Students’ and Mathematicians’ Playful Math Engagement

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Mathematical play is hypothesized to foster student dispositions that mirror authentic disciplinary engagement, such as exploring, conjecturing, and strategizing. However, most research on mathematical play investigates the mathematics that can emerge from children's natural play or play in informal spaces. We introduce the term "playful math" to highlight the potential of playifying school mathematics tasks and investigate both students' and mathematicians' problem-solving activity through this lens. Drawing on teaching experiments and mathematician interviews, we found evidence of playful engagement in both populations and identified similar activity structures across both groups, which we call Explore-Focus Cycles. We present two such cycles and discuss implications of playful math for student activity.

Keywords: Playful math, cognition, mathematician interviews, teaching experiments

Motivation and engagement are critical factors in supporting students’ abilities to understand, perform, and persist in mathematics (e.g., Durksen et al., 2017). Students’ experiences of self-efficacy can foster motivation, which in turn predicts persistence in the STEM fields (Simon et al., 2015). Despite the importance of these factors, however, studies show ongoing challenges with students’ self-efficacy and positive engagement in mathematics (e.g., Martin & Marsh, 2006). Finding ways to help students experience mathematics as enjoyable and worthwhile is an important part of addressing attrition rates (Wilson et al., 2012).

One way to foster enjoyment is to provide opportunities for students to engage in more authentic disciplinary practice, such as exploring, conjecturing, defining, and proving (Jasien & Horn, 2018). In contrast to school mathematics, which can be procedural and routinized, these practices can be simultaneously challenging, creative, and playful. In fact, much of authentic disciplinary engagement involves many of the same features as play (Gresalfi et al., 2018), and professional mathematicians have been described as engaging in mathematical play as part of their practice (Lockhart, 2009; Mason, 2019; Su, 2020). Mathematical play has the potential to center student voices enabling them to create their own challenges, investigate novel ideas, and develop a disposition for innovation (Barab et al., 2010; Gresalfi et al., 2018).

Mathematical play is, therefore, a promising avenue for fostering productive practices (Mason, 2019). However, the bulk of existing research on mathematical play is situated either in early childhood or in informal spaces such as video games. Less is understood about the potential benefits of incorporating play into school mathematics, particularly for older students. Furthermore, the degree to which “playifying” classroom mathematics fosters engagement that is more similar to that of mathematicians is not well understood. Therefore, this study investigates both mathematicians’ problem-solving activity and middle-school students’ activity in playful math tasks to ask, how does students’ playful math engagement compare to mathematicians’ problem-solving engagement? We use the term “playful math”, rather than “mathematical play”, to highlight the opportunities of “playifiying” classroom mathematics tasks.

Background Literature: The Benefits of Playful Math

Playful math offers both conceptual and affective benefits. Playifying mathematics can favor social interaction and communication (Edo & Deulofeu, 2006) and has been shown to foster increased enjoyment (Plass et al., 2013; Sengupta-Irving & Enyedy, 2015), engagement (Barab et al., 2010), and to engender positive attitudes towards mathematics (Holton et al., 2001). Su (2017) noted that it can also foster connection and community: "It's why the MAA supports programs like the American Mathematics Competitions and the Putnam Competition. We help kids flourish through building hopefulness, perseverance, and community" (p. 486).

Researchers have also identified a number of learning benefits of playful math for children, including increased mathematical fluency (Plass et al., 2013), improved knowledge in geometry (Levine et al., 2012; Sedig, 2008), improved spatial skills (Casey et al., 2008; Levine et al., 2012), and improved number development (Siegler & Ramani, 2008; Wang & Hung, 2010). The degree to which these learning benefits extends to older students is unclear, but researchers have identified a number of ways in which playful math can foster forms of engagement that is thought to more closely mirror mathematicians' disciplinary practice. This includes activities such as abstracting and generalizing (Featherstone, 2000; Melhuish et al., 2021), exploring patterns within axiomatic structures (Fernández-León et al., 2021; Su, 2017), conjecturing (Jasien & Horn, 2018), and "extending, varying, simplifying, and complexifying" (Mason, 2019, p. 95), among others. However, the manner in which this actually occurs, particularly with classroom tasks, is not well understood. Therefore, we sought to better understand both the playful nature of mathematicians' problem-solving activity, as well as the activity of students when engaging in playified tasks.

