (thoughts on the implementation of the session and thoughts on next steps). The analysis shown in the results section represents the themes from the debrief sessions in a Sankey diagram (Figure 1), where the numbers in the diagram represent the occurrences of the themes in the conversations (Riehmann et al., 2005).

Results

The Session 1 [S1] debriefing was attended by six observers and the Session 4 [S4] debriefing was attended by five observers (including the instructors). Figure 1 depicts the eight coding themes that emerged from analysis across the debriefing sessions and their relationships with the two categories of reflection thoughts. We elaborate on these results by first identifying themes that were exhibited in one debrief but not the other, and then identifying themes that were present across the two debriefing sessions. In this way, we identify shifts in instructors' attentions over the course of the lesson study.

Divergent Themes Across Debrief Sessions

In the first debriefing session, *technology use and its issues*, *student self-evaluating*, and *student-instructor dialogue*, were three main points of discussion when talking about the effectiveness of the lesson. In one of these conversations about technology use, an observer mentioned, "...but they seem to get into precision and locked in on precision very quickly—they had to use GeoGebra. Because it wouldn't be precise, if they just sketched something." Another noted, "I also think sometimes the technology might have limited their own drawings." A co-instructor spoke about a norm of students self-evaluating their work in her class (the ones in this observation):

And I mean, when it's really off that I would rather me say you're off task, I would give them the framework and tell me evaluate yourself: where do you see there's issues with what's happening and what's in relation to how effective this should be? And they're like, okay, yeah, this is off. And they kind of would make a note, but they would self-assess...because I hold them accountable.

Finally, when discussing the dialogue between students and teachers, the discussions focused on the necessity for these whole-class discussions to be more effective toward the goals of the lesson, as well as brainstorming ways to keep the class engaged. For example, a participant

noted, “Because I think too often, we're always mesmerized by, you know, word choice. But is it mathematically sound? Is it accurate? What are they emphasizing? And if you're not careful, it'll become non-productive.”

The fourth debriefing session highlights a shift away from identifying local issues with the lesson and towards more global issues, such as ideas for how the lesson fits more broadly with *meeting the goals of the course*. When it came to meeting the goals of the course, the debriefing participants discussed where to place the lesson in the course's topic sequence: “I feel like this for me, would be a an initial lesson, as I don't see it as a heavy content-oriented lesson and I was thinking like where would I go after this if I took the class time?” and “[G]ood idea, because at the moment we don't have that follow up lesson and, yes, students might be left to think, what is it that I gained from this?”

Convergent Themes Across Debrief Sessions

A number of themes emerged across both debrief sessions. One example was how instructors could work towards aligning with other important mathematics education topics, such as *defining and precision*. When speaking about the role of the lesson and its relationship to defining and precision, an observer in S1 offered,

I was thinking for the part where we would be offering a definition of mutuality and then looking at everyone else's sorting or their creation of their new symbol to see if that those fit the definitions, we could ask, ‘[W]hat makes a good mathematical definition—what is a mathematical definition?’

Another S1 observer commented that

[I] thought that it was interesting that they were talking about precision, and in terms of like, being able to produce a very exact diagram. But, you know, you can be precise and have a very sketchy, inaccurate diagram. It's about like, how you're actually defining what's part of the diagram.

The other themes that were present across debriefing sessions, *student collaboration*, *technology and its issues*, *time management*, and *scaffolding*, were often intertwined with the *defining and precision* theme. For instance, an observer in S1 suggested,

At [component three of the lesson], they are just trying to make something that has the same aesthetic as that figure. So [right now] it's still very open. So yeah, the idea that ... it has to have ovals and triangles, and it has to have squares is valid. I mean, there's nothing to say that shouldn't be.... How soon do we want to tell them that that's wrong? Or guide them? ... I think that if we did early, we might have been more apt to get more discussion about the transformations or the actual definitions.

Another S1 observer suggested,

I'm wondering if we had gone into the groups, if we had been able to prompt them with a question. Like not, necessarily given it away, but had been able to ask them a question that could have, you know, prompted for their diagrams to get a little bit closer.

In S4, a different observer commented that,

We provide only one example given to them and say this shape exhibits something we call mutuality. I'm wondering if we found a second shape that could be used, and after a certain point within the lesson said, ‘Okay, here's another shape that exhibits mutuality,

how would you change your definition?’ If it’s a well-chosen second shape, it might start pushing them in the direction of using more of a transformational definition.