Theoretical Framework: Defining Playful Math and Principles for Task Design

What makes mathematical engagement playful? Definitions of play vary, but all emphasize the learner's freedom and autonomy (Holton et al., 2001; Su, 2017). Jasien and Horn (2018) described mathematical play as entailing agency in exploration, self-selection of goals, and self-direction in how to accomplish them. Davis (1996) described play as an acceptance of uncertainty and a willingness to move, which is similar to Mason's (2019) depiction of play as trying out actions and exploring consequences. Play is not aimless, but rather, it is goal driven and occurs within a structure (Featherstone, 2000; Huizinga, 1955; Su, 2017). Following these researchers, we define playful math as a particular form of engagement in mathematics, one that entails (a) agency in exploration and goal accomplishment, (b) self-selection of mathematical goals, and (c) a state of immersion, investment, and/or enjoyment.

One common feature of playified tasks is that they are typically goal-free (Kapur, 2015), open-ended problems (Cai & Cifarelli, 2005). In goal-free problems, learners only have to attend to the present state and consider how to make any possible move from that state. This is in contrast to goal-specific problems, in which learners must consider a problem's givens, the pre-determined goal, and how to connect the givens to the goal. Open-ended problems are similar, in that one could construct many different problems and solutions within the given situation. In constructing goal-free and open-ended problems, we followed four design principles for playifying our tasks (Plaxco et al., 2021): (1) allow for free exploration within constraints; (2) allow the student to act as both designer and player; (3) engender anticipating within the task; and (4) provide a method for authentic feedback.

Methods

Study 1: Middle-School Teaching Experiments

We conducted two teaching experiments (TEs) over the course of 5 weeks during the summer of 2020 (Steffe & Thompson, 2000). Sessions for the TEs occurred weekly for approximately one hour; the third author was the teacher-researcher. We video and audio recorded both TEs, with the exception of the second session, when a technical error prevented video collection. We also collected student work at the end of each session. The first TE was with two rising 7th grade students, Stewie and GJ, and the second TE was with three participants, Artemis, a rising 7th grade student, and Apollo and Frances, rising 6th grade students (all students chose their own pseudonyms). The students had limited graphing experience and had received no prior instruction on linear functions.

To investigate students' activity when exploring rates of change within playified tasks, we drew on a set of established research-based linear and quadratic growth tasks (Ellis et al., 2020; Matthews & Ellis, 2018). In these activities, students investigate shapes via dynamic geometry software, determining the rate of change of a shape's area compared to its changing length as it sweeps out from left to right, and then graphing that relationship. In following the above-described four design principles to playify the tasks, we developed a series of activities called *Guess My Shape*, in which the students created secret shapes of their choice (design principles #1, #2), constructed graphs comparing length and area (design principles #2, #3), and then challenged one another or the teacher-researcher to determine the secret shape based on the graph alone (design principles #2, #3, #4). Because playful math describes a form of engagement rather than the task itself, we hypothesized that the playified tasks would encourage playful engagement, but we did not assume that they would. For Session 1, we only used the extant covariation tasks, which we call standard tasks. For Sessions 2 - 5, we used both the standard tasks and the playified math tasks. For each of those sessions, we spent the first two-thirds of the session working with the standard tasks and the last one-third implementing the playful math tasks

Study 2: Mathematician Task-based Interviews

Additionally, we analyzed task-based interviews with 13 mathematicians (see Lockwood et al., 2016 for details). We gave each mathematician one or two tasks and asked them to work on the task(s) while sharing their thinking out loud. The interviewer only interrupted to ask clarifying questions or to address participant questions. We audio recorded the interviews with Livescribe pens, which also captured the mathematicians' written work as they spoke. All mathematicians worked on the *Interesting Numbers* task, and 7 of the mathematicians also worked on the *Selfish Sets* task. The Interesting Numbers tasks asked, "Most positive integers can be expressed as a sum of two or more consecutive integers. For example, 24 = 7 + 8 + 9 and 51 = 25 + 26. A positive integer that cannot be expressed as a sum of two or more consecutive positive integers is therefore *interesting*. What are all the interesting numbers?" The Selfish Sets task asked, "Define a *selfish* set to be a set which has its own cardinality as an element. Find, with proof, the number of subsets of $\{1, 2, . . . , n\}$ which are minimal selfish sets, that is, selfish sets none of whose proper subsets is selfish."

Analysis

We analyzed the TE data for aspects of playful mathematics using a combination of a priori and emergent codes (Strauss & Corbin, 1990). We drew the a priori codes from our three-part definition of playful math, and we developed emergent codes to address aspects of students' playful engagement that were not captured by the initial set of codes. All three authors coded the transcripts independently and met weekly to refine codes and to adjust and resolve discrepancies. This process continued through ten rounds of adjustments until all codes had stabilized. This analysis of the TEs also produced a cyclic model of exploring and focusing actions (Ellis et al., 2022), which we then used as an analytic tool for the mathematician data. We discuss this Explore-Focus Cycle in more detail in following sections.

We then turned to the mathematician data and analyzed the interviews for playful engagement using the codes we developed from the TE analysis. We also developed descriptive notes and diagrams that described each mathematician's actions in terms of the Explore-Focus Cycles. The first and second author coded the interviews separately and met weekly to resolve discrepancies until the coding scheme had stabilized.

Results

Explore-Focus Cycles

In the Explore-Focus Cycle, one shifts back and forth between exploring actions and focusing actions in order to make sense of a problem and determine a solution path. *Exploring actions* occur when the problem solver either (a) engages in an action with little or no anticipation of an associated outcome or (b) notices properties of outcomes and wonders about the connection between those outcomes and the actions that led to them. As such, exploring actions are empirical; they entail gathering information without a pre-determined aim, in order to develop an idea, direction, or conjecture. In contrast, *focusing actions* are ones that a problem solver makes with a specific anticipated outcome in mind.

For example, in the Interesting Numbers task, many of the mathematicians engaged in exploration by generating examples of interesting and non-interesting numbers in order to see if any useful patterns would emerge. Others explored by generating the algebraic formula for the non-interesting numbers and then seeing if they could identify a useful structure in the algebra. These actions typically resulted in the development of a conjecture that the interesting numbers are the powers of 2. At this point, the mathematicians shifted to focusing actions, pursuing strategies with anticipated outcomes such as an algebraic proof that powers of 2 are interesting.

In examining the students' activity, we also found evidence of Explore-Focus Cycles but only in the playified tasks. In the standard tasks, the students engaged in focusing actions but not exploring actions. In order to exemplify the Explore-Focus Cycles for both the students and the mathematicians, we present a truncated version of one cycle for a trio of students, Apollo, Frances, and Artemis, and for one mathematician, M10. The students' cycle occurred during a Create a Shape playified task, and M10's cycle occurred during the Interesting Numbers task (Figure 1). In the figure, the exploring actions are in the green cells and the focusing actions are in the blue cells.

Let us first consider the mathematician's engagement. M10 began with an exploration of possible approaches in order to determine which approach might be most productive (A1). She did not attempt any of the approaches but just listed and considered which ones might be feasible, and so there was not yet an anticipated outcome. She then focused on one strategy that allowed her to generate an algebraic formula for non-interesting numbers (A2). M10 then switched away from the algebraic form and instead generated and explored examples of non-interesting numbers to "see if it makes sense" (A3). After generating examples, she did not notice any patterns, and so she returned to the algebraic formula (A4). Here, she explored the formula to determine helpful structures that might indicate restriction on non-interesting numbers. From here, she switched again to the generated examples to look for patterns and structures and determines empirically that all odd numbers will not be interesting (A5).