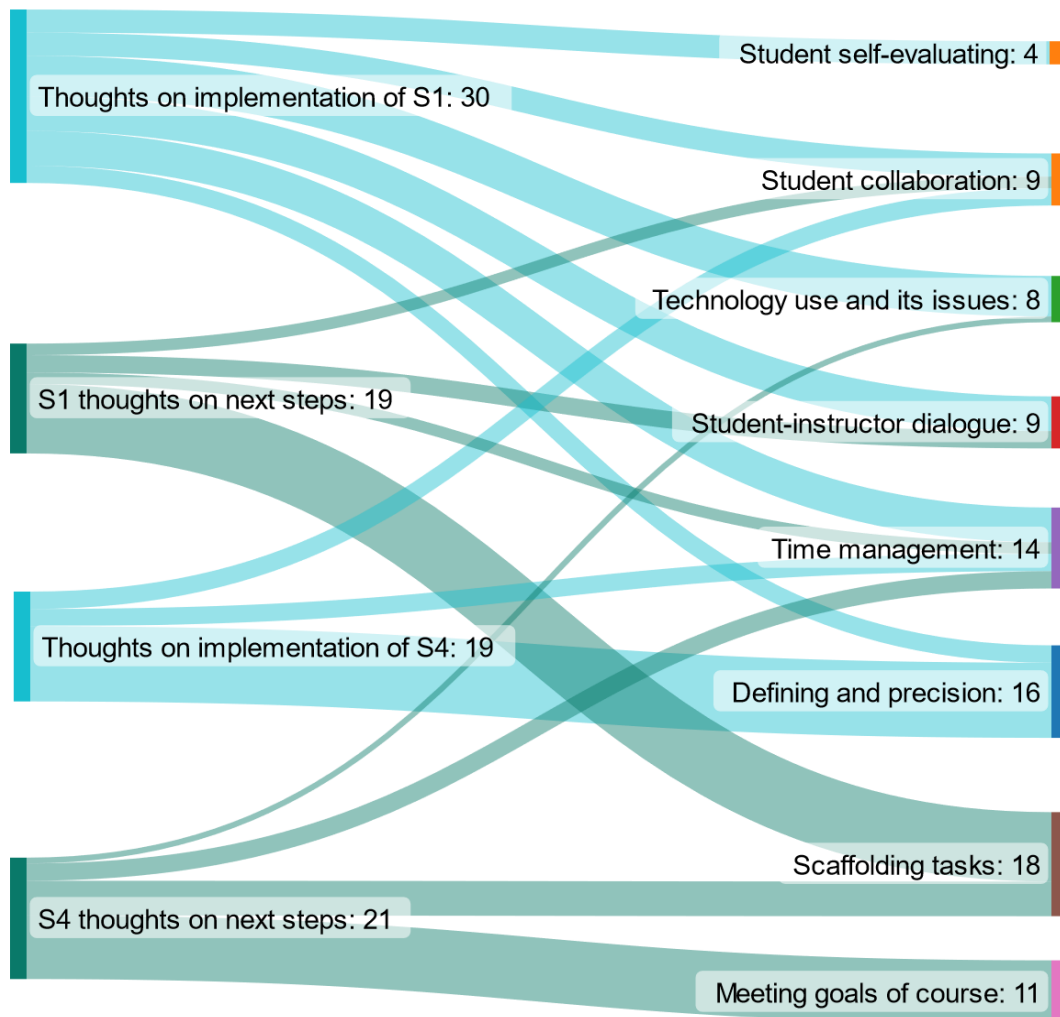


Figure 1: Sankey diagram of themes across debriefing sessions

Discussion

In this report, we have described how the foci of debriefing sessions changed over a one-year span within a lesson study. We have illustrated how observers’ attention regarding implementations and ideas for modifications evolved during debriefing sessions. These shifts accompany new ideas for lesson modifications. Our next steps in our lesson study include enhancing opportunities for instructors to make connections between the mathematical content found in the lesson we created to topics included in individual courses. An important outcome of these discussions is consideration of the placement of the lesson within a particular course. One instructor is planning for a next version of the lesson to be used as a springboard into a lesson about definitions in mathematics. The instructor wants to embed this topic into the course to better align the course offering with student learning outcomes developed by a group of mathematicians and mathematics educators. Those outcomes include that students will understand the role of definitions in mathematical discourse (GeT: A Pencil, 2022a). As a result, the next time the lesson is taught in this particular course, it will be followed by a lesson that will

lead students to determine the need for precision in definitions by engaging them in an activity in which they will compare and contrast their constructed definitions during the lesson. In particular, they will examine if all constructed definitions of mutuality lead to the same symbols satisfying the definitions.

Furthermore, the next version of the lesson is planned to be modified to include more or alternative Adinkra symbols in order to give students sufficient opportunities to notice features that relate to transformational geometry topics of translations, reflections, rotations, and dilations. This is also a topic aligned to a targeted learning outcome whereby students will use transformations to explore definitions and theorems about congruence, similarity, and symmetry (GeT: A Pencil 2022b). In previous versions of the lesson, none of the symbols presented to students exhibited connections specifically to translations and dilations. Also, given that the change in Adinkra symbols will include more than one example that exhibits the concept of mutuality, the instructor will include time for students to reflect upon and discuss how the meaning of an Adinkra symbol might influence the existence of, or make more apparent, mutuality in the symbol itself.

As we continue to seek ways to make geometric concepts meaningful and engaging for our prospective teachers and their students, it behooves us to go beyond what has been done traditionally and venture into uncharted terrain. On the basis of his familiarity with Adinkra symbols, having conducted research on its uses in instruction and education by extension, the fourth author joined this project after the S4 debriefing to not only make meaningful contributions, but also emphasize that by designing and implementing a lesson study, students can be engaged in studying mathematics concepts from a non-western perspective. The task at hand includes working to provide a format for identification of geometric ideas and the development of working definitions in anticipation of student engagement with Adinkra symbols from the standpoint of its aesthetic qualities. The importance of the sorting activity to this lesson study will inform suggestions for the inclusion or deletion of specific Adinkra symbols when being revised. He will serve as a resource person for meanings, translations, and context of Adinkra symbols selected for use in this lesson study. By doing the aforementioned, we hope that prospective teachers will be exposed to different ways of teaching mathematics concepts and connecting to cultural knowledge for the benefit of learners.

Cerbin claims that “[a] lesson is a place where our instructional goals come to life” (2011, p. 3). Living that “life” with others serves to make the teaching experience deeper and more meaningful. A lesson study challenges us to not only individually decide what is important, but necessitates the communication of our beliefs to others. Through the process, we learn about ourselves and about others. We grow. Clearly, participating in this type of undertaking requires trust and respect. Without a doubt, the time spent on becoming a community and developing common goals and solutions has been time well spent for us. In the end, the work impacts our students. Focusing on a lesson as a shared experience allows us to nibble our way into broader changes with respect to what we teach and how we teach it. The experience also provides us a community where we can discuss student difficulties (in general) with others who care as much about the outcome as we do.

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