For the *Create a Shape* challenge, the students began by exploring possible shapes to use in making a challenge for the teacher-researcher (TR) (B1). Here, they generated options without considering the associated length/area graph. However, once the options were suggested, they considered the reasonableness of the associated graph. They anticipated that some shapes were beyond their ability to graph and returned to exploring other possible shapes (B2). Apollo suggested a spiral and then a C-shape (B3, B4). Then, they applied a grid structure and a "connect the dots" structure to the C (B5). They were not anticipating the outcome of the graph or measurements but were looking for structures that might help them to do so. Artemis then considered the connect the dots method and suggested the need to number them (B6). Artemis was now anticipating how to use this structure for measurement and thus was engaged in a focusing action. Not included in the diagram below, the girls continued exploring and narrowing in on their chosen shape and ended with creating a "wave" shape and corresponding graph that introduced shapes shifted off the horizontal axis (Figure 2).

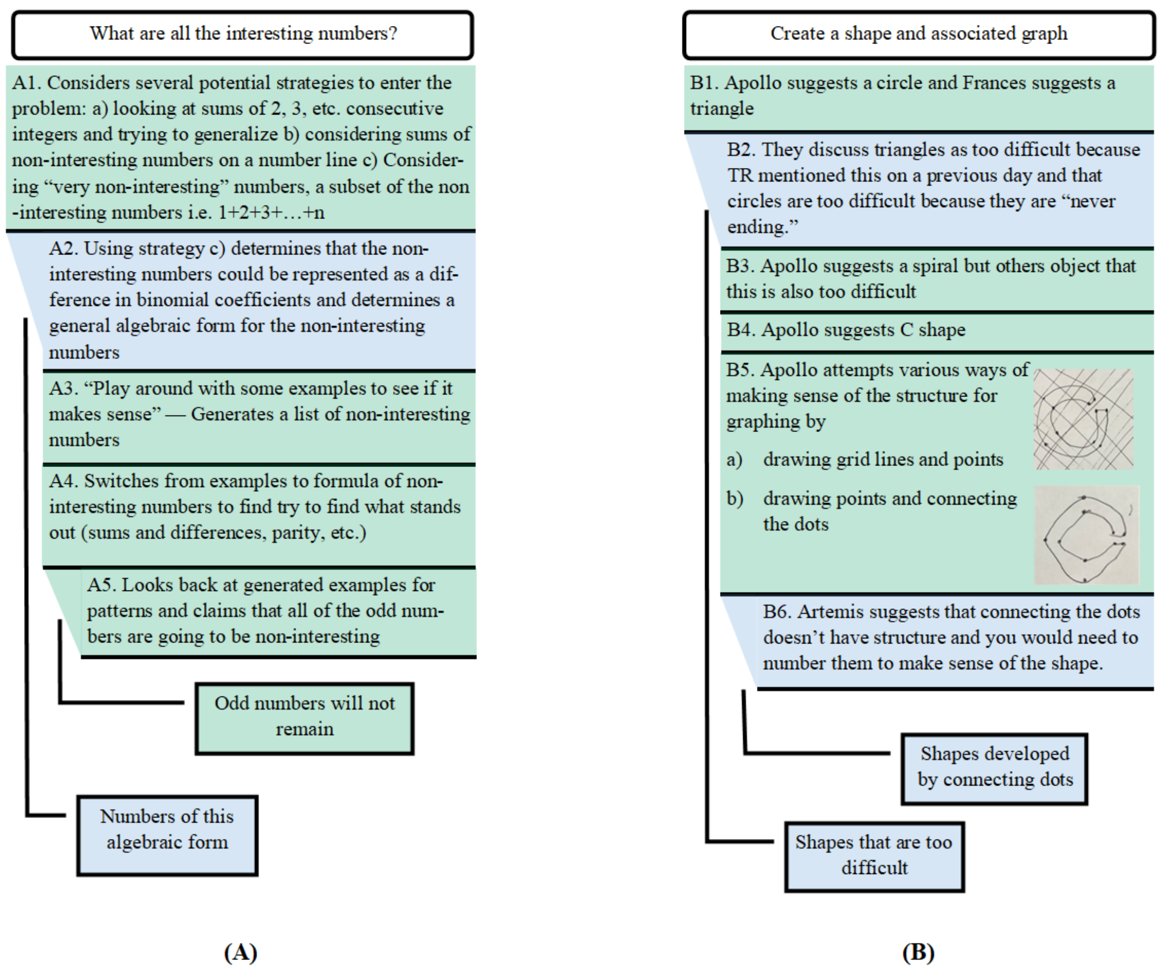

Figure 1. (A)Mathematician Explore-Focus Cycle and (B) Middle-School Explore-Focus Cycle

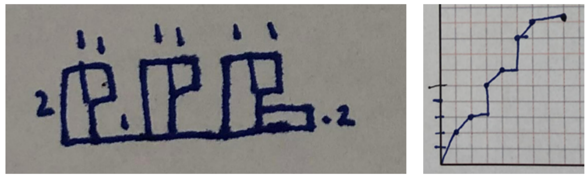

Figure 2. Artemis, Apollo, and Frances's final shape and graph for the Create a Shape Challenge on Day 3

For both the mathematicians and the students, we saw similarity across the presence and the function of exploring actions. Exploring actions can serve to help determine intial goals to begin narrowing in on the problem. M10 began with an exploration of possible approaches in order to determine which approach might be most productive to begin the problem, and then, she focused on one strategy that allowed her to generate information about non-interesting numbers. The exploration allowed her to select an initial strategy. Similarly, the students began by exploring possible shapes to use in their challenge. The exploration of shapes allowed the students to choose a goal for graphing and also helped them focus on the constraints of particular shapes. We saw this when Frances mentioned that triangles were too difficult, and when Artemis suggested, "Circles are never ending and that's impossible to graph a circle." Additionally, we saw that sometimes exploration serves as a way to search for structures. M10 explored structures when looking back at the algebraic formula of non-interesting numbers and when looking across her generated list of non-interesting numbers. The students did this when trying to work with a shape but were unsure of how to graph it. Wanting to graph the C-shape, Apollo tried overlaying a grid and "connecting the dots" between predetermined points. Here, she was trying to make sense of the structure of the C in a way that may be productive for graphing, as using points or grids may be natural ways of creating measureable areas and lengths.

Although similarities exist across the cycles, the cycles only occurred for the students during the playified tasks; whereas, they modeled the mathematicians' engagement across all of the tasks and interviews. One difference in the cycles is the novel, creative, and difficult mathematics that the students created for themselves. In this example, Apollo, Artemis, and Frances had only experienced linear and piecewise linear changes with shapes that were rectangles. However, to make the problem sufficiently challenging for the TR, they worked hard to create something different than what they had seen before, suggesting triangles and curved shapes such as circles, spirals, and the C-shape. Even though they moved on from using curved shapes, the desire to construct something novel and challenging persisted throughout the sessions. The mathematicians did not demonstrate creativity or make the mathematics more challenging, likely because the nature of the tasks did not call for designing or exploring their own questions.

Playful Engagement Codes

Recall that our three-part definition of playful math is (a) agency in exploration and goal accomplishment, (b) self-selection of mathematical goals, and (c) a state of immersion, investment, and/or enjoyment. Three additional codes emerged during our analysis: (d) taking on authority, (e) creative/unusual, and (f) harder math. *Taking on authority* refers to either a shift in authority from the TR to the students, or to an instance of students taking on mathematical authority with one another or with the TR. *Creative/unusual* applies when students have developed an idea, representation, connection, or task that we perceived as creative, novel, or atypical, and *harder math* occurred when students created a goal or task that introduced new challenges for them and represented mathematical ideas that surpassed the TR's intended set of topics or concepts.

Across the 10 sessions of the two TEs, we implemented 40 standard tasks and 9 playified tasks. Students completed one to two playified tasks at the end of each session, beginning with the second session. We coded any evidence of the six codes across both the playified and standard tasks. We then examined the rate of occurrence of each of the codes with respect to task type (Figure 3). For instance, self-selection of goals occurred on average 0.075 times per standard task and 3.11 times per playified task. We found that all six codes occurred with greater frequency on average within the playified tasks compared to the standard tasks. Furthermore, two of our codes, *creative/unusual* and *harder math*, occurred only in the playified tasks.

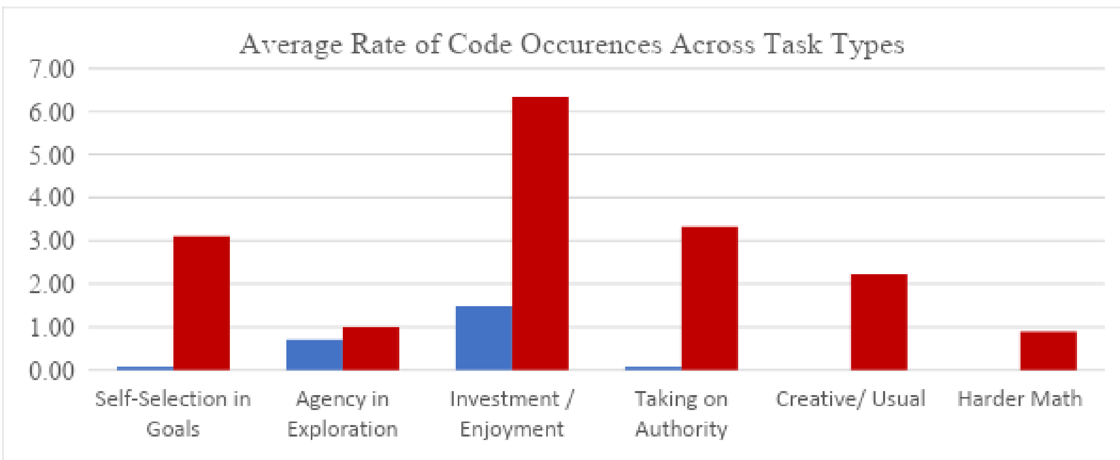

Figure 3. The Average Number of Occurrences of each code per Standard and Playified Task

We also investigated the presence of the playful math codes (a – c) in the mathematician interview data. We did not include the three emergent codes (d – f), as the nature of the interview tasks, unlike more authentic research activity, did not necessitate creative, unusual, or harder math (and the mathematicians already held mathematical authority). We found evidence of (a) *self-selection of goals* and (b) *agency in exploration/goal accomplishment* for all 13 mathematicians, and evidence of (c) *immersion, investment, and/or enjoyment* for nine of the 13 mathematicians. The presence of these codes and the Explore-Focus Cycles in both data sets suggests that playful math can be one way to support the development of productive practices.

Discussion

Playful math is a dispositional stance towards an activity, rather than a description of the activity itself. Thus, we did not know whether our design principles for playifying tasks would necessarily support more playful engagement. We found that they did, and they also supported students' development of novel, creative, and challenging ideas and strategies. It is possible that other open tasks may do the same thing; more research is needed to understand the unique affordances of a play-based approach. Nevertheless, this study offers a proof of concept that playful math can be one fruitful avenue for necessitating (Harel, 2008) more spohisticated concepts and connections.

We also found that mathematicians' problem-solving activity is not only playful, it also entails open exploration, with mathematicians shifting between phases of exploring and focusing. Notably, the students demonstrated similar cycles, but only in the playified tasks.

These findings provide evidence to support theoretical arguments that incorporating playful math into classroom activities can foster forms of engagement similar to mathematicians' activity (e.g., Mason, 2019).

